

SUPPLEMENTING MATH INSTRUCTION IN THE ELEMENTARY CLASSROOM WITH COGNITIVELY
GUIDED INSTRUCTION (CGI)

By
Wai Yang

A capstone project submitted in partial fulfillment of the requirements for the degree of
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Capstone Project Facilitator: Kelly Killorn
Content Reviewer: Ger Kue
Peer Reviewer: Mary Beth Groff

Project Description

My project addresses my research question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?*. The project is meant to supplement current math instruction at the elementary school level, specifically fourth-grade. The project is not meant to replace a teacher's current math curriculum, but rather is an additional option that teachers may want to incorporate into their current math curriculum. The project is particularly geared towards students who may have had significant difficulties utilizing the common standard written algorithm for mathematics.

The nine lessons are intended to be short, focused lessons that would ideally be implemented during an after-school program at a school. Many academic after school programs are geared toward addressing students who are in need of additional instruction. The math lessons in my project are meant to be used to address areas of fourth-grade multiplication math that students may have difficulty with. Some of these topics include understanding multiplication as repeated addition, decomposing numbers, and other flexible strategies that students can transfer to their math schoolwork during normal school hours.

The lessons follow the framework of cognitively guided instruction (CGI). The lessons include examples of potential solutions that students may arrive at. There are also instructions in the lessons that guide teachers with how to utilize the principles of CGI math, including letting students approach the math problems how students see fit, while also making teachers aware of the potential strategies that students may decide to use. CGI math is student-centered, and the lessons reflect this principle of the CGI framework by suggesting students describe their math thinking, including extensive use of modeling with manipulatives, as well as use of visuals.

The nine lessons can, and should, be modified or differentiated to match student abilities. Although there is a timeline with regards to how long the nine lessons will take, if students are struggling to master a multiplication strategy the teacher should modify the numbers used in the lessons as needed for the student to achieve mastery of the math strategies and skills.

Learning Activities	
Time	Learning Activities
3 min	Lesson Launch
	<p>Instructor will tell students: "I am going to show you some addition problems, and you write on your white board what it would look like as a multiplication problem." Instructor will show flashcards of the repeated addition examples, and students will write the corresponding multiplication equivalent. If students need assistance, instructor should say, "how many <i>times</i> do you see (state the multiplicand)." Instructor should remind students that they don't need to solve the problems, just to see if they can write it as repeated adding or multiplication.</p> <p>Instructor will then show students flashcards of multiplication problems, and ask students to write the equivalent repeated addition form on students' small whiteboard. Instructor can demonstrate for students as needed.</p>
10 min	Instructional Task(s) Sequence
	<p>Students will practice using the term "group of." Instructor will ask students to draw pictures of various multiplication problems (the instructor can use the supplemental example multiplication story problems included in this lesson.) As students draw a picture, the instructor should prompt students to describe what they are drawing, or what they are grouping. Instructor should make sure to guide students to describe how many items are in each "group." Students would be asked questions such as "draw a group of 4 students" and students should describe what they drew (such as "I drew a group of 3").</p> <p>The instructor will then assess students by using multiplication flashcards to see what facts students are able to recall quickly, and which ones students have difficulty with (either taking a long time, having to use strategies such as counting fingers, or unable to answer). The instructor will use the flashcards from this lesson. The purpose of this task is to get a better understanding of which multiplication problems students have difficulty with. These multiplication problems will be used in later lessons. As instructor asks students to give the solution to the flashcards, instructor should place the cards into an "answered correctly" or "answer incorrectly" pile.</p>
5 min	Lesson Summary and Closure
	<p>The instructor will wrap up the lesson by checking in to make sure students retained the objectives. The instructor will ask the question "What is another equivalent way to say or write $\# + \# + \#$." The instructor will then ask what is another way to say or write $\# \times 4$." Which word do we use to describe adding a group with the same amount again and again (instructor can hint that the word starts with "r"/repeated adding) The instructor will then ask students what is something called when we have many of the same things together in one place? (group)</p>
Management and Safety Issues:	
<ul style="list-style-type: none"> None for this lesson 	
Curricular Resources / Citations:	
The lesson launch is adapted from <i>Math Intervention</i> by Jennfier Taylor-Cox, pages 82-83.	

For the Lesson 1 Launch (cut-out)

Students answer how these would be written as a multiplication problem.

$2 + 2 + 2$	$3 + 3$	$5 + 5 + 5 + 5$	$4 + 4 + 4 + 4 + 4$
$10 + 10 + 10$	$7 + 7 + 7 + 7$	$6 + 6 + 6$	$1 + 1 + 1 + 1 + 1 + 1 + 1$
$9 + 9 + 9 + 9 + 9$	$4 + 4$	$8 + 8 + 8 + 8$	$5 + 5 + 5 + 5 + 5$

Students answer how these would be written as a repeated-addition problem.

8×3	3×8	5×5	9×4
7×2	9×5	2×6	3×3
14×5	6×3	300×4	575×3

LESSON PLAN 2/DAY 2

	Lesson Title: Use What You Know
(K-12) Course Name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Solving Out-of-Reach Problems	Day in Lesson Sequence: 2
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> 	
Content Objectives:	
<ul style="list-style-type: none"> Students will be able to use their current knowledge of multiplication facts to solve for more complex multiplication problems, specifically by using number facts Students will be able to count on from known multiplication facts in order to solve multiplication problems that may appear out of their abilities 	
Academic Language:	
<ul style="list-style-type: none"> Instructor should tell students “quick-recall” means to be able to answer a math question quickly in their head 	
Assessment	
<ul style="list-style-type: none"> Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> For this lesson, the instructor may choose to quickly review the relationship between repeated addition and multiplication with students. The instructor may also use the multiplication fact flashcards from the previous lesson to get a refresher on students’ current multiplication facts knowledge and ability. 	
<ul style="list-style-type: none"> Formative Assessment: Informal and Formal <ul style="list-style-type: none"> As students work on solving multiplication problems that they aren’t able to immediately recall, instructor should look to see that students are able to start from a multiplication fact that is close to the one they are looking to solve. Students should be prompted to write out their solutions to the multiplication problems. 	
<ul style="list-style-type: none"> Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> Summative assessment can include posing multiplication problems that students may believe are out of reach (for example, some students may state they haven’t learned multiples of 9s). Students can be given prompts to start from multiplication problems they <i>do</i> know (for example, a student who knows $9 \times 3 = 27$ could start at that product, then add 9 to get to 9×4). 	
Materials:	
<ul style="list-style-type: none"> -small whiteboards -dry erase markers -cloth (for erasing whiteboards) 	

