IMPLEMENTING MATH DISCOURSE AND RICH MATH TASKS TO DEVELOP MATHEMATICAL THINKING

by

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CAPSTONE PROJECT

The development of mathematical thinking, which includes procedural fluency, strategic competence, adaptive reasoning, and productive disposition, is essential for students to be equipped for success in the classroom and beyond (Baroody et al., 2007; Boaler, 2018; Katz, 2014; Schoenfeld, 2016). Despite a long standing awareness of the need for math reform in policy and practice (Smith, 1996) for fifty years American students' performance on domestic and international tests has remained stagnant, consistently ranking below average (NCTM, 2020; Nations Report Card, 2019). Classroom practices that emphasize quick computation and the pursuit of correct answers can no longer be the norm. For math education and outcomes to change, classroom practices must evolve and provide opportunities for students to experience math as sense-making.

How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking? Explicitly cultivating a math mind set and a high quality discourse culture is imperative in the development of mathematical thinking. Through discourse, students learn to explain, analyze, modify, and justify their own thinking, as well as the thinking of others. In addition, open-ended, rich math tasks provide students an avenue to explore and develop strategies to make sense of the mathematics.

This project is a fourth grade supplemental math curriculum designed to transform classroom practices in math. It focuses on building discourse routines, establishing mathematical mindsets, and shifting math experiences from procedure based computation to sense-making opportunities with the purpose of developing mathematical thinking. It provides explicit skills and strategies necessary for high quality discourse and specific tasks to promote conceptual development and understanding.

The project contains two units for fifteen days of instruction. The first unit focuses on transforming math mindset, establishing discourse norms and routines, and experiences with sense-making tasks. The second unit is a review of place value, addition and subtraction concepts. The units are laid out in a three stage plan. Stage one identifies the desired results, essential questions and student skills. Stage two illuminates the assessment evidence for the unit. Lastly, stage three provides a detailed learning plan for each lesson. The lessons include student objectives, a daily discourse routine, a group launch, individual and cooperative exploration, and a discussion/reflection of the exploration. Each unit concludes with a summative assessment.

Unit 1: Intro to Math Talk & Mathematical Thinking

Stage 1: Desired Results

Established Goals:

Students will…
4.8.1.1 **Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 4 topics and texts, building on others' ideas and expressing their own clearly.**

b. Follow agreed-upon rules for discussions and carry out assigned roles.
c. Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others.
d. Review the key ideas expressed and explain their own ideas and understanding in light of the discussion. Cooperate and Problem solve as appropriate for productive group discussion.

NCTM Process Standard: Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others.

Essential Questions:
How can I effectively communicate and justify my mathematical thinking and reasoning to others?
How does the thinking of others impact my mathematical thinking?
How does understanding mindset affect my learning and experience in math?
What strategies can I use to make sense in math?

Student skills:
I can explain my mathematical thinking to others.
I can justify my mathematical thinking to others.
I can apply multiple strategies to solve a math problem.
I can modify my strategy(ies) using information from others.
I can describe how the brain forms pathways for learning.
I can use mental math strategies to solve math problems.
I can use multiple representations to explain my solution strategies.

Stage 2: Assessment Evidence

Performance Tasks:
Brain Messages Poster Students will create a poster illustrating new understanding of how the brain learns and the impact of that information on math learning.
Fewest Squares This task will demonstrate the different ways and attempts students are making to solve the problem. It will include several attempts to find a solution, illustrating the development of thinking.
Fewest Squares Performance Task Rubric
Crossing a rectangle: self-assessment

Other Evidence:
Math Mindset Inventory (pre and summative assessment) This inventory will be used to compare students' math beliefs prior to learning about the brain and mindset with their beliefs after the week of learning and exploring different math strategies and experiences.
Exit Ticket Students will identify what they have learned, found interesting and an area that requires clarification or interest in deeper learning.
Discussion Participation Form: Observations during Math Talks
Discussion Observation Notes This observation form will be used for observation notes from math talks and group task discussions. The growth/participation can be tracked throughout the week by color coding observations each day.

Stage 3: Learning Plan

Lesson #1: Math and the Brain

Student Objective(s):
I can explain my mathematical thinking to others.
I can use mental math strategies to solve math problems.
I can describe how neuroplasticity affects math mindset.

Discourse Routine: (10-15min)
Discourse Goal/Focus: Whole group engagement
Mathematical Goal/Focus: determining multiple strategies

Number Talk: Dot Talk #1

1. Gather the students around the screen/board/poster so that everyone can see. Tell them that they will be learning a new routine that will be used regularly in class.
2. Before embarking on the first number talk, the number talk norms need to be displayed and discussed. After a review of the norms, the hand signals should quickly be practiced.
3. Tell the students that an image of dots will be displayed for a short amount of time and their job is to figure out how many dots appear on the screen. The goal is to figure out how many dots there are without counting them one by one. If students find one strategy to determine the number of dots, they should display a thumbs up signal next to their chest.
4. Encourage students to look for more than one way to determine the number of dots.
5. Display the image. Give students a relatively short amount of time to look at the dot

image. Watch for most students to report a solution strategy with their hand signals.
6. Call on students to share the number of dots they saw. Use the questions: *How many? How did you see it?*
7. Record all the possibilities on the board. Give them another chance to look at the dot image and determine another way to see a solution.
8. When all of the different solutions are recorded, call on students to share how they saw the results. Record their strategies on the board. Remind them that it is ok to change their thinking if they see it in a new way as strategies are shared.
9. Reflect on the activity with the questions: *What surprised you about this? How did your thinking change as you listened to other mathematicians share their strategies?*

Lesson:

Big idea(s) for developing norms/values:

- Your math abilities are not "fixed" - your brain can grow and change.
- Mathematicians explain their thinking.

Launch: (20min)

Pre-assessment:

- Students are given access to the google form Math Mindset Inventory. Read each question out loud and have students respond independently on their google form. The results will be compiled and used to inform the math discussions for the week and for student comparison at the end of the unit.

