# PROMOTING STUDENT TO STUDENT DISCOURSE IN MATHEMATICS CLASSROOMS

By

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A capstone submitted in partial fulfillment of the requirements for the degree of Master of Arts in Teaching.

Hamline University

Saint Paul, MN

August 2021

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## PROJECT DESCRIPTION

This project was created with the following question in mind: *How can I promote student to student discourse in mathematics classrooms?* The purpose of this project is to get students discussing their mathematical ideas with one another. Students learning to effectively communicate their ideas will not only benefit them in the math classroom, but effective communication is a needed skill that will carry over throughout all subjects and into their everyday life. After conducting a thorough literature review of content related to the research question, activity guides were designed to help foster discourse in the mathematics classroom. The various activities in the guide were created with the three varying learning formats in mind; in-person, hybrid, and distance. The activity guides are meant to be used throughout the school year. The activities are designed to be implemented as ways to enhance the lesson, not replace the lesson. They are for use with an audience of high school math teachers and math students. The activity guides included, specifically, are intended to be implemented in geometry classrooms. However, the activity ideas can be modified to be used within other math classrooms as well.

The guides in this project were put together using the Understanding by Design (UbD) planning method (Wiggins &McTighe, 1998). The UbD planning model has three stages: identify the desired results, identify evidence that is acceptable to determine learning goals have been met, and plan for instruction and learning activities (Bowen, 2017; Wiggins & McTighe, 1998). In using this backward design approach to planning, teachers have the benefit of having established goals before planning any activities or learning experiences. This is immensely different from a forward design approach. When a teacher uses the planning forward design typically the lessons and activities come first,

then they move onto planning assessments, and finally connections between these two and the course goals are made (Bowen, 2017). The forward design approach can lead itself to having activities that do not align with any course goal because those connections are made last in planning. Teachers may find an activity they want to use but it does not necessarily align. Whereas in UbD, backwards planning gets rid of the chance of having students participate in activities just for the sake of doing them; this is where intentionality comes in (Bowen, 2017). If an activity does not align with any of the pre-established goals, then it should not be used. While putting the guides together, intentionality and the UbD planning method was at the forefront of every decision.

For each of my activity guides, I decided to include all three stages of UbD. In the first stage, desired results, you will find all of the Minnesota standards that correlate with that particular geometry unit (Minnesota Department of Education, 2008). The first stage of the guides also includes the essential questions and objectives for those standards. I included these two sections because as a Collaboration Team (CT) at my school these are shared and communicated with the students repeatedly. In the second stage, assessment evidence, the small group and whole group activity resources are listed and linked. Small group activities are shorter activities in length that promote mathematical conversations for students in groups of two to four. Whole group activities are group-worthy learning activities that promote discourse. In several of the activities students are given group roles so that every student has a space in the conversation to ensure all voices are heard. These roles help groups work collaboratively with each other on various problems while promoting discourse. Lastly, in the third stage, titled discourse activity, the small and

whole group discourse routines are described and explained with the learning model with a list of materials needed for each.

# Guide 1 - Shapes and Transformations

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.4.6** Use numeric, graphic and symbolic representations of transformations in two dimensions, such as reflections, translations, scale changes and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate grid.
**9.3.3.3** Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results.
**9.3.1.2** Compose and decompose two- dimensional figures; use decomposition to determine the perimeter and area.
**9.3.1.3** Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.

| Essential Question(s): | Objective(s) Students will be able to: |
|---|---|
| ● Why are lines of symmetry helpful in shapes?<br>● How can I describe changes to shapes?<br>● What connections can I make between the transformations? | 1.1: Students will be able to identify reflective symmetry and lines of symmetry. They will also complete shapes given a line of symmetry.<br>1.2: Students will be able to perform, identify, and label transformations.<br>1.3: Students will be able to mark, describe, and compare traits of shapes. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Desmos Venn Diagram Activity
  - Partner Transformation Logo Exploration (E. K, 2020)