Learning Activities	
Time	Learning Activities
3 min	Lesson Launch
	The instructor will purposely pose a multiplication problem that is just out of the student's current quick-recall or quick-solve ability (using information from the previous lesson or prior assessments). For example, if a student isn't able to quickly solve multiples of "7s," the instructor can pose questions such as "7x6." The instructor will prompt students to think about ways they can solve the problem, even if students are not immediately able to state the answer. A list of strategically selected multiplication problems are included in this lesson.
20 min	Instructional Task(s) Sequence
	<p>The instructor will post multiplication problems that are just out of quick-recall for the student. To start, the instructor will ask the student to solve "7x6." The instructor should let the student attempt to solve the problem whatever way the student can.</p> <p>-if the student states they cannot solve the problem or don't know where to begin, instructor can prompt students to use the repeated addition strategy they know from the previous lesson</p> <p>-if students indicate that the numbers are too high, instructor should ask students to state what they can multiply to (such as $7 \times 2 = 14$, or $7 \times 3 = 21$, which are number fact strategies). Students should then try to get the solution by whichever method makes sense to them, which could be repeatedly counting to seven, counting by 7 (skip counting), and doubling up ("7x2 plus 7x2").</p> <p>Different students will know different multiplication facts, so the instructor should adjust multiplication problems as needed. Some of the more difficult multiplication facts to present to students include multiples of 6s, 7s, 8s, and 9s. It is important to remind students that the purpose is to get the correct answer confidently, using repeated adding at this point if necessary, and to take as much time as needed to arrive at the correct answer.</p> <p>Questions that may be asked are attached to this lesson. The questions are specifically chosen so that students can use solutions they have found to solve new math problems.</p>
5 min	Lesson Summary and Closure
	This lesson will have given students opportunities to find ways to solve multiplication problems that may be just out of students' reach. Instructor should let students know that if students can use what they know to solve multiplication problems that appear out of reach initially. Instructor should let students know that students used repeated addition, as well as multiplication facts they knew (derived facts), to solve difficult multiplication problems.
Management and Safety Issues:	
<ul style="list-style-type: none"> Some students may not be used to having to show their work as much, including creating visual representations, so students should be reminded to display as best they can how they are solving a problem. 	

“Just Out-of-Reach” Multiplication Problems

Students should be instructed to show how they arrived at the solution, whether verbally or written. Instructor should prompt students to use previous solutions to solve new ones as students see fit.

$9 \times 3 = \underline{\quad}$

$3 \times 11 = \underline{\quad}$

$3 \times 12 = \underline{\quad}$

$9 \times 4 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$

$12 \times 5 = \underline{\quad}$

$12 \times 6 = \underline{\quad}$

Potential strategies instructor may want to suggest or go over with students.

$9 \times 3 = \underline{\quad}$

Students may start from 9 doubled, then add 9

$3 \times 11 = \underline{\quad}$

Students should be able to quickly count by 11/repeatedly add 11

$3 \times 12 = \underline{\quad}$

Prompt student to see if student can start from the “3 x 11” solution

$9 \times 4 = \underline{\quad}$

Student could be prompted to see if they can start from the 9 x 3 solution

$6 \times 5 = \underline{\quad}$

Students may be able to double 6 twice, then add 6

$12 \times 5 = \underline{\quad}$

Students may use repeated addition or standard algorithm

$12 \times 6 = \underline{\quad}$

Students should be prompted to see if they can use the 12 x 5 solution (60).
Students may find that adding another group of 12 to 60 gives them the solution.

LESSON PLAN 3/DAY 3

	Lesson Title: Decompose & Add Groups of Groups
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Derived Facts Strategies	Day in Lesson Sequence: 3
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> • 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> • 4.1.1.3, <i>Multiply multi-digit numbers, using efficient and generalizable, based on knowledge of place value, including standard algorithms</i> 	
Content Objectives:	
<ul style="list-style-type: none"> • Students will be able to use their current known multiplication facts to solve and verify their solution to multiplication problems. • Students will be able to decompose a multiplication problem into smaller, easier to work with numbers • Students will be able to add groups with equivalent amounts to each other, and to relate it to multiplying. 	
Academic Language:	
<ul style="list-style-type: none"> • Instructor will need to discuss what a derived fact is, which in this case is using a multiplication fact you know, to help you solve for one you do not immediately know. 	
Assessment	
<ul style="list-style-type: none"> • Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> • Students should have completed Lesson 1 first. If needed, the instructor can quickly run the Lesson 1 flashcards prior to the start of this lesson. 	
<ul style="list-style-type: none"> • Formative Assessment: Informal and Formal <ul style="list-style-type: none"> • Instructor should listen for students use of the term “group of” as students are counting and counting groups. 	
<ul style="list-style-type: none"> • Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> • Instructor can have students complete multiplication problems with work showing and verifying multiplication facts. The objective is for students to understand the repeated adding of groups as being equivalent to multiplying, so having work shown or drawn by students is important. 	
Materials:	
<ul style="list-style-type: none"> • -small whiteboards -dry erase markers -cloth (for erasing whiteboards) • -flash cards from Lesson 1 if instructor wants to quickly review 	

Learning Activities	
Time	Learning Activities
5 min	Lesson Launch
	The instructor will quickly summarize Lesson 1 with students. Some questions instructor can ask students include “ When we add the same amount in a group over and over, what do we call that? It starts with the letter ‘r.’ (repeated adding)” Instructor will let students know that they will get more practice by adding groups to groups. Instructor will have students complete the first page of multiplication problems attached to this lesson as a review.
15 min	Instructional Task(s) Sequence
	Instructor should refer to the attached worksheets for students to get practice with using their current multiplication knowledge to solve seemingly out-of-reach multiplication problems. Instructor can support students by reiterating that repeated adding of the same amount or group can be written as multiplication, as well as multiplication being able to be shown as repeated adding. In the second worksheet instructor will guide students with deriving multiplication facts using the facts students currently know.
3 min	Lesson Summary and Closure
	To close, instructor should discuss with students how they were able to solve multiplication problems that they may have not been asked to solve before, and how they were able to solve them. Some students tend to look at more difficult multiplication problems, such as “7 x 6,” and say things such as “I don’t know my times up to 6’s yet.” Instructor should reiterate that the student was able to use smaller groups to get to the larger groups.
Management and Safety Issues:	
<ul style="list-style-type: none"> • None for this lesson. 	
Curricular Resources / Citations:	
<i>Children’s mathematics: Cognitively guided instruction.</i> 2015	

Write the multiplication equivalent of the repeated adding.