Discussion:

1. Ask the question: *What do you feel or think when I say the word 'math'?*
2. Give students time to think quietly for about a minute then have students turn to a partner and share their thoughts.
3. Ask for volunteers to share with the entire class and record the thoughts on a poster.
4. Explain to students that before doing math, there is some important learning that needs to happen about how the brain works and the impact of one's math beliefs on their experiences and success in math. Ask students if they have ever heard someone described as "math minded" and the belief that some people are just born good at math and some are not. Use a think, pair, share discussion to engage the class on this topic.
5. Show the video: Brains Grow and Change.
6. Discuss the video. Students can first share thinking with a partner, practicing restating what their partner said to a table group. Conclude the discussion with a whole class sharing opportunity.
7. The following questions can be used to support the discussion: *What is new information for you in the video? Has your thinking changed? What is neuroplasticity? What does neuroplasticity have to do with math? What is the connection between neuroplasticity and the belief that you are either born good at math or you are not? What are the messages about math learning and the brain that are important to remember to improve math learning?*
8. Record the brain messages that students are sharing on a poster.

Explore: (30min)

1. Students should be given a couple of minutes to record the most interesting or personally important messages that they heard in the video and the discussion in their math notebooks. Encourage students to use words and/or pictures to capture their thinking. After a few minutes of personal reflection, distribute the brain messages poster template to students. Their job is to choose an important message from today's learning and create a poster that represents the idea and states it clearly for others.
2. Before beginning the poster creation, allow for a few minutes of discussion/sharing at each table. At the end of the discussion, students should choose one message that stands out for them from today's information.
3. For support or further independent review and investigation, provide access to the following videos:
    a. Brains Grow and Change
    b. Neuroplasticity video
    c. Math Anxiety Video: TedEd
    d. Learning and the Adolescent Mind

Discuss/Reflect: (5min)

1. Give students time to hang their brain posters up around the room.
2. Do a gallery walk of the brain images and math messages that will guide the math culture for the year.
3. Discuss how their thinking has changed or been challenged with today's experiences. Go back to the initial poster that displayed their feelings about math and ask them to consider if these new math messages could have an impact on those statements.

Resources:

Charles A. Dana Center at the University of Texas & Agile Mind. Inc. (n.d.). *Learning and the adolescent mind.* http://learningandtheadolescentmind.org/
Fury, L. (n.d.). *Dot card and number talks.* https://www.youcubed.org/
Golden, J. (2010, September 3). *Preassessment, Part 1.* http://mathhombre.blogspot.com/
Rubinstein, O. & Adriatic Animation. (2017, March 27). *Why do people get so anxious about math?* Ted-Ed. https://www.youtube.com/watch?v=7snnRaC4t5c
Sentis. (2012, November 6). *Neuroplasticity.* https://www.youtube.com/watch?v=ELpfYCZa87g
Stanford University. (n.d.). *Youcubed.* https://www.youcubed.org/

Lesson #2: Strategies for Doing Math

Student Objective:
I can apply multiple strategies to solve a math problem.
I can use multiple representations to explain my solution strategies.

Discourse Routine (5-10min):
Discourse Goal/Focus: whole group engagement
Mathematical Thinking Focus: There are multiple ways to solve problems.

Number Talk: Dot Talk #2

1. Review the math talk norms and hand signals with students before displaying the dot image. Again, remind students that their goal is to find multiple ways to see the solution.
2. After displaying the image, discuss the different ways that students saw the dot formations and arrived at their solution.

Lesson:

Big idea for developing norms/values(s):

- There are many ways to find solutions.
- Mathematicians collaborate.

Launch: (20min)

1. Video: Strategies for Learning Math.
2. Engage in a group discussion about the ideas presented by the youcubed team in the video.
3. Students record the suggestions in their math notebook while the teacher records the big ideas on a poster.
4. Guide the discussion to the strategy of working in a group. Ask students to reflect on past experiences working in math groups, specifically things that they like/do not like classmates to say and do when working in a group. Each student should jot down a couple of ideas to bring to their group.
5. Introduce the activity: Good Group Work.
6. Give students a minute or two to quietly think about the questions: *What do you like people to say or do when you're working in a group on math? What do you NOT like people to say or do when working in a group on math?*
7. Break students into groups to discuss their thoughts.
8. After about 5 minutes of sharing time, reconvene for a whole group discussion on the topic. Record the reported "What we don't like" and "What we do like" ideas on posters to display in the room.

Explore: (25 min)

Fewest Squares

1. Tell students that they are now going to have an opportunity to practice some of the strategies that have been discussed with an activity called Fewest Squares.
2. In this activity, students are working to determine the fewest number of squares that can be drawn in a specified area.
3. Before moving students to work in groups and share thinking, students should be given 5-7 minutes to explore the problem independently. Encourage students to use drawings, words, graph paper, color coding, etc. as they work. Supply ample amounts of graph paper to use in the explorations.
4. After about 5 minutes of independent time, move students into cooperative groups that include a vertical space to write and share ideas to continue to find a solution.
5. Circulate and ask probing questions and make observations.
6. Fewest Squares Performance Task Rubric

Discuss: (5min)

1. As a whole group, debrief different solutions that were found and agreed upon by groups. Have several students show their strategies and explain their thinking on the white board.
2. Revisit the experience of collaboration as well, asking if there is anything from today's experience that should be added to either poster, thinking about the questions: *What went well in your collaboration? What could be improved?*

Resources:

Stanford University. (n.d.). *Fewest squares.* https://www.youcubed.org/
Stanford University. (n.d.). *Good group work.*. https://www.youcubed.org/
Stanford University. (n.d.). *Strategies for learning math.* https://www.youcubed.org/

Lesson # 3: The Benefit of Mistakes (Mistakes=learning)

Student Objective(s):
I can use mental math strategies to solve problems.
I can modify (revise) my thinking.

Discourse Routine:(5-10min)
Discourse Goal/Focus: whole group engagement, justifying answers
Mathematical Thinking Focus: Flexibility
Unit Talk: #1 Avocados

- This number talk builds on the experiences from the dot talks from the past two days. Students will apply the same number talk norms, hand signals and strategies to this number talk. The unit talk gives students an image to examine that presents opportunities for multiple interpretations due to the concept of unit. They first notice what they see and then determine how many and how they saw that amount.

Lesson:

Big idea for developing norms/values(s):

- Mistakes are important for learning.
- Mathematicians are persistent.
- Mathematicians justify their thinking.