- Whole Group Assessment Evidence:
  - Transformation Ask an Expert

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Transformation Ask an Expert:** After a short lesson on transformations task each group with becoming experts on their assigned transformation. Students will discuss questions on the assigned worksheet for their transformation and put together expert advice posters to share with their classmates. This activity gets students talking in their own small group to better prepare to be an “expert” and also gets students hearing about transformations from their classmates' perspectives. This activity should take 25-30 minutes.
- Hybrid Learning:
  - **Desmos Venn Diagram Activity:** Place the students in mixed groups (in-person with online students) using breakout rooms to help facilitate. In the Desmos activity students will be asked to sort letters in the alphabet based on what sort of symmetry they have (Desmos, 2021). They need to discuss their thoughts with one another and justify each letter's placement. This gets students talking in small groups about how they see symmetry and listening to others justify their ideas as well. This activity should take 10-15 minutes.
- Distance Learning:
  - **Partner Transformation Logo Exploration:** This idea was inspired by an online resource by Mrs. E Teaches Math (E. K, 2020). Using breakout rooms pair students together and give them 10 minutes to find a brand or logo that uses each of the major transformations. Ask them to copy and paste their logo onto their jamboard page and be ready to justify their thinking to the class. This activity has students seeing math used in real world images and gets them discussing their ideas with one another (Clements, Battista, & Sarama, 2001). They may even be able to use the same logo in a few locations!

**Materials needed:**

- Transformation Ask an Expert: Tracing paper (if needed), poster paper, graph paper, and markers/colored pencils
- Desmos Venn Diagram Activity: Desmos link and breakout rooms. Person in the room pen and paper.
- Transformation Logo Exploration: Device to access internet for logo pictures and Jamboard

**Sources:**


Clements, D., Battista, M., & Sarama, J. (2001). Logo and Geometry. Journal for Research in Mathematics Education. Monograph, 10, I-177. doi:10.2307/749924

Desmos classroom activities. https://teacher.desmos.com/custom.

E , K. (2020, November 10). Transformations - Logo Project. Mrs. E Teaches Math. https://www.mrseteachesmath.com/2017/01/transformations-logo-project.html.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

# Guide 2 - Angles and Measurements

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.3.1** Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results.

**9.3.3.2** Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results.

**9.3.3.3** Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results.

**9.3.3.4** Apply the Pythagorean Theorem and its converse to solve problems and logically justify results.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| ● What words can I use to describe angles in relation to each other?<br>● What patterns help me identify angle relationships?<br>● How can I find an area without a grid?<br>● What is the relationship between the sides of triangles?<br>● Why is a right triangle special? | 2.1: Students will be able to identify relationships between angles formed by intersecting lines and angles within a triangle to apply properties and find missing values.<br>2.2: Students will be able to calculate the area of triangles, parallelograms, and trapezoids and use these areas to find missing information.<br>2.3: Students will be able to identify relationships in triangles and use the Pythagorean Theorem and Triangle Sum Theorem to answer questions about right and non-right triangles. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Find the Mistake
- Whole Group Assessment Evidence:
  - I Have… Who Has (Holly, 2021)
  - Number Talk - Which One Doesn't Belong (Parrish, 2011)

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **I Have… Who Has:** This was adapted from an online blog post with a list of fun classroom activities (Holly, 2021). In this activity students will each be given a card with an angle or definition on one side and a clue on the back. Have students turn to the angle or definition side. The student who has the word "STARTER" on their card will go first. They read their clue out and whoever has the answer to their clue stands up and reads their answer. They then flip their card over and read their clue. This continues until you get to the student with the word "FINISHED". Students should be leading the activity and discussing their thoughts as a class. This should take 5-10 minutes.
- Hybrid Learning:
  - **Number Talk - Which One Doesn't Belong:** Adapted from an article on Number Talks where student's will look at four images of triangles and decide which picture does not belong (Parrish, 2011). They need to justify their idea by writing it down on scratch paper. If kids finish quickly, challenge them to find a way every image could be "the one that doesn't belong" (Newell & Orton, 2018). Students will discuss their different perspectives to the problem as a whole class. This should take 5 minutes.
- Distance Learning:
  - **Find the Mistake:** Students will be given four different problems where there is a mistake made. They will discuss in their teams, in breakout rooms. The problems will gradually increase in difficulty so they will start with

number 1. Students will use their jobs to help guide the conversation. The Resource Manager will share their screen so everyone can see the problem. The Recorder/Reporter will circle the mistake and write a statement with how their group decided this was the error. The Task Manager will write the correct steps down to solve the problem. The Facilitator will be tasked with hitting the ask for help button when their group is stuck or needs clarification. This should take 15-20 minutes.