Examples:

$$9 + 9 + 9 = 9 \times 3 \text{ or } 3 \text{ groups of } 9$$

$$4 \times 5 = 4 + 4 + 4 + 4 + 4 \text{ or } 5 \text{ groups of } 4$$

$$7 + 7 + 7 + 7 = _ \times _ \text{ or } _ \text{ groups of } _$$

$$22 + 22 + 22 = _ \times _ \text{ or } _ \text{ groups of } _$$

$$19 + 19 = _ \times _ \text{ or } _ \text{ groups of } _$$

$$25 + 25 + 25 + 25 + 25 = _ \times _ \text{ or } _ \text{ groups of } _$$

$$353 + 353 + 353 = _ \times _ \text{ or } _ \text{ groups of } _$$

Decomposing and Using What Is Known to Derive Other Multiplication Facts

For this portion of the lesson, instructor will work with students to show how the students can use their known multiplication facts to solve new ones. Students will learn that they can decompose a multiplication problem into easier to work with numbers. Students may have previous experience with decomposing numbers, but they may not have done so for multiplication. Students will work on recognizing how they can use their current multiplication facts knowledge to derive facts for unknown multiplication problems.

1. Instructor should pose this question on the whiteboard for students:

$$8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = \underline{\quad} \times \underline{\quad}$$

2. Once students have indicated it is “8 x 12,” instructor should then ask students how they would say it in terms of groups. Students should respond with “12 groups of 8.”

3. Instructor should then ask students if we remove 2 groups of 8, how many groups of 8 will remain (10). Instructor should then write out the sentence:

“10 groups of 8” and “2 groups of 8”

$$10 \times 8 \quad + \quad 2 \times 8$$

$$\begin{array}{r} \backslash \quad / \\ 80 \quad + \quad 16 \end{array}$$

$$\backslash \quad /$$

$$\text{“12 groups of 8”} = 96$$

4. Instructor should let students know that groups can be counted too, as long as there are the same amount in the groups, and that breaking larger multiplication problems into smaller groups can be helpful. Instructor will hand out the attached worksheet for students to practice adding groups, reminding students to create drawings as needed.

Decomposing and adding groups worksheet

Instructor should have gone through the first example to show students a way they can break the multiplication problem apart, to be in smaller groups resulting in multiplication facts that students can confidently recall. Numbers in **RED** are the multiplicands or quantity in one group, while numbers in **BLUE** are the multiplier or number of groups. *The last couple of questions require the students to complete more of the work.

Example:

$$\begin{array}{c}
 8 \times 12 \\
 \diagdown \quad \diagup \\
 8 \times 10 = 80 \quad 8 \times 2 = 16 \\
 \diagup \quad \diagdown \\
 80 + 16 \\
 = 96
 \end{array}$$

*Instructor should state this to student as “10 groups of 8 plus 2 groups of 8 equals 12 groups of 8, which is 96

For students:

$$\begin{array}{c}
 9 \times 12 \\
 \diagdown \quad \diagup \\
 9 \times 10 = \quad 9 \times 2 = \\
 \diagup \quad \diagdown \\
 \quad + \\
 =
 \end{array}$$

*Instructor should prompt for a description of the student’s work, listening for “10 groups of 9 plus 2 groups of 9 equals 12 groups of 9 or 108.

$$\begin{array}{c}
 7 \times 6 \\
 / \quad \backslash \\
 7 \times 3 = \underline{\quad} \quad 7 \times 3 = \underline{\quad} \\
 \backslash \quad / \\
 = \underline{\quad}
 \end{array}$$

*Instructor should prompt for a description of the student's work, listening for "3 groups of 7 plus 3 groups of 7 equals 6 groups of 7 or 42.

$$\begin{array}{c}
 13 \times 11 \\
 / \quad \backslash \\
 13 \times 10 = \underline{\quad} \quad 13 \times 1 = \underline{\quad} \\
 \backslash \quad / \\
 \underline{\quad} + \underline{\quad} \\
 = \underline{\quad}
 \end{array}$$

*Instructor should prompt for a description of the student's work, listening for "10 groups of 13 plus 1 group of 13 equals 11 groups of 13 or 143.

$$\begin{array}{c}
 22 \times 13 \\
 / \quad \backslash \\
 22 \times 10 = \underline{\quad} \quad 22 \times 3 = \underline{\quad} \\
 \backslash \quad / \\
 = \underline{\quad}
 \end{array}$$

*Instructor should prompt for a description of the student's work, listening for "10 groups of 22 plus 3 groups of 22 equals 13 groups of 22 or 286.

$$31 \times 10 = \frac{31 \times 12}{\quad} \quad 31 \times 2 = \underline{\quad}$$

$$= \underline{\quad}$$

*Instructor should prompt for a description of the student's work, listening for "10 groups of 31 plus 2 groups of 31 equals 12 groups of 31 or 372."

$$11 \times \underline{\quad} = \frac{11 \times 12}{\quad} \quad 11 \times \underline{\quad} = \underline{\quad}$$

$$= \underline{\quad}$$

*Instructor should prompt for a description of the student's work. Some variations of combinations could be $11 \times 6 + 11 \times 6$, or $11 \times 10 + 11 \times 2$.