Launch (15-20min):
Video: The Importance of Struggle

1. After the video, provide time for students to think about the message of the video and add to their math and the brain list/notes/doodles in their math notebook.
2. Discuss as a class and add the new messages to the class poster from lesson #1.

2-4-6 Puzzle

1. Present the 2-4-6 puzzle to the class. Allow them to ask questions and record their guesses in the "correct" or "incorrect" category on a class poster/list on the white board.

2. Remind students that as a class, they only get one guess as to what the rule is each day, so use the guess wisely.
3. After students ask an ample amount of 3 number strings, allow for discussion time to determine the "rule" guess.
4. Students could have initial conversations in small groups with a facilitator leading the sharing and then return to the whole group discussion and facilitators can share the group's ideas with the whole class.
5. If the class figures out the rule, be sure to discuss how the incorrect sequences (mistakes) helped determine the solution as much if not more than the correct sequences.

Explore (25-30min):
Crossing a rectangle In this activity, students have the task of finding the shortest route through the grid without touching either side.

1. Students should have ample copies of the grid paper available to them and be given 5 - 10 minutes of independent exploration time before working in groups to share and discuss their solutions and strategies.
2. Students should also help each other identify if their strategies have stayed within the task constraints.
3. During this exploration time the teacher should be circulating to observe strategies and ask questions to help students analyze their different attempts. Questions: *Why did you make that change in your next attempt? How did your last attempt help you find a shorter path?*
4. Provide other templates for students who move through the task more quickly and ask students to look for any connections between the different sized templates. *What is the same? What is different? How did the first experience help you figure out how to solve the second challenge more efficiently?*
5. Encourage students to make conjectures for a rectangle of any size.
6. Have students complete the crossing a rectangle self-assessment.

Discuss (5min.):

1. As a class, discuss the connection between the video's message at the start of the lesson to their two math experiences today. *How did that message come into play in their work? What feelings surfaced as you encountered strategies that did not work? How did you push through (persevere)?*

Resources:

Finkel, D., Cook, K., (n.d.). *2-4-6 puzzle.* https://mathforlove.com/
Stanford University. (n.d.). *Crossing a rectangle.* https://www.youcubed.org/
Stanford University. (n.d.). *The importance of struggle.* https://www.youcubed.org/

Lesson #4: What about Speed?

Student Objective:
I can explain and justify my mathematical thinking.
I can use multiple representations to explain my solution strategies.

Discourse Routine: (5-10min)
Discourse Goal/Focus: whole group engagement, justifying answers
Mathematical Thinking Focus: Flexibility
Unit Talk: Unit Talk #2: Pinwheels

1. Display unit talk #2 for the class to view for a minute or two. Students should continue to use hand signals to communicate their thinking during the number talk.
2. Record students' observations on the board. Record all of the varied totals students saw in the image.
3. Call on students to explain and justify each response that is recorded on the board. Remind students to use the connection hand signal if they saw the same thing in the picture.
4. Provide time for students to ask any questions of each other for clarification.

Lesson:

Big idea for developing norms/values(s):

- Digging deeper is more important than speed.
- Mathematicians think flexibly.
- Mathematicians look for patterns.

Launch (5-10min)
Video: Speed is Not Important

1. After viewing the video, ask the students to identify the different messages presented in the video. Invite students to share any experiences or feelings they have had in regard to the message.
2. Provide time to record the new messages in their math notebooks and on the class poster.

Explore (30min):
Game of Totals

1. Explain the constraints and goal of the game to the students.
2. Practice one round of the game together as a whole class so that all students understand how to play.
3. Place students in groups of two to play several rounds of the game. Remind students to record their turns using numbers and in visual form.
4. After about 10 minutes of game play time, call the group together for a conversation.
5. Discuss the questions: *Which player seems to win most frequently, player 1 or player 2? Is there a way a player can make sure to win every time?*
6. If needed, give pairs a couple more minutes to try again and pay attention to those questions.
7. Introduce the finger perception concept to the class. Distribute the handout for students

to use as they incorporate and play the game using their fingers.

8. Invite students to change the total and numbers used for the game. *Does the strategy change?*

Discuss (5min):

1. Discuss as a group how their experience with "Game of Totals" connects to the message from the video about speed in math.
2. Provide time for students to share strategies, connections and conjectures that they discovered during their game play.

Resources:

Finkel, D., Cook, K., (n.d.). *Pinwheels.* https://mathforlove.com/
Stanford University. (n.d.). *Game of totals.* https://www.youcubed.org/
Stanford University. (n.d.). *Speed is not important.* https://www.youcubed.org/

Lesson #5: Making Conjectures

Student Objective:

I can explain and justify my mathematical thinking.
I can make conjectures.

Discourse Routine: (5-10min)

Which One Doesn't Belong #1

1. Display slide #1 from the "Which One Doesn't Belong" slide and give students 1-2 minutes to think about the image.
2. Students should use the number talk hand signals to convey their thinking to the group. At this point in the unit, the hand signals may or may not need to be reviewed during the math talk. Signal with your own use of the gestures when students need a reminder.
3. Record all of the different responses from students on the white board for the class to analyze and discuss.
4. Invite students to share their thinking and reasoning with the group. Students may ask clarifying questions and communicate connections they make during the discussion.

Lesson Format:

Launch: (10-15min)

1. Give students access to the math beliefs post-assessment and read each question aloud to the class as they select their answer in google forms.
2. Game introduction: Horseshoes Game - Explain the rules of the game to the whole group and model a round with an example game so that everyone is familiar with the guidelines.
    a.

Explore: (25-30min)

1. Break students into groups of 3 or 4 to play the game. Introduce the concept of golf scoring (the goal is to have the lowest score). Points are earned by calculating the

difference between the target number and a student's closest solution for the round.

2. Distribute the materials needed to play the game (Card Decks, Recording paper). Be sure to have math manipulatives available for students who prefer to use them (counters, rekenreks, base 10 blocks, etc.)
3. After about 15 minutes of play time, pose the following questions: *Are you noticing any patterns? What strategies are you using? Are your strategies changing the longer you play? Is it always possible to reach the target number? Can you make any conjectures about reaching the target number, odd number or even numbers?*
4. After the brief discussion and the introduction of questions, give students 10 to 15 more minutes of time to play the game.