**Materials needed:**

- I Have… Who Has: The Answer/Clue cards
- Number Talk - Which One Doesn't Belong: Access to the 4 images and scratch paper
- Find the Mistake: Device to access internet and a shared google document of the assignment

**Sources:**


Holly. (2021, May 24). 20 Fun Classroom Angles Activities and Teaching Resources. Teach Starter.

https://www.teachstarter.com/us/blog/20-fun-angles-activities-resources-us/.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

Newell, C., & Orton, C. (2018). Classroom Routines: An Invitation for Discourse. Teaching Children Mathematics, 25(2), 94-102.

Retrieved March 10, 2021, from https://www.jstor.org/stable/10.5951/teacchilmath.25.2.0094

Parrish, S. D. (2011, September 30). Number Talks Build Numerical Reasoning. Teaching Children Mathematics.

https://eric.ed.gov/?id=EJ943575.

# Guide 3 - Justification and Similarity

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.2.1** Understand the roles of axioms, definitions, undefined terms and theorems in logical arguments.
**9.3.2.2** Accurately interpret and use words and phrases such as "if…then," "if and only if," "all," and "not." Recognize the logical relationships between an "if…then" statement and its inverse, converse and contrapositive.
**9.3.2.3** Assess the validity of a logical argument and give counterexamples to disprove a statement.
**9.3.3.6** Know and apply properties of similar figures to solve problems and logically justify results.
**9.3.4.7** Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| • How do I set up and solve a proportional equation?<br>• How does proportional reasoning correspond with similarity statements?<br>• What information do I need to prove similarity? | 3.1: I can identify similar polygons and use their properties to find missing information.<br>3.2: I can choose and apply triangle similarity theorems to prove that two triangles are similar. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Similar Figures Drawing (Example to show)
  - Love it OR Lose it
- Whole Group Assessment Evidence:
  - Similar Figures Word Dump

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Similar Figure Drawing:** Each person will be given graph paper and a ruler. They will make two similar shapes on their graph paper and come up with a title for their drawing. They then will pass the paper to the left, and the next person will add two new similar figures to the drawing. This process will continue around their groups. Then students will have time to color the pictures and make them fun and creative. We will post them around the room and have students share their drawings. This should take 15 minutes.
- Hybrid Learning:
  - **Similar Figures Word Dump:** After discussing similar triangles in class, either as a closure activity or warm up activity students will be asked to write down any words or phrases they can think of regarding two sets of similar figures on the board. Then students will share out as a whole class. This activity allows students to share their ideas and there is not a correct answer. They could be making observations, doing calculations, or justifying why the shapes are similar. This is low stakes and allows for a large variety of students to have an entry point into the activity. This should take around 5 minutes.
- Distance Learning:
  - **Love it OR Lose it:** Each group will be placed in a breakout room where they will move through a slideshow presentation with pairs of figures. They need to decide together if they "Love" the pair, meaning they are similar or if they should "Lose" the pair, meaning they are not similar. The Task Manager will need to share their screen with the group. The Facilitator needs to read the questions aloud and make sure everyone gets a chance to share their ideas. The Recorder/Reporter will be typing into their team's document. The Resource Manager will be in charge of asking for help if any issues arise in the team. This activity requires students to discuss important features to look for in similar figures and encourages the use of vocabulary terms. This should take 20 minutes.

**Materials needed:**

- Similar Figure Drawing: Graph paper, ruler, pencils, and markers.
- Similar Figures Word Dump: Scratch paper and images
- Love it OR Lose it: Google slides with pairs of figures and worksheet

**Sources:**

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

# Guide 4 - Special Triangles and Trigonometry

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.3.5** Know and apply properties of right triangles, including properties of 45- 45-90 and 30-60-90 triangles, to solve problems and logically justify results.

**9.3.4.1** Understand how the properties of similar right triangles allow the trigonometric ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.