LESSON PLAN 4/DAYS 4 & 5

	Lesson Title: Double-Up
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Multiply by 2 by adding	Day in Lesson Sequence: 4 and 5
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> • 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> • 4.1.1.3, <i>Multiply multi-digit numbers, using efficient and generalizable, based on knowledge of place value, including standard algorithms</i> 	
Content Objectives:	
<ul style="list-style-type: none"> • Students will be able to double a number in order to solve multiplication problems • Students will be able to verify and understand that doubling a number is the same as adding the same number to itself. 	
Academic Language:	
<ul style="list-style-type: none"> • Instructor may have to refresh with students that <i>double</i> in this lesson means to add a number by itself, or to multiply that number by 2. 	
Assessment	
<ul style="list-style-type: none"> • Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> • Students at this stage should be comfortable adding numbers quickly. If there is prior evidence that a student has difficulty with adding larger numbers, small numbers (such as 1-9) should be utilized for this lesson during the “doubling” of doubled numbers. 	
<ul style="list-style-type: none"> • Formative Assessment: Informal and Formal <ul style="list-style-type: none"> • As students work on these problems, the instructor should check for automaticity with doubling numbers with the number cards. If students have difficulty with 1-9 doubling, the word problems should be modified to have numbers that are easier to work with. 	
<ul style="list-style-type: none"> • Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> • Future assessment could include posing a large, multi-digit number and asking students to multiply it by 2. Students who may not be able to solve a math problem such as “6,988 x 2” would then solve the problem by way of “6,988 + 6988.” 	
Materials:	
<ul style="list-style-type: none"> • White board, dry-erase markers, cloth for erasing, paper and pencil, cards individually labeled 1 through 10, can use playing cards if needed • 	

Learning Activities	
Time	Learning Activities
5 min	Lesson Launch
	Instructor will have a conversation with students regarding what the term “double” means. Some things the instructor can bring up include the “Daily Double” from Jeopardy!, the idea of “double or nothing,” and earning double points or “double XP” in video games. Instructor should verify students understand that “doubling” has the same effect as “taking something times two” before moving on to the instructional task sequence.
15 min	Instructional Task(s) Sequence
	Students will work on doubling numbers. The instructor should <u>start the lesson each day</u> by showing students a card (either typed or from a deck of cards), and then asking students to double the number. Instructor should spend around 3 minutes with this part. The instructor will then read the word problems from the attached worksheet or print each question out. Instructor should prompt students to illustrate as best they can how they are approaching and solving the problems. The story problems should be split into 2 days or 2 sessions. Instructor should encourage students to spend time on showing all their work and to be able to describe what the student is doing at each stage of problem solving. Because different students may solve the problems differently, instructor should allow students to go over how they solved the problems. There are notes on a few problems regarding potential strategies instructor can go over with students after the problems are solved.
5 min	Lesson Summary and Closure
	To wrap up, instructor will ask students what happens when a number is doubled. Instructor should check to see if students are able to describe adding a number by itself as the same as multiplying by 2.
Management and Safety Issues:	
<ul style="list-style-type: none"> The language demands of some of the story problems may have to be modified for some students. Instructor should differentiate the story problems to reduce the language demands as needed. For example, instead of saying “Students are going to their bus to go home, and notice the number of wheels...” instructor could simply say “The front of the bus has # of wheels...” 	
Curricular Resources / Citations:	
None for this lesson.	

Double-Up Instructional Task Word Problems

Many of the numbers used in these numbers can and should be modified as needed for students. The objective is for students to be able to double a number. Students should be advised to use adding, and to also write out their math thinking, which could include visuals. Students should be advised that the strategy they may have used in the previous lesson, decomposing numbers, would work here too.

Day 1/Part 1: Multiplication Problems Involving Groups

1. At the end of the school day a student is walking to the bus. The student notices there are 2 wheels at the front of the bus, but the back of the bus has double the number of wheels. How many wheels are at the back of the bus?
2. The small-group table in a classroom can seat 6 students. The teacher added another of the same small-group table to the classroom. How many students can the two small-group tables seat?
3. During class transition time, a fourth-grade class of 24 students go wait in the hallway for the teacher to be ready. Another fourth-grade class of 24 students are also waiting in the same hallway for their teacher. How many students are now waiting in the hallway?
4. The teacher has asked you and a classmate to get crates of milk for breakfast. There are 16 milk cartons in a crate. If you grab a crate of milk, and your classmate grabs a crate of milk too, how many milk cartons will be brought back?
5. You are using the stairs to go from the first floor to the third floor. The first set of stairs has 14 steps, and the second set of stairs has 14 steps, too. How many steps are there from the first floor to the third floor?

Day 2/Part 2: Multiplication Problems Involving Measurement

1. It is busy at lunchtime. In 1 minute, the lunch staff can check out 38 students' lunches. How many student lunches can the lunch staff check out in 2 minutes?
2. In the morning, the temperature checker can take 47 students' temperature in 1 minute. How many students can have their temperature taken in 2 minutes?
- 3A. In gym class students ran 16 times around cones. The next day, students ran another 16 times around cones. How many times did students run around cones in these 2 days?
- 3B. If students continued running the same 16 times around cones each day, how many times will students have run around the cones after 4 days?
- 4A. You are helping with sharpening colored pencils for art class. If you can sharpen 19 colored pencils a minute, how many colored pencils can you sharpen in 2 minutes?
- 4B. With the same 19 colored pencils sharpened a minute, how many could you sharpen in 4 minutes?
- 5A. There is a typing test in technology class today. If you can type 16 words in 1 minute, how many words can you type in 2 minutes?
- 5B. With the same typing speed of 16 words in 1 minute, how many words can you type in 9 minutes?

***Potential student responses (instructor should work through with students who may not have used these specific strategies):**

Students may add these quantities to get to 8 minutes, then add "16" for the 9th minute.

32 words in 2 minutes	32 words in 2 minutes
32 words in 2 minutes	32 words in 2 minutes

+

16 words in 1 minute

Some students may find that they can double 32 in 2 minutes to get 64 in 4 minutes, then double again to get 128 in 8 minutes, then add 16 for the 9th minute. The important thing is to have students be able to walk-through their problem solving process.