Discuss: (10min)

1. Use a think, pair, share to discuss the conjectures, strategies or patterns students noticed while playing the game.

Exit Ticket: Give students time to reflect on their math learnings from the week and complete the Exit Ticket. Their notes from their math journals and the posters that were created during the week can be used as resources for them as they reflect.

Resources:

Finkel, D., Cook, K., (n.d.). *Horseshoes game.* https://mathforlove.com/
Forest, A. (2013). *Shape 20.* https://wodb.ca/shapes.html.

Unit 2: Review of Place Value and Addition/Subtraction Strategies

Stage 1: Desired Results

Established Goals:

Minnesota K-12 Academic Standards in Math

Students will...

3.1.1 Compare and represent whole numbers up to 100,000 with an emphasis on place value and equality.

3.1.1.1 Read, write and represent whole numbers up to 100,000 (extend to 1,000,000 for 4th grade district benchmark.)

3.1.1.2 Use place value to describe whole numbers between 1,000 and 100,000 (1,000,000 for 4th grade district benchmark) in terms of ten thousands, thousands, hundreds, tens and ones.

3.1.1.3 Find 10,000 (100,000) more or 10,000 (100,000) less than a given five-digit (six-digit) number.

3.1.1.4 Round numbers to the nearest 10,000, 1,000, 100 and 10 (and 100,000). Round up and round down to estimate sums and differences.

3.1.1.5 Compare and order whole numbers up to 100,000 (1,000,000 extension for 4th grade).

3.1.2 Add and subtract multi-digit whole numbers; solve real-world and mathematical problems using arithmetic.

3.1.2.1 Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.

3.1.2.2 Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.

Essential Questions:
How does the value of a digit change based on its placement in a number?
What conclusions can I make about place value in our base ten number system?
Why are place value strategies important for addition and subtraction?
How can whole numbers be compared and ordered?

How do number relationships help me to develop efficient strategies for adding/subtracting multi-digit numbers?

Student skills:
I can read, write and represent numbers up to 1,000,000.
I can use place-value to decompose numbers up to 1,000,000.
I can round numbers to the nearest place-value up to 100,000.
I can compare and order whole numbers up to 1,000,000.
I can estimate solutions in math problems.
I can add multi-digit whole numbers.
I can subtract multi-digit whole numbers.
I can use a variety of strategies and representations to solve real world addition and subtraction problems using whole numbers.

Stage 2: Assessment Evidence

Performance Tasks:
Deca Tree
Fill the Stairs: student form
Ink Blots
Getting Closer: Mystery Number
Estimate Photos
Open Middle: Sum to 10,000: student recording sheet
Student Form: 3 Act Task
Subtraction Advice,
Open Middle: Closest Difference
Math Foursquare
Word problem
Word problem Task Cards

Other Evidence:
Observations
Task rubric - modify for specific skills/task
Self Assessment
Show What You Can Do: Self-Assessment
Exit Ticket
Exit Ticket: Would you rather have?
Summative Assessment: Lessons 1-5
Summative Assessment: Lessons 6-10

Stage 3: Learning Plan

Lesson #1:

Student Objective:
I can read, write and represent numbers up to 1,000,000.
I can use place-value to decompose numbers up to 1,000,000.

Discourse Routine: (5-10min).
Choral Counting

1. Invite students to engage in choral counting. Start at 1 and count to 20, then again starting at 100 and stopping at 120.
2. Do the same counting again, but use the video Counting by 1 and Counting Above 100 as the class counts together. Ask the questions: *What do you notice? What do you wonder?*
3. Continue further with discussion *What is the same about counting from 1 - 20 and counting from 100-120? What is different? How many groups of 10 are in 100? How does the picture show you? Why does the number 114 look like it has the number 11 in front of it?*

Lesson Format:
Launch: (10 min).

1. Which is quicker? Pose the following questions on the board?
    a. Which is quicker, counting up to 30 by 1's or counting up to 300 by 10's?
    b. Which is quicker, counting up to 10 by 1's or counting up to 10,000 by 1,000's?
    c. Which is quicker, counting up to 20 by ones or counting up to 100,000 by 10,000's?
2. Give students some independent work time to formulate their answers and justifications. Students should have a visual to support their thinking.
3. Place students into groups (select the best format from the Grouping Structures based on class dynamics/needs) to share their answers and critique each other's solutions.
4. Each group/partnership should come to a consensus for an answer to each question and agree on a visual to represent their thinking (compare the visuals they created during independent work time).
5. If there are groups who move more quickly through the process, pose the following questions for consideration as well:
    a. Which is quicker, counting up to 20 in ones or counting up to 140 by 7's?
    b. Invent another example and then find a solution.
6. Call the whole group back together to share what they noticed and concluded from the different questions. Be sure to discuss the intersection with this activity and the math message about speed. Use this as an opportunity to introduce the idea of finding and using efficient strategies.

Explore: (30-40min)
The Deca Tree

1. Place students into groups(grouping structures and techniques).
2. Display the Deca Tree task on the screen and read it aloud for the class.
3. Each group should work together to determine a solution. Remind students to use visuals to track their thinking. Each group should have their math notebooks, grid paper, a vertical white board surface and ample time to explore how to find a solution to the problem.
4. The teacher should be circulating, asking probing questions and recording observations of students' mathematical thinking, discourse strategies and demonstrations of math values in action.

Discuss: (10min)

1. Gallery Walk - Each group should post their solutions and visuals for the class to see. The students should circulate with post-its two different colors of post-it notes. They can place a small yellow post-it on solutions/visuals that connect to their own strategies or write a question/ask for clarification on the green post-its.
2. After a few minutes of circulating, gather the group as a whole and highlight any connections, strategies or questions from the gallery walk.

Resources:

Everett, B. (n.d.). *Counting by 1. Counting above 100.* https://mathvisuals.wordpress.com/
Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks: Engaging students in doing math. Grades 4-5.* Corwin
University of Cambridge. (n.d.). *Which is quicker?* https://nrich.maths.org/
University of Cambridge. (n.d.). *The deca tree.* https://nrich.maths.org/

Lesson #2:

Student Objective:

I can read, write and represent numbers up to 1,000,000.
I can use place-value to decompose numbers up to 1,000,000.
I can compare and order whole numbers up to 1,000,000.