**9.3.4.2** Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.

**9.3.4.3** Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| ● What is the connection between the slope ratio with the angle?<br>● How can I find slope ratio for any angle?<br>● How are the three trigonometric ratios similar and different?<br>● What does inverse mean? | 4.1: I can use a tangent ratio to find the measure of a missing leg in a right triangle.<br>4.1/5.1 – I can choose and apply trigonometric ratios to find missing side lengths in right triangles. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Think-Ink-Pair-Share (Krall, 2018)
  - Target Trig
- Whole Group Assessment Evidence:
  - Trigonometry Relay

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Trigonometry Relay:** This activity involves the students communicating in a relay within their team. Each person of the team has a different marker and will complete one step of the problem before passing the paper. They can talk to one another but at any time all of them should be able to answer where they are at in the problem and WHY they are doing what they are doing to solve it. This should take 5-10 minutes depending on the number of problems done.
- Hybrid Learning:
  - **Think-Ink-Pair-Share:** In this activity students will get a question involving trigonometry. The type of question here can vary immensely but keeping it open ended can lead to better discussion. Krall starts the process with students having one minute to simply think about the problem (Krall, 2018). Give them one minute to "ink" or write their own ideas down. Then pair people at home together and people in the room together to discuss their ideas. This allows students to see multiple approaches to the same problem and could also be a time they can ask clarifying questions to one another. Then students will share ideas with the whole class. This should take 5 minutes.
- Distance Learning:
  - **Target Trig:** Spin a virtual wheel that determines what problem number from the homework you will either explain to your group or ask questions on if you are not sure. Go around having every student explain or ask for

help on one question. This way they are able to see and hear how other students may have approached a problem differently and they are able to get help from peers. This should take 10 minutes.

**Materials needed:**

- Trigonometry Relay: Problem Worksheet or a sheet of paper and different colored markers
- Think-Ink-Pair-Share: Device to help pair students at home, scrap paper, and pencil
- Target Trig: Virtual spinner and homework problems

**Sources:**

Krall, G. (2018). Necessary conditions: Teaching secondary math with academic safety, quality tasks, and effective facilitation.

Portland, ME: Stenhouse.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

# Guide 5 - Congruent Triangles

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.3.6** Know and apply properties of congruent and similar figures to solve problems and logically justify results.
**9.3.3.2** Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results.
**9.3.3.1** Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| ● How can I use the definition of congruence?<br>● What combinations of information is enough to say two triangles are congruent?<br>● What visual clues can I use to decide if triangles are congruent? | 6.1 – I can identify if two triangles are congruent and use triangle congruence theorems to prove it.<br>7.2 – I can write an effective proof using triangles. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Jamboard Word Wall with Helpful Tips (Coppens, 2018)
- Whole Group Assessment Evidence:
  - Human Flowchart
  - How Do You See it - Triangles (Newell & Orton, 2018)

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Human Flowchart:** Give students either a flowchart bubble or a reason statement. Have pictures posted around the room of congruent triangles. Students with the bubbles first need to decide which triangle pair they belong to. Then students with the reason statements will walk around the room to help fill in the remainder of their human flowchart. This should be a student led activity. They will need to discuss with one another which theorem to use to prove congruence for every pair. This should take 10-15 minutes.
- Hybrid Learning:
  - **How Do You See it - Triangles:** This is an adaptation of Krall's Number Talk examples (Krall, 2018). Post an image of three triangles on the board with two statements. Two of the triangles are congruent and the other triangle is similar. Give them 1-2 minutes of quiet time to think on their own about the statements and a strategy to prove congruence and/or similarity. Have them hold their hand to their chest (for kids in the room) and use the thumbs up button on google meet (for kids at home) when they have a strategy. Ask students to share out and keep a list of strategies on the board. This should take 5 minutes total.
- Distance Learning:
  - **Jamboard Word Wall with Helpful Tips:** Create a Jamboard page with every Congruence Theorem on it. Allow kids to come up with three ideas of tips, tricks, pictures, or phrases that they could add to the jamboard. Have them present their ideas to their small group and in the small group they will choose the five best ideas and add them to the class jamboard. Students will have an opportunity to discuss their additions and ask questions of other groups (Coppens, 2018). This should take 15 minutes.

**Materials needed:**

- Human Flowchart: Triangle pairs around the room. Fact bubbles and reason statements printed out.

- How Do You See it - Triangles: Image of the three triangles and the two statements written out
- Jamboard Word Wall with Helpful Tips: Jamboard that students have access to edit

**Sources:**

Coppens, K. (2018). Strengthening your word wall. Science Scope, 42(4), 28-31. Retrieved August 4, 2021, from https://www.jstor.org/stable/26611886

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

Newell, C., & Orton, C. (2018). Classroom Routines: An Invitation for Discourse. Teaching Children Mathematics, 25(2), 94-102. Retrieved March 10, 2021, from https://www.jstor.org/stable/10.5951/teacchilmath.25.2.0094

# Guide 6 - Proof and Quadrilaterals

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.2.3** Assess the validity of a logical argument and give counterexamples to disprove a statement.