Some students may approach it as 16 words in 10 minutes is 160 words. Minus 1 minute or 16 words equals 144.

LESSON PLAN 5/DAYS 6 & 7

	Lesson Title: Double Double
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Multiplying by Organizing and Counting Groups	Day in Lesson Sequence: 6 and 7
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> • 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> • 4.1.1.3, <i>Multiply multi-digit numbers, using efficient and generalizable, based on knowledge of place value, including standard algorithms</i> 	
Content Objectives:	
<ul style="list-style-type: none"> • Students will be able to solve story multiplication problems using a variety of support materials such as counters and drawings. • Students will be able to demonstrate and verify that doubling a number twice is the same as multiplying by 4. 	
Academic Language:	
<ul style="list-style-type: none"> • Instructor will need to confirm with students that groups of groups means counting groups that have the same quantity in each group, and being able to add those groups as long as the groups have the same quantity or amount in the group. 	
Assessment	
<ul style="list-style-type: none"> • Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> • Students on this lesson should have completed the prior two lessons, and should be comfortable adding 2-digit numbers. 	
<ul style="list-style-type: none"> • Formative Assessment: Informal and Formal <ul style="list-style-type: none"> • As students work on the later problems, instructor should check to see that students are viewing groups as a whole and counting the groups. Instructor may need to prompt students to indicate how many groups they see, to prompt students to count groups in addition to individual units 	
<ul style="list-style-type: none"> • Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> • Instructor can pose problems for students that involve multiplying by 4. Some students may not be adept at multiplying by 4. The objectives achieved in this lesson would allow students to solve problems such as “37 x 4” by doubling the original multiplicand through adding (37) and doubling through adding again 	
Materials:	
<ul style="list-style-type: none"> • White board, dry-erase markers, cloth for erasing, paper and pencil, cards individually labeled 1 through 10, can use playing cards if needed, small manipulatives for counting (if student ability warrants it) 	

Learning Activities	
Time	Learning Activities
5 min	Lesson Launch
	The instructor will use the 1-10 numbered cards or playing cards to get students to practice doubling the numbers. For this warm up, instructor should also occasionally ask students to once again double the sum (so if a student sees 5 and answers 10, the instructor could also ask the student to double the sum)
15 min	Instructional Task(s) Sequence
	Instructor will read the math problems from the attached worksheet. Instructor may also decide to give students a copy of the problems too, edited to remove the instructional suggestions/tips. Students should be encouraged to draw visuals or use manipulatives as needed. The 2 sets of questions should be spread out over 2 days.
5 min	Lesson Summary and Closure
	Instructor will go over with students how the number of banana bunches in the math problem increased when the first bunch was doubled, and then doubled again in the classroom. Instructor should guide students to see that with one bunch or group doubled, you have 2 groups, and with another double, 2 groups and 2 more groups would be 4. Instructor can use a visual to show that 1 group of something doubled and doubled again results in getting 4 groups. If students are still unclear, instructor can explain that since 1 group doubled 2 times results in 4 groups, it is the same as "1 group + 1 group + 1 group + 1 group" or 4 times (referencing Lesson 1's repeated adding concept).
Management and Safety Issues:	
<ul style="list-style-type: none"> • None for this lesson. 	

Counting Groups Instructional Task Word Problems

Many of the numbers used in these numbers can and should be modified as needed for students. The objective is for students to be able to double a number, and then double it again. Students should be advised to write out their math thinking, which could include visuals. The use of manipulatives will be useful for students to visually see how a number doubled, and its sum doubled again, is equivalent to multiplying the original number by 4. Visuals or manipulatives/counters will especially be useful for these 2 lessons, and students should talk through their verification of the solutions.

Day 1 Questions

1A.

Students are playing a game of “Four Corners” in the classroom, in which students can choose to go to one of the 4 corners of the classroom. In the first corner, there were 7 students. The second corner also had 7 students. How many students are in these 2 corners? (Instructor may advise students to draw a square or use manipulatives to visualize the students in the corners).

1B.

During the same moment of the same “Four Corners” game, in corner 3 and corner 4 there were also 7 students each. How many students were in the 4 corners altogether? (If the student individually counts or illustrates students for corners 3 and 4, let the student finish, and then pose the question to the student: “Could you solve this problem without having to count the students in corners 3 and 4 separately?”)

2A.

The afternoon fruit snack for the day is bananas, which all come in bunches, or groups, of 9 bananas. You grab a bunch of 9 bananas from the lunchroom, and your classmate also grabs a bunch of 9 bananas, doubling the bananas you took. How many bananas do you both have altogether?

2B.

When you and your classmate returned to the classroom, you saw that your teacher had brought some bananas too. Your teacher says that with the bananas that he brought, the total number of bananas in the classroom is double what you and your classmate brought. How many bananas are there now altogether, if you double the amount of bananas you and your classmate brought?

2C.

Your teacher’s bananas also come in bunches of 9. How many bunches, or groups of bananas, are there in the room?

2D.

What do you notice about the number of bunches of bananas when your original 1 bunch of bananas was “doubled” 2 times? (Student should indicate that there were 4 bunches of bananas). Instructor should refer to the lesson summary/closure section once answer is complete.

Day 2 Questions

Most of these questions are intended to be solved by repeated adding, in this case repeated doubling, in place of multiplying by four. Many students who have difficulty multiplying by 4 the traditional way may find this method easier to follow, and helps the student keep a sense of adding groups when multiplying. Instructor should instruct students to draw out their math thinking and solution.

1A.

The school vans, which carry 6 students each, are arriving in the school parking lot. 2 vans arrive first, at the same time. How many students have arrived in these 2 vans?

1B.

After a little while, 2 more vans, each carrying 6 students, arrived. How many students have arrived altogether?

2.

The class is playing a game of "Who Wants To Be A Millionaire?" The class is on the \$16,000 question. The next 2 questions were answered correctly, so now the question is worth 4 times the \$16,000 question. How much money is the question worth now? (For questions such as this one, students might have " $32,000 + 16,000 = 48,000$." Then students may do " $48,000 + 16,000$," which creates additional borrowing if added the traditional way. Instructor should cue students to see if they could solve it by doubling, which would be " $32,000 + 32,000$," which is easier to compute mentally).