Discourse Routine: (5min)

Number String: #1

1. Display the first problem in the number string and direct students to solve the problems on the first slide mentally.
2. Ask the students: *How does solving the first expression help you solve the second? Third?*
3. Flip to the next slide and repeat the process.
4. Encourage students to discuss the strategy.

Lesson Format:

Launch: (5-10min)

1. Introduce the game "Highest Score" to the students.
2. The students will compete against the teacher to get the highest score. For each round, a card will be drawn that has two different numbers on it. The class will get to choose which number they want added to their score. That score will be added to the class score and the other score will be added to the teacher score. The trick is that each number stands for an amount in a different place value (Choose your digit: 3 or 7? They will naturally choose 7, but what they don't know is that it's 3 hundreds and 7 tens).
3. Lead a discussion with the class to determine how they will come to a consensus about which score to choose each round. When making those decisions, keep in mind that it is important for everyone to participate.
4. Draw a t-chart on the board labeled class, teacher.

5. Begin playing with Highest Score Game Cards: Set 1

Explore:

1. Record the scores from the first round on the t-chart.
2. Play a couple of rounds, continuing to record the scores on the chart.
3. Ask students to reflect on the score with the following questions: *Is that the score you expected? Why or why not? Who is winning right now? What issues are you having with playing this game?*
4. Students should be given some wait time to think about their own response to the questions, then discuss in table groups or with partners.
5. Call on students to share ideas from the discussion. Record the conjectures or connections on the board.
6. As students conclude that they don't have enough information to truly play this game, ask them if they would like to try a different version of the game where they get to choose the place value first.
7. Repeat the game with: Highest Score Game Cards: Set 2
8. As each choice is made, ask students: *Why did you choose…?*
9. Record the scores and encourage students to reflect with the questions from step 3.
10. Again, students can share first with partners and then as a whole group.
11. Record their observations and conclusions on the board.

Discuss:

1. As a group, discuss the following questions: *How did you know which score was greater in each round? Why was game two more successful for you than game one? At the beginning of game 1, how did you choose which score? Why did you choose that one? How did your strategy change? What was the difference in your strategy from game 1 to game 2?*
2. Distribute the Exit Ticket for students to complete.

Resources

Everett, B. (n.d.). *Making 10 by filling the ten frame.* https://mathvisuals.wordpress.com/
Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks: Engaging students in doing math. Grades 4-5.* Corwin

Lesson #3:

Student Objective:
I can compare and order whole numbers up to 1,000,000.

Discourse Routine: (5min)
Focus: Justifying thinking, asking clarifying questions
Would You Rather: jellybeans comparison

1. Introduce the "would you rather" image to the class.
2. Give students time to think and determine their response.
3. Invite students to share their thinking and engage in questioning each other on how

they came up with their solutions and their reasoning for their choice. Highlight justifications or strategies related to the student outcomes.
4. Facilitate the conversation, but allow the students to take the lead in the conversation.
5. If needed, provide sentence stems or question stems for the students to use.

Lesson Format:
Launch: (10min).
Fill the Stairs

1. Distribute a fill the stairs template for each student.
2. Explain that this is a game that students will be able to play with each other in the classroom, but today the entire class will play together to learn the goal and the procedures.
3. The goal of the game is to be the first person to fill in all the stairs on the page. Each number must be greater than the number before it and less than the number after it (numbers increase going up the stairs, decrease going down). If a number can't be placed on the stairs, write it below the stairs and no number gets filled in on your staircase for that turn.
4. Choose two different colored ten-sided dice to determine the numbers. One color is the tens-place and the other is the ones-place.
5. Play the game as a class, asking the following questions as you play: *Where are you going to put that number? Why there? Are you hoping for a specific number on your next turn? What number? How can you tell when a number is greater than another number?*

Explore: (30-35min)
Ink Blots:

1. Share the following story with the class:
    a. *Mr. Marty wrote some numbers and expressions on index cards for his math class. The next day, he noticed that some ink had spilled on the cards. He showed the stained cards to his class, and the students noticed that they could still put the cards in order from least to greatest. How could they do it?* The first number on all five cards is the same.
2. Show the class one of the cards.
3. Ask: *What do we know about the value of this card even though there is a stain covering part of it? How do you know that?*
4. After some wait time for students to think, encourage students to share in a *Pair-to-Pair* share first; then have volunteers share their ideas with the class. Elicit justifications for their thinking through questions, encouraging student to student questions as well.
5. Organize students into small groups and distribute a set of inkblot cards to each group.
6. Direct students to work together to put the cards in order from least to greatest.
7. As student groups work on the task, move through the room to make observations about discussions, explanations and justifications.
8. As groups work, ask probing questions to determine the mathematical thinking being discussed in each group. Use the following questions: *How do you know which expression has the least/greatest value? How did you decide that this expression has a*

greater/lesser value than this one? What if this value was greater/less, how would that impact the order?

9. Groups who complete the task can partner together to share their ordered sets and discuss their thinking and their process.
10. Extension - For groups who move very quickly, have them create their own set of expression cards for another group to determine the order and repeat the process.

Discuss: (5-10min).

1. Gather the class together and display each set of ordered cards on the board for examination and discussion.
2. Ask students to share strategies (strategically highlight specific strategies from the observations during their exploration time).
3. Revisit the cards used in the introduction of this activity and have the class quickly work to put them in order as a group.

Resources:

Finkel, D., Cook, K., (n.d.). *Fill the stairs.* https://mathforlove.com/
Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks: Engaging students in doing math. Grades 4-5.* Corwin
Stevens, 009. (2018, April 23). *Would you rather…* https://www.wouldyourathermath.com/

Lesson #4:

Student Objective:
I can round numbers to the nearest place-value up to 100,000.

Discourse Routine: (5min).
Would You Rather: place value

1. Display the image for the class to examine. Before determining an answer, ask students to share ideas for what the unit could be for this situation (cookies, dollars, toys, video games, etc.). After jotting down suggestions, come to a consensus as a class.
2. Allow students a minute of time to think of their solution and justification.
3. Ask students to share their position and strategies.
4. Encourage clarifying questions, connections and critique from fellow students.