**9.3.2.4** Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations.

**9.3.3.6** Know and apply properties of congruent and similar figures to solve problems and logically justify results.

**9.3.3.7** Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.

**9.3.4.4** Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| ● What can triangles tell me about parallelograms, kites, rhombi, rectangles, and squares?<br>● What is special about Trapezoids?<br>● How do I identify specific quadrilaterals and apply their properties?<br>● What special properties should I look for in each shape? | 7.2 – I can write an effective proof using quadrilaterals and triangles.<br>7.3 – I can use coordinate geometry to find midpoints and to justify conclusions about geometric figures. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Coordinate Geometry Desmos Activity
- Whole Group Assessment Evidence:
  - Quadrilateral Headbands and Example Questions
  - Class Traits v.s. Shape Traits and Survey

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Quadrilateral Headbands:** These headbands would be made specifically for the quadrilateral unit, so they all will be part of the quadrilateral family. They need to put it on and walk around the room asking questions to try to figure out what shape they are. They can ask each person one question before moving on. This gets students discussing different characteristics of the various quadrilaterals and working on asking questions. When they are being asked a question they are also being challenged to analyze the shape looking at the picture. Have example questions or sentence structures on the board to help students who may be struggling to come up with a question. This activity should take 5 minutes.
- Hybrid Learning:
  - **Coordinate Geometry Desmos Activity:** Have students work in mixed groups of three, meaning kids at home mixed with kids in the classroom. They will be given a link to a desmos activity that has them investigating quadrilaterals on a coordinate grid (Desmos, 2021). Students will be looking to identify shapes based on things like parallel or perpendicular lines or segment length. Have groups assign new roles based on whose birthday is closest to today's date. The Closest birthday is Task manager that will share their screen and type into the activity. The next closest birthday will be the Facilitator who will read the questions and keep the group on task. The last person will be the Recorder/Reporter who will need to check in with the teacher halfway through and ask any questions their group may have. This should take 15 minutes total.

- Distance Learning:
    - **Class Traits v.s. Shape traits:** After taking a short survey the night before or as warm up for the day, students will fill in a chart to show various class traits. Then have students brainstorm a list of traits shapes could have. Share out with the whole class. After the brainstorm the class will fill in a shape trait chart discussing why different shapes should go in each section. This should take 15-20 minutes total.

**Materials needed:**

- Quadrilateral Headbands: The headbands students will wear and helpful tips slide (optional)
- Coordinate Geometry Desmos Activity: Desmos link
- Class Traits v.s. Shape Traits: Survey of class traits and chart

**Sources:**

Desmos classroom activities. https://teacher.desmos.com/custom.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

# Guide 7 - Polygons

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.1.2** Compose and decompose two- dimensional figures; use decomposition to determine the perimeter and area of various figures.

**9.3.3.7** Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.

**9.3.3.4** Apply the Pythagorean Theorem and its converse to solve problems and logically justify results

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| ● What is the relationship between the number of sides in a polygon and the sum of the interior angles?<br>● What is the relationship between the sum of the exterior angles in polygons?<br>● How can I use triangles to help me find the area of regular polygons? | 8.1 – I can apply properties of polygons to answer questions regarding interior angles, exterior angles, and areas.<br>8.2 – I can use similarity properties to determine areas for similar figures. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Polygon Toothpicks ("Ideas For Teaching Your Students About Polygons", 2019)
  - Red Light - Green Light Polygon Game (Gaglione, 2000)

- Whole Group Assessment Evidence:
  - Notice and Wonder - Polygons (Newell & Orton, 2018)