3.

These next questions are intentionally posed to allow students quick practice with doubling twice instead of multiplying by 4 traditionally. If instructor notices students setting up the problems the traditional way, instructor should prompt students if they can compute it in a more efficient way, and whether they really need to use the standard written algorithm. Some students who use the standard written algorithm for large numbers will sometimes err during repeated noting of numbers above the original multiplicand. In those cases, students should be prompted to try repeated addition through doubling instead (this can also be used to check their work). Some students may decompose the numbers as well, such as " 252×4 " solved as "product of 250×4 added to product of 2×4 "

$$1,050 \times 4 = \underline{\hspace{2cm}}$$

$$252 \times 4 = \underline{\hspace{2cm}}$$

$$2,055 \times 4 = \underline{\hspace{2cm}}$$

$$225 \times 4 = \underline{\hspace{2cm}}$$

$$21,250 \times 4 = \underline{\hspace{2cm}}$$

$$125 \times 4 = \underline{\hspace{2cm}}$$

$$15,150 \times 4 = \underline{\hspace{2cm}}$$

$$5,052 \times 4 = \underline{\hspace{2cm}}$$

LESSON PLAN 6/DAY 8

	Lesson Title: Area and Array 1
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Area and Array	Day in Lesson Sequence: 8
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> 	
Content Objectives:	
<ul style="list-style-type: none"> Students will be able to calculate area and visually represent area with multiplication problems. Students will be able to demonstrate how different combinations of lengths/widths can result in the same area. 	
Academic Language:	
<ul style="list-style-type: none"> Instructor will need to review the term <i>area</i>, which is measurement of 2-dimensional or “flat” space, counted in square units. The term rectangle may have to be reviewed with students. A rectangle is simply a polygon with only right or 90 degree angles. 	
Assessment	
<ul style="list-style-type: none"> Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> Students should have completed the first few lesson plans first. Instructor should be aware of students’ current multiplication facts recall knowledge from previous lessons, so that the numbers selected are not too difficult for students. 	
<ul style="list-style-type: none"> Formative Assessment: Informal and Formal <ul style="list-style-type: none"> As students work on the multiplication problems, instructor should make sure students are making visual representations. Instructor should look for students’ attempts to make the individual units in the areas as equal and uniform as students can. 	
<ul style="list-style-type: none"> Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> Future assessments could include visuals of rectangle areas with individual units, where students would have to utilize the sides of the rectangle to calculate the area. The area would be made so that individually counting would not be ideal. 	
Materials:	
<ul style="list-style-type: none"> White board, dry-erase markers, cloth for erasing, paper and pencil, linking cubes or any small, square manipulative that can be used to model square units 	

Learning Activities	
Time	Learning Activities
5 min	Lesson Launch
	To refresh the concept of area, instructor should remind students that area is measured in square units, and that each unit is equal. Instructor should also bring up that this lesson will look at areas of rectangles. Instructor should prompt students to list rectangle-shaped areas that they are familiar with or know, and give students a few minutes to write their answers on the whiteboard (some examples could be their bedroom, the gym, the soccer field).
15 min	Instructional Task(s) Sequence
	Students will get practice with multiplication facts by way of calculating the area of rectangles. Additionally, students will also get more practice with counting groups of groups, by way of areas and rectangles. Instructor will use the attached lesson instructions.
5 min	Lesson Summary and Closure
	Instructor should wrap up the lesson by prompting students to share what they saw with their creating of areas with the manipulatives or drawings. Instructor needs to make sure to reiterate to students that the same area does not mean the rectangle will all be similar or look the same.
Management and Safety Issues:	
<ul style="list-style-type: none"> If using small manipulatives, instructor may have to instruct students on proper use of the material, such as to not throw the material, or to ask other students before borrowing other manipulatives. Students may be tempted to use the manipulatives for non-academic purposes (such as building other things) 	

Area Questions to Work Through

Instructor should tell students to draw out their solutions as best they can. Students should use manipulatives and drawings as needed, and be allowed to form their own rectangles. Students need to read the questions closely, and instructor may help unpack the questions as needed. Once students are done, they should share their answers with each other, so that students can see the various possible solutions.

1.

Students are sitting on the classroom rug. All students are sitting shoulder-to-shoulder, within their own marked square on the rug. Each of the 7 students in the front row had 3 more students sitting directly behind them. How many students are sitting on the classroom rug?

2.

The small milk cartons are lined up in a large crate box. The milk cartons have a square bottom. There were 7 milk cartons tightly in a row, and 5 rows total. All the milk cartons are touching each other. How many milk cartons are there in the large crate box?

3.

Today's breakfast is Rice Crispy bars. The cook has to put 24 square bars in a rectangular-shaped pan. What is one way the 24 square bars can be arranged?

4.

The class will be watching a movie on the Smartboard. You are helping arrange the 28 chairs into a rectangle so students can all view the Smartboard. What are some possible rectangle arrangements?

5.

You are helping set up dinner for the afterschool program kick-off. Dinner comes in square, take-out boxes. You are given 64 take-out boxes to arrange into a rectangle, with each take-out box touching. What is one way you can arrange the take-out boxes? (Note: a square is also a type of rectangle).