Lesson Format:

Launch:

1. Gather the class to introduce the activity: Getting Closer.
2. They are going to play a game where they will try to figure out a mystery number by making educated guesses.
3. Tell the class that you are thinking of a mystery number and give the following clue: *The mystery number is less than 1,000 and it does NOT have a zero in the one's place.* (Ask students if they know what that means (it is not a multiple of 10).
4. Ask: *Based on that clue, what do you know about the mystery number? What are some possibilities for the mystery number? What are some numbers you can rule out? How do you know?*

5. Record the numbers that students share on the clothesline (use index cards folded in half).
6. Give one more clue about the number: *The mystery number is closer to 1,000 than to 0.*
7. Ask: *Now what do you know about the mystery number? Are any of your guesses still possibilities for the mystery number? Which ones can you eliminate for the mystery number? Why?*
8. Introduce the vocabulary word 'benchmarks on the numberline' for students to use as reference after a student shares the strategy of using benchmark numbers as a guide.
9. Ask the class to think of a question that could help them get closer to guessing the mystery number. Use the sentence frame, *Is the mystery number closer to… or …?* Answer the question and repeat

Explore: (30min)

Getting Closer

1. After the abbreviated group "sample" round, break students into small groups to play.
2. Each team will work together to generate questions and make guesses about the number. One team will act as the student leaders for the round and work together to determine a number and give clues.
3. Students can keep record of their thinking/process on the student recording sheet: Getting Closer
4. Teams will take turns asking their questions to the student leaders or they can use their turn to make a guess if they think they have the number.
5. The teacher's role is to observe the strategies being used by each of the groups. Pay attention to how students refine their guesses and how they are reasoning about range. As they ask about particular ranges, do students consider numbers that are less than the lower number of the range? (For example, if the number is closer to 200 than 300, do they presume it is larger than 200?)
6. Continue to move discussion with the questions from step 4 and 7 in the launch of the lesson.
7. Repeat the activity with new student leaders as time allows.

Discuss:

1. Ask students to share strategies, based on observed strategies from the group game time.
2. Ask the question: *Did your strategy change as the activity went on? How did it change? What realization or pattern did you notice that helped you make the change?*
3. Highlight strategies that narrowed the range of numbers through their questions. Introduce the clothesline on the whiteboard as an open number line visual. Remind students that on a number line, movement to the right represents an increase, while moving to the left represents a decrease.
4. Demonstrate using the clothesline to narrow in on the number and eliminate certain values. Call attention to the connection between knowing that a number is "closer to" one value than another and rounding (paying attention to the "halfway point" between numbers).

Resources

Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks: Engaging students in doing math. Grades 4-5.* Corwin

Stevens, 009. (2016, April 2). *Would you rather…* https://www.wouldyourathermath.com/

Lesson #5:

Student Objective:

I can estimate solutions in math problems.

Discourse Routine: (5min)

Which One Doesn't Belong: #2

1. Gather students in a location where the image on the screen is highly visible to everyone.
2. Give students about a minute to look at the image and determine their solution.
3. Ask for students to share their justifications for each picture being determined to be the one that doesn't belong.
4. Facilitate student leadership in guiding the probing questions, asking for clarification and for making connections between strategies and justifications.

Lesson Format:

Launch: (10min)

1. Tell students that sometimes we use photographs to solve problems in the real world.
2. Display one of the crowd photos (from the estimate photos slideshow) and ask students if they can come up with a mathematical question related to the photo.
3. Show students the photos for the lesson and tell them that often when there is a big event, we want to know how many people attended. The same thing can be done with photos of animals for scientists to monitor animal populations.
4. Pose the following question: *How could you estimate how many people or animals are in one of the photos?*
5. Today's challenge will be to develop a method to estimate and record your method so that others can understand your strategy for estimation.

Explore: (30-40min).

How crowded is the crowd? (Boaler et. al., p.205, 2017)

1. Break students into groups of two or three. Each group may select a photo of their choice from the estimate photos slideshow.
2. Students work together to develop a method for estimating the size of the crowd.
3. Record your process (counting, calculations, visual representations) on a poster in a clear manner so that other students could follow your thinking/reasoning. It may be valuable to color code the different stages in the process to keep track of the different stages.
4. It may be necessary or beneficial to have copies of the photos available so that students can write on them as they develop strategies and methods for solving.
5. After completing the process for one photo, groups may select another photo and try their method again, paying attention to their efficiency and strategy changes on the

second attempt.

6. As students complete photos, they can be displayed around the room for examination from other groups.

Discuss: (10min).

1. Give the students some time to look at the estimates from other groups that are posted around the room.
2. Direct the discussion with the following questions: *How did you arrive at your estimates? What strategies did you use or methods did you develop? How did you combine the numbers you were observing? Why did you use the operation that you did? Did you try anything that didn't work? How did you know it wasn't working? How did that help you develop something that did work?*
3. At the end of the discussion time, highlight some of the methods and organizational strategies that were shared. Reiterate that ultimately, the students were working to make sense of the problem. This situation is similar to other math problems where the problem doesn't tell you how to solve it, you have to choose based on how you make sense of the problem.

Summative Assessment: Lessons 1-5

Resources

Boaler, J., Munson, J., Williams, C.(2017). *Mindset mathematics. Visualizing and investigating big ideas. Grade 4*. Jossey-Bass.
Bourassa, M. (2013). *Shape 21*. https://wodb.ca/shapes.html.

Lesson #6:

Student Objective:
I can add multi-digit whole numbers.

Discourse Routine:
Number String: #2

1. Follow the same number string procedure as was introduced in lesson 2.
2. Use the video after the number string to provide a strong visual connection to the strategy.
3. Allow students to share their own connections to the video images.

Lesson Format:
Launch:

1. Write the following problem on the board, horizontally: 635 + 264
2. Ask students to solve the problem mentally.
3. Give students some wait time to solve the problem. Encourage students who solve it quickly to look for another mental strategy to use to solve it.
4. Call on students to share their solutions. Record them on the board.
5. Invite students to defend the different solutions and share the strategies that they used

to arrive at the solutions.
6. Draw attention to the different representations students are using.