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Polygon Toothpicks:** Adapted from an online resource called Top Notch Teaching that provided the idea of this activity and some of the guided questions ("Ideas For Teaching Your Students About Polygons", 2019). Pair the Task Manager with the Recorder/Reporter and the Facilitator with the Resource Manager. Give every group eight toothpicks and have them see how many polygons they can make. One person needs to sketch their drawing on a notebook paper. Challenge students to name the shape and decide if the shape is regular or irregular. This should take 5-10 minutes.
- Hybrid Learning:
  - **Red Light - Green Light Polygon Game:** This activity was adapted from a Relay Review activity by Gaglione (2000). In groups of two to three, give students a link to a problem. They will discuss the problem with their partner or team and come up with an answer on their shared jamboard. When they have their answer circled they will either put out their red cup (students in the room) or hit the ask for help button (students online). This will signal that the teacher will come over to check their work before they can continue. The teacher will randomly call on one student in the group to share what their team did. Once the answer is checked they will get the link, or the "green light", to move to the next problem. Pair students in mixed ability groups and make sure they are aware that any of them could be called on to explain. The problems and length of the activity can vary here drastically depending on what the students in the class need more practice with.
- Distance Learning:
  - **Notice and Wonder - Polygons:** Have students take 2 minutes to write down what they notice or wonder about different polygons on the board. Encourage them to use vocabulary words and express their ideas in full sentences (Newell & Orton, 2018). Do a whip around share with students unmuting and sharing their ideas, if

students are unable to unmute have them type in the chat. This is a low stakes activity that allows all student's voices to be heard. This should take 5 minutes total.

**Materials needed:**

- Polygon Toothpicks: Toothpicks and paper
- Red Light - Green Light Polygon Game: Polygon problems, Jamboard, and red cups
- Notice and Wonder - Polygons: Polygon images

**Sources:**


Gaglione, J. (2000). RELAY REVIEW. The Mathematics Teacher, 93(4), 282-283. Retrieved August 5, 2021, from http://www.jstor.org/stable/27971382

Ideas For Teaching Your Students About Polygons. Top Notch Teaching. (2019, December 26). https://topnotchteaching.com/lesson-ideas/polygons/.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

Newell, C., & Orton, C. (2018). Classroom Routines: An Invitation for Discourse. Teaching Children Mathematics, 25(2), 94-102. Retrieved March 10, 2021, from https://www.jstor.org/stable/10.5951/teacchilmath.25.2.0094

# Guide 8 - Surface Area

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.1.1** Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.

**9.3.1.2** Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures.

**9.3.1.4** Understand and apply the fact that the effect of a scale factor k on length, area, and volume is to multiply each by $k$, $k^2$ and $k^3$, respectively.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| • How can we find the area of common 2D shapes?<br>• How are the faces of a shape related to the total surface area of the shape?<br>• Are there connections between faces of a shape and the equation for surface area? | 2.2: I can calculate the area of triangles, parallelograms, and trapezoids and use these areas to find missing information.<br>9.1 – I can calculate the surface area and volume of 3-D objects including prisms and cylinders.<br>11.1 – I can calculate the surface area and volume of 3-D solids including pyramids, cones, and spheres. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Wrapping Paper Game
  - Surface Area Team Quizizz
- Whole Group Assessment Evidence:
  - Nets Matching Game

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Wrapping Paper Game:** Hand out a different size box to every group. Tell them to think about having to cover the whole box with wrapping paper. Give them 5 minutes with their box and a ruler to try to determine how much wrapping paper they will need. This is a challenge between the groups. Whichever group gets closest to the correct amount will win a prize. During this time students should be discussing ideas and talking with their teammates to come up with the best strategy. After the 5 minutes is up take the boxes away. Give students wrapping paper and tell them to cut the amount they think will cover their box. When they are done, have them check their work and see if they were successful. Discuss different strategies the teams used and why some were closer than others. This should take 15 minutes total.
- Hybrid Learning:
  - **Nets Matching Game:** Give out virtual sets of cards. Some of the cards will have 3D shapes and the rest will have nets. Ask students to match each net with the correct shape. After 5 minutes of individual work time, group them and have them discuss the strategies they used to match them. Set a timer and have them record the time that they finished. This helps add to the game aspect but also reminds them that the most important aspect is making connections between the shape and the net (Bright, Harvey, & Wheeler, 1985). Then have students share different strategies with the whole class. This should take 10 minutes total.
- Distance Learning:
  - **Surface Area Team Quizizz:** In breakout rooms break students up into teams of three students. Students will have a link to a live quizizz that has them practicing surface area for all kinds of shapes and discussing their thought process with other students (Quizizz, 2021). Have them decide their own roles. The Resource Manager

will open the link to the quizizz and share their screen. The Task Manager will read the questions. The Facilitator will ask for help if needed. This should take 10 minutes total.