LESSON PLAN 7/DAY 9

	Lesson Title: Area and Array 2
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Using Geoboards to Find Area	Day in Lesson Sequence: 9
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> 	
Content Objectives:	
<ul style="list-style-type: none"> Students will be able to utilize their current known multiplication facts to find possible side lengths for a given rectangular area. 	
Academic Language:	
<ul style="list-style-type: none"> Instructor may need to build on the vocabulary used from the previous lesson on finding areas of rectangles. Vocabulary that needs to be reviewed include <i>area</i> which is measurement of 2-dimensional or “flat” space, counted in square units. Additionally, instructor may choose to review length and width with students. The phrase “# by #” should also be reviewed as a way to describe the dimensions of the students’ rectangles. Depending on the level of the students, instructor may include <i>factor</i> as a word to go over. In this lesson, factor would be one of dimensions or sides in the rectangles 	
Assessment	
<ul style="list-style-type: none"> Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> Students should have been assessed for current multiplication facts recall abilities. Instructor should use numbers that are multiplication facts that are within students quick-recall. Formative Assessment: Informal and Formal <ul style="list-style-type: none"> As students work on creating the various possible length by width combinations of the area, instructor should check to see that students are counting and then multiplying the units on the lengths and widths, and not the individual square units. Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> A future summative assessment could include showing a number, and having students find combinations of numbers that can multiply to get to the number. It would be a good start to factoring, which could occur at a later date. 	
Materials:	
<ul style="list-style-type: none"> Geoboards with rubber bands OR an electronic device that can access a digital Geoboard at https://apps.mathlearningcenter.org/geoboard/ 	

Learning Activities	
Time	Learning Activities
5 min	Lesson Launch
	Instructor will let students know that they will all be playing a game with the Geoboards. If students are unfamiliar with Geoboards, instructor should demonstrate by showing how a Geoboard works. Essentially, rubber bands (whether physical or virtual) can be wrapped around pins to create geometric shapes. Instructor should give students several minutes to try out the Geoboards.
15 min	Instructional Task(s) Sequence
	Instructor should let students know that a card with a number will be presented. The students will create a rectangle on their board that has the same area as the number on the card. The first student to create a rectangle with a correct corresponding length/width combination gets 3 points, the second student gets 2 points, and the third gets 1 point. When students say they are done, the instructor should keep track of who said they were done first, and students who say they are done cannot touch the Geoboard until all answers are reviewed. *IMPORTANT* Instructor needs to show students an example and describe the dimensions as “# by #.” Students have to use that phrase when they say their answer. *Some numbers that have several more factors include “24, 36, 48, 60.” For example, 36 factors to 12 by 3, 9 by 4, and 6 by 6.
5 min	Lesson Summary and Closure
	To summarize, instructor can ask students what strategies they used to solve for the two numbers that would result in the given area. Instructor should probe to see if students used the connection between the area and dividing to get a multiplicand or multiplier.
Management and Safety Issues:	
<ul style="list-style-type: none"> If using physical Geoboards, proper safe use of rubber bands should be addressed by instructor, including not shooting the rubber bands and not over-stretching them. 	
Curricular Resources / Citations:	
https://apps.mathlearningcenter.org/geoboard/ -This is an online, virtual Geoboard students can use in place of physical boards.	

LESSON PLAN 8/DAY 10

	Lesson Title: Be Square
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Understanding the Meaning of “Squaring” a Number	Day in Lesson Sequence: 10
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> 	
Content Objectives:	
<ul style="list-style-type: none"> Students will be able to square a number and show it visually. 	
Academic Language:	
<ul style="list-style-type: none"> The mathematical meaning of square will need to be defined or reviewed for students. In this lesson, instructor should confirm students understand square to mean a special type of rectangle (polygon with only right angles) 	
Assessment	
<ul style="list-style-type: none"> Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> Students should have gotten practice over the past few lessons with multiplication facts, although students do not need to be able to quickly recall numbers 1-9 multiplied by itself for this lesson. 	
<ul style="list-style-type: none"> Formative Assessment: Informal and Formal <ul style="list-style-type: none"> Instructor will check to see if students are able to square a one-digit number. Instructor should look for whether students need to model out the entire area of the square with manipulatives, or if students use other strategies such as skip counting. If students model the entire area, instructor should ask if students can model just 2 sides of the square, and use skip counting to solve for the area. 	
<ul style="list-style-type: none"> Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> Future assessments could include showing students a row or column of a number of square units, and asking students if the row was “squared,” how much would the area be. 	
Materials:	
<ul style="list-style-type: none"> Small square manipulatives or square cut-outs, small whiteboard with erasers, paper and pencil Either actual, or virtual, examples of a Rubik’s cube, a checker’s/chessboard, a Bingo card and a Sudoku puzzle 	

Learning Activities																																				
Time	Learning Activities																																			
5 min	Lesson Launch																																			
	Instructor should open the lesson by discussing the properties of a square. Important points to make sure students or instructor bring up is the fact that all sides are equal. Before launching into the instructional task sequence, instructor should ask students what it might mean to “square” a number.																																			
15 min	Instructional Task(s) Sequence																																			
	<p>5 mins. Instructor should show students a Rubik’s cube and ask students to discuss it, including what they notice. Instructor should make sure that students notice how many squares are on each side, and how many square units there are in all. Once the Rubik’s cube has been shown, instructor should show the checker/chess board. Instructor should ask if students notice anything similarities from the Rubik’s cube that could also apply to the board (instructor is listening for things such as “it is a square board, with 8 squares on each side”). If students do not find the similarities, instructor can then tell about the similarities. Finally, instructor should show students a copy of a Sudoku puzzle. Students should then be asked to describe similarities they see with the puzzle (it is squared, nine-by-nine). Once all objects have been looked at, instructor should show all the items and ask students what they notice about all the items (they are square, but have a different number of units on each side). Instructor can then tell students that when you square a number, you take the number and create a row with that many units, and then add rows until it “squares” up. If students do not notice that there are the same amount of rows as columns, instructor can count each row as a group, and then count up each column, resulting in the same number of groups as is in one row.</p> <p>10 mins. The rest of the instructional task sequence involves students “squaring” numbers, using manipulatives. Instructor will say to students “Make a row of # (students would use manipulatives to make that row). Now, square it.” Students should then add manipulatives to assemble a square. Instructor will then ask students, “what is the total when you square #?” Instructor should repeat for numbers 1-10. As students get to the larger numbers, instructor should prompt students to try to find ways to <u>not count the squares individually</u> for the area. Instead, students should count by groups that they are comfortable or able to.</p> <p>Example of NOT counting individually:</p> <table border="1" data-bbox="212 1465 626 1860"> <tbody> <tr> <td>10</td><td>10</td><td>20</td><td>20</td><td>30</td><td>30</td> </tr> <tr> <td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td> </tr> </tbody> </table> <p>In this example, a student would count a 6 by 6 as groups of 10s first, so 10, 20, 30, then 31, 32, 33, 34, 35, and 36.</p>	10	10	20	20	30	30	10	10	20	20	30	30	10	10	20	20	30	30	10	10	20	20	30	30	10	10	20	20	30	30	31	32	33	34	35
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10	10	20	20	30	30																															
31	32	33	34	35	36																															

Example of NOT counting individually:

5	10	15	20	25	30
5	10	15	20	25	30
5	10	15	20	25	30
5	10	15	20	25	30
5	10	15	20	25	30
31	32	33	34	35	36

In this example, a student would count by fives, then individually by ones until 36 is reached.