Explore:
1. Tell students that they are going to continue to use different strategies to add multi-digit numbers, but the problem is going to look a little different than what they are used to.
2. Explain that their job is to find two four-digit addends who have a sum as close to 10,000 as possible. The catch is that each digit between 1-9 may only be used 1 time in the process.
3. Distribute the student sheet: Open Middle: Sum to 10,000
4. Students should work for about 10 minutes independently to find a solution.
5. After ample time working independently, move students to strategic partnerships or groups of three to continue refining their solutions, strategies and process.
6. Circulate and observe.
7. Use probing questions to guide students who are stuck or having trouble persisting. Use the question: *Which digits will be the most helpful to focus on to make the solution as close to 10,000 as possible?*
8. This task can be modified by changing the goal to adding two, three-digit addends to make a number close to 1,000 or two, five-digit addends to make a number close to 100,000.

Discuss:
1. Gather the class together to share solutions and strategies.
2. Ask students to describe how their method changed as they progressed through the problem. *What was the first thing that you tried? How did you change it from there? Were mistakes helpful in this process? How? Why?*

Resources
Everett, B. (n.d.). *Making 10, 7 + 8 and 27 + 8.* https://mathvisuals.wordpress.com/
Goree, J. (n.d.). *Sum to 10,000.* https://www.openmiddle.com/

Lesson #7:

Student Objective:
I can add multi-digit whole numbers.

Discourse Routine: (5-10min)
Target number: #1
1. Display the target number slide for students.
2. This number talk strategy allows students to find as many solution strategies as they can. They can use as many or as few of the numbers from the number bank to find solutions. They are also able to use any operations as well.
3. After some wait time for students to think and use their hand signals to communicate

the number of strategies they have found, call on some students to share.
4. Record their process on the board for other students to see.
5. Encourage classmates to ask clarifying questions, make connections, build off of each other's strategies or challenge misconceptions that are shared.
6. This is a good opportunity to record their mathematical thinking with the corresponding math symbols and draw attention to new concepts (parentheses, etc.).

Lesson Format:

Launch: (15min)

1. Three Act Task: The Arcade part 1 Show the Act 1 portion of the video to students.
2. Ask the questions: *What do you notice? What do you wonder?*
3. Record the students' thoughts on the board. Do not limit students' contributions to math related ideas.
4. Replay the first act video for students to take one more look and add any noticings or wonderings to the list.
5. Distribute the student form: Student Form: 3 Act Task and have students record their noticings and wonderings. They may add ideas from the class generated list.
6. Ask students to generate questions about this situation.
7. Highlight the question: *How many tickets does he have?*
8. Have students record a "too high" estimate and a "too low" estimate on the open number line.

Explore: (30min)

1. Tell students that we are now moving to Act 2. In Act 2, students will receive a little more information and work to find an answer to the question: *How many tickets does he have?*
2. Show the image for Act 2. Three Act Task: The Arcade part 1
3. Give students a short amount of time to work on a solution. Remind students to use visual representations as well as numbers as they work toward their solution.
4. Assign partners for students to compare their solutions and their strategies.
5. Call the entire class together to share solutions and strategies. Write the different solutions on the board and discuss the different approaches and methods students used to answer the question. Draw connections between the strategies.
6. Tell the class there is one more layer to this task, called Act 3. Show the next image (the calculator) and pose the following questions: *Is the attendants total correct or incorrect? Would you be happy or unhappy with this total? Why? The attendant said that he "rounded up", what did he mean by that? What did he round to? The nearest 10? 100?*
7. Post the questions for the class to see and give each partnership/group time to answer the questions.
8. Pair up partners/groups to share their answers and conclusions with another group.

Discuss: (5min)

1. As a class, discuss the different ways the groups answered the Act 3 questions.
2. Guide students to critique, add on and connect to each other's process, strategies and mathematical thinking.

Resources
Castillo, C. (2018, Feb. 25). *The Arcade.* https://mscastillosmath.com/
Fletcher, G. (n.d.) *3-5 Recording Sheet.* https://gfletchy.com/

Lesson #8:

Student Objective:
I can subtract multi-digit whole numbers.

Discourse Routine:
Number String: #3

1. Follow the number string routine from earlier in the unit.
2. There is a video that provides a visual to support the strategy development.

Lesson Format:
Launch:

1. Introduce the math class scenario from the form: Subtraction Advice
2. Give students a couple of minutes of wait time to think about the situation.
3. Ask the students to do a Turn and Talk to discuss a reasonable estimate for Noelle's problem.
4. Record the estimates on the board and come to a consensus as a class about one for the class to use. Encourage students to justify their reasoning.

Explore:

1. Organize students into pairs or small groups. Each group should have access to chart paper to record their thinking, the student form: Subtraction Advice, and base ten blocks.
2. Each group will need to answer the following question: *What should Felicity say or do to help Noelle?*
3. Each group should create a response and use both visual representations and numbers to display their thinking.
4. Circulate and observe as each group works through the challenge. Encourage them to carefully consider how to organize their thinking and clarify their ideas through tools and pictures.
5. Questions to probe thinking: *What does Noelle need to do next since there aren't any tens to regroup? How does using base ten blocks help you explain how to regroup? How does drawing the problem help you explain how to regroup in the hundreds place? In the number representation, what does the 10 written above the crossed out zero mean?*

Open Middle: Closest Difference - Depending on the amount of time needed for this activity, there may or may not be time for an open middle activity with subtraction. This activity follows the same procedure as the open middle addition problem used in an earlier lesson.

Discuss:

1. After groups have concluded their discussion and creation of a visual, gather the class for a group discussion.
2. Specific groups can present their process to the class, using the board or a document camera to explain.
3. It is important in this task for students to practice restating others' explanations and refine the use of the math vocabulary.

Resources:

Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks: Engaging students in doing math. Grades 4-5.* Corwin
Trifilette, T. & Goree, J. (n.d.). *Closest difference to 200 Problem 2.* https://www.openmiddle.com/

Lesson #9:

Student Objective:

I can use a variety of strategies and representations to solve real world addition and subtraction problems using whole numbers.

Discourse Routine:

Math Flip:

1. Display the first math flip slide, image A. Ask the students: *How many? How do you know?*
2. Students can turn to a partner to share their responses. Have a few volunteers share.
3. Show image B on the same slide and ask: *How many NOW? How do you know?*
4. Repeat the process with a few more slides.
5. After the students have the hang of it, ask: *How does side A help you figure out side B? What is the same/different about side A and side B*? *Do you notice any patterns?*

Lesson Format:

Launch:

1. Introduce a story problem to the class one sentence at a time, stopping after each statement to have students visualize and make a "bet" about what they think will happen next.
2. As students make their bets, encourage them to also think of questions that could be asked based on the information that they currently have.
    a. *During the first week of school, the daily temperatures were hotter than normal.*
    b. *On Wednesday, the high temperature was 89 degrees.*
    c. *Weirdly, the temperatures for that week consistently decreased by 4 degrees each day.*
    d. *How much hotter was the high temperature on Friday than it was on Monday?*
3. After the actual question is revealed, go back and read each statement again, allowing students to visualize what is happening and create a doodle image to represent the

situation.

Explore:

1. Break students into smaller groups (partnerships, threes or groups of 4, based on observed class dynamics and collaboration experiences).
2. Give each student a math foursquare paper to use as they work to solve the problem.
3. Circulate and observe solution strategies. Ask questions to probe thinking and justifications.
4. Gather the group to share solutions and strategies, calling on groups to present their thinking to the entire class.
5. Return students to small groups to work on solutions for the next problem, encourage them to visualize and discuss the problem before determining solution strategies.
6. Distribute the form: Word problems
7. During the work time, the teacher should again circulate and observe.

Discuss:

1. Once again gather the glass for a group discussion of the strategies that were used to solve the problem.
2. Highlight strategies that were observed during the exploration time. Be sure to focus on connections, mathematical representations and organizational methods used by different students.
3. Allow students to ask questions and guide the conversation.
4. Distribute an Exit Ticket for students to share their own learning from the day.

Resources:

Candler, L. (n.d.) *Math mindset challenges: Editable word problems.* https://www.lauracandler.com/turn-a-word-problem-into-a-rich-math-task-part-one/
Everett, B. (n.d.). *Math Flips.* https://berkeleyeverett.com/math-flips/

Lesson #10:

Student Objective:

I can use a variety of strategies and representations to solve real world addition and subtraction problems using whole numbers.

Discourse Routine:

Which One Doesn't Belong: #3

1. Students should be given about a minute to look at the image and determine which number does not belong and formulate a justification for their choice.
2. Students may select more than one solution and justification during this number talk.
3. Have students share their reasons and critique each other's choices and justifications.
4. Pay attention to connections that can be made among strategies and justifications as well.

Lesson Format:

Launch:

1. Tell students that today we will be working on story based problem solving.
2. Present the following numberless problem to the students:
    a. *Isabella is preparing for the spelling bee. Her teacher gave her two lists of words to study. The Level A list contains _____ words and the Level B list contains _____ words. She wants to spend _____ days studying all the words.*
3. After reading the problem, have students close their eyes and visualize the problem. Ask: *What is happening in the problem?*
4. Invite students to make a sketch of their ideas.
5. Read the problem again. Encourage the students to generate questions that could be asked about the situation.
6. Have students share some of their questions with the whole group and record them on the whiteboard for everyone to see.
7. Reveal the entire problem and read through it together. Encourage students to focus their attention on the quantities and their relationships
    a. *Isabella is preparing for the spelling bee. Her teacher gave her two lists of words to study. The Level A list contains 230 words, and the Level B list contains 250 words. She wants to spend 4 days studying all the words. She wants to study the same amount of words each day. How many words does Isabella need to study each day?*
8. Ask: *What do we know? What don't we know?*
9. Explain that we can use a letter to represent an unknown in an equation. If there are two unknowns then we use two different letters.
10. Break students into pairs and have them work to write equations for the situation. Encourage visual representations as well.
11. During the group work time, pause the discussions to have groups share some strategies and solutions.

Explore:

1. Distribue the Word Problem Task Cards, chart paper and markers to each pair and remind them that they are working to write equations for each situation. Encourage them to visualize and focus on quantities and relationships after they have a strong visual of what is happening in each problem.
2. Circulate, observe, and probe students to show how the equations represent the problems (color coding could be a good tool).
3. Each group will display their equations and process on the chart paper.

Discuss:

1. Distribute two colors of dot stickers to students.
2. Each group should display their poster around the room for a gallery walk.
3. As students look at their peers' equations and solutions, they should mark the posters with the coordinating color for something that is the same as their own and something that is different.
4. Based on the observations and class responses, select some pairs of students to explain their equations to the class. This is a good time to model using color coding to help

organize different parts of the word problem and which part of the equation represents the words.
Summative Assessments: Self Assessment - Students complete this assessment, reviewing previous work and looking for evidence. This can also be administered via a student/teacher interview process. Summative Assessment: Unit assessment (use after lessons 6-10)
Resources: Wilson, P. (2013). *Number 1*. https://wodb.ca/shapes.html. Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks: Engaging students in doing math. Grades 4-5, Task 1*. Corwin

Appendix

Understanding by Design Template

Adapted from Wiggins & McTighe (2011)

Stage 1: Desired Results
Established Goals: Students will…
Essential Questions:
Student skills: I can…
Stage 2: Assessment Evidence
Performance Tasks:
Other Evidence:
Stage 3: Learning Plan
Lesson #:
Student Objective(s):
Discourse Routine: (10-15min) **Discourse Goal/Focus:** Whole group engagement **Mathematical Goal/Focus:** determining multiple strategies
Lesson: **Big idea(s) for developing norms/values:** (this section may be omitted in future lesson units/lesson plans) **<u>Launch:</u> (20min)** **<u>Explore:</u> (30min)** **<u>Discuss/Reflect:</u> (5min)**
Resources:

References

Harkness, K. C. (n.d.). *Strategies for classroom dialogue.* https://www.katherinecadwell.com/katherine-cadwell-publications

Kazemi, E.& Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions.* Stenhouse Publishers.

Liljedahl, P. (2021). *Building thinking classrooms in mathematics.* Corwin Mathematics.

Minnesota Department of Education (2007). *Minnesota K-12 academic standards in math.* Retrieved from https://education.mn.gov/MDE/dse/stds/Math/

Minnesota Department of Education (2010). *Minnesota K-12 academic standards in english language arts.* Retrieved from https://education.mn.gov/MDE/dse/stds/ela/

National Council of Teachers of Mathematics. (n.d.). *Executive summary.* https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_ExecutiveSummary.pdf

Wiggins, G., & McTighe, J. (2011). *The understanding by design guide to creating high-quality units*. ASCD.