**Materials needed:**

- Wrapping Paper Game: Wrapping paper, some boxes, scissors, and tape
- Nets Matching Game: Virtual cards with image of 3D shape and cards with corresponding nets
- Surface Area Team Quizizz: Link for Quizizz

**Sources:**

Bright, G., Harvey, J., & Wheeler, M. (1985). Learning and Mathematics Games. Journal for Research in Mathematics Education. Monograph, 1, I-189. doi:10.2307/749987

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

Quizizz. (2021). https://quizizz.com/admin/quiz/60f21e4e3095a4001b72895f/surface-area-team-quiz.

# Guide 9 - Volume

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.1.1** Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.

**9.3.1.2** Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures.

**9.3.1.4** Understand and apply the fact that the effect of a scale factor k on length, area, and volume is to multiply each by $k$, $k^2$ and $k^3$, respectively.

| **Essential Question(s):** | **Objective(s):** |
|---|---|
| ● How is volume different from surface area?<br>● Why does the base of the shape affect the total volume? | 9.1 – I can calculate the surface area and volume of 3-D objects including prisms and cylinders.<br>11.1 – I can calculate the surface area and volume of 3-D solids including pyramids, cones, and spheres. |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Real World Shapes
- Whole Group Assessment Evidence:
  - Water Pouring Activity with Videos (LameEStorage, 2012)
  - Volume Blooket

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Water Pouring Activity with Videos:** Have two student volunteers come up to the front of the room. Up front have a pyramid and a rectangular prism with the same base and same height. Tell the class we will be filling the pyramid with water and pouring it into the prism. Ask them to predict how many pyramids it will take to fill the prism. Then have your two volunteers do the experiment. Have students write down their notice and wonderings. After the demonstration, students discuss how they can alter the volume formula for a prism to create a new formula for the pyramid. This should take about 10 minutes, if you do not have access to the materials, instead play the videos linked that show the demonstration.
- Hybrid Learning:
  - **Real World Shapes:** In their small groups give students 5 minutes to list as many examples of each of the main shapes they have been discussing which includes prisms, cylinders, pyramids, cones, and spheres. Have students share what real world examples they have. Keep track and make a class board to put up in the room and post a picture in the google classroom.
- Distance Learning:
  - **Volume Blooket:** Have students play a warm up game of blooket that has them practice a mixed assortment of questions involving volume (Blooket, 2021). Depending on the students in the room the game mode can be switched to better meet their interests. This is a fun low stakes way to get students practicing and interacting with their peers virtually. It should take around 5-7 minutes.

**Materials needed:**

- Water Pouring Activity: Pyramid and rectangular prism with same base and height or link to the videos
- Real World Shapes: Chart for them to create list on
- Volume Blooket: Link to blooket game

**Sources:**

Blooket. (n.d.). https://www.blooket.com/.

LameEStorage. (2012, May 17). Volume of Three Square Pyramids Fitting into a Cube. YouTube. https://www.youtube.com/watch?v=OUDjY6vJ8pw.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

# Guide 10 - Circles

## Stage 1 - Desired Results

**MN State Standards:**

**9.3.3.8** Know and apply properties of a circle to solve problems and logically justify results.

**9.3.4.5** Know the equation for the graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$ and justify this equation using the Pythagorean Theorem and properties of translations.

**9.3.2.2** Accurately interpret and use words and phrases such as "if…then," "if and only if," "all," and "not." Recognize the logical relationships between an "if…then" statement and its inverse, converse and contrapositive.

| Essential Question(s): | Objective(s): |
|---|---|
| ● How are the parts of a circle similar and different?<br>● How are central angles, chords, and arcs related to each other?<br>● How are inscribed angles related to their intercepted arcs?<br>● How are chord, tangent, and secant segments related?<br>● What do the various parts of a circle correspond to in the equation? | 8.3/10.1 – I can find area, circumference, arc length, and sector area in circles.<br>10.1 - I can identify and solve problems involving chords, tangents, secants, and arcs of circles.<br>12.1 – I can write the equation of a circle and use equations to answer questions about a circle on a grid |

## Stage 2- Assessment Evidence

- Small Group Assessment Evidence:
  - Circles on Graphs
  - Area of Sectors Pizza
- Whole Group Assessment Evidence:
  - How Do You See it - Circles

## Stage 3 - Discourse Activities with Learning Model

- In- Person learning:
  - **Circles on Graphs:** Break students into groups. Give each of the groups four circles graphed with their equations. Ask students in their groups to answer questions and discuss how the equation connects to the graph. The task manager is responsible for