3 min**Lesson Summary and Closure**

To close the lesson, instructor should ask students what they remember about numbers being squared. Instructor should let students know that there is nothing different about these numbers being squared, but that when numbers are multiplied by their own value they are given a special name: squares.

Management and Safety Issues:

- None for this lesson.

LESSON PLAN 9/DAY 11

	Lesson Title: Multiplication Challenge/Combination Problems
(K-12) Course name: Afterschool Math CGI Intervention	Grade Level: 4
Topic: Using Flexible Strategies to Solve Multi-Digit Multiplication Problems	Day in Lesson Sequence: 11
Content Standards: (Minnesota State Standards):	
<ul style="list-style-type: none"> • 4.1.1.1, <i>Demonstrate fluency with multiplication facts</i> • 4.1.1.3, <i>Multiply multi-digit numbers, using efficient and generalizable, based on knowledge of place value, including standard algorithms</i> 	
Content Objectives:	
<ul style="list-style-type: none"> • Students will be able to solve combination multiplication problems by using a variety of strategies. • Students will be able to state the commutative property in multiplication. 	
Academic Language:	
<ul style="list-style-type: none"> • Instructor will need to elaborate to students what a “unique combination” is. Examples of how to elaborate are included in the Lesson Launch section of this lesson. 	
Assessment	
<ul style="list-style-type: none"> • Prior Knowledge Assessment: (in this lesson or previous) <ul style="list-style-type: none"> • Students should have shown some mastery of the previous lessons, especially with unpacking word problems. Although this lesson is more hands-on and fun, the concept of the commutative property of multiplication may be more difficult for some students to grasp initially. 	
<ul style="list-style-type: none"> • Formative Assessment: Informal and Formal <ul style="list-style-type: none"> • Instructor should check to see how students are solving the problems, which may include many drawings initially, some some students possibly able to not rely on drawings toward the end of the lesson. • Instructor should also ask students to respond verbally unpack the story problems first, to make sure students understand the math combination task. 	
<ul style="list-style-type: none"> • Summative Assessment: (planned for the future, if any) <ul style="list-style-type: none"> • Future assessments will include combination problems that encourage students to solve without having to draw out visual representations of the combinations. 	
Materials:	
-Blank white paper -Colored pencils -Scissors (optional, can tear paper instead if needed)	

Learning Activities	
Time	Learning Activities
5 min	Lesson Launch
	<p>To launch the lesson, instructor will go over what a “unique combination” is when solving combination problems. An example that can be given is to describe choosing an item at an ice cream shop. Instructor can say: “Dairy Queen has many ice cream choices. Dairy Queen serves vanilla, strawberry, or chocolate ice cream in either a cone or a bowl. You can have your own “unique combination” by choosing a flavor and either a cone or a bowl. You can decide on the flavor first, or the bowl or cone first, and you’ll still end up with your own unique combination.”</p> <p>Before moving on to the instructional sequence, instructor should ask for responses from students regarding other places or things that can come in combinations.</p>
15 min	Instructional Task(s) Sequence
	<p>Instructor should hand out paper and colored pencils to students. Students should be told they can use the materials however they want, including cutting or tearing up the paper as needed. Instructor will tell students that the task is to find how many unique combinations can be made from the choices given. Instructor should refer to the attached hand out for the combination problems.</p>
5 min	Lesson Summary and Closure
	<p>Instructor should discuss with students if they noticed anything about the number of choices they were given, and the total amount of unique combinations that could be made. If students have not found the relationship, instructor should write out some of the numbers to see if students can come up with the answer. For example, with 2 crust options and 5 toppings, instructor can write “with 2 of an option, and then 5 of the next option, we get 10 total unique combinations.” Instructor can let students know “10” is reached when the first two numbers are multiplied. If students were not able to complete the challenge question, instructor may ask students to see if they can now solve the problem.</p>
Management and Safety Issues:	
<ul style="list-style-type: none"> • A review of scissors safety may done if instructor feels it is needed. 	

Math Combination Problems

For these questions, instructor should prompt students to unpack the problems verbally by restating what the question is asking the students to do. Instructor should let students solve the problems how they want, but should also be sure to remind students that it doesn't matter if one category of the combinations is selected first, the result will still be counted as 1 unique combination. The colored pencils are intended to help students keep visual track of the combinations they have come up with, but students can use them as they see fit. If students are still drawing out or modeling all possible combinations, instructor should prompt students to see if they noticed a pattern with the combination results, and whether they may be able to solve the problems without modeling all the results.

1.

It is Friday, so that means lunch will be pizza. There are 2 crust options: original crust, and gluten-free crust. For toppings, the choices are pepperoni, mushroom, onions, or pineapple. How many different combinations of one-topping pizzas can the lunch staff order?

2.

Class picture day is coming up. Your friend has 6 pairs of pants. Your friend also has sweaters which are colored red, yellow, and blue. How many possible combinations can your friend dress up for on class picture day?

3.

For each of the 5 school days of the week your class may attend art, gym, or music. How many different combinations of each day having one of those 3 classes are possible? Note: This is NOT asking about unique week combinations.

4.

There are food crates that are red, purple, blue, white, and green. The crates can be used to bring back a type of drink that is milk, juice, soda, or water. How many combinations can there be of a crate color and drink type?

5.

*Challenge Question: Students should think about how they may solve this without directly modeling it. The numbers posed would make it solvable easily without writing anything down.

For the last day of after school the teacher is bringing in soda and paper cups. There are 8 different sodas brought in. The paper cups have 10 different art designs on them. How many different combinations of sodas in paper cups with art designs are there? Note: There is no mixing the sodas!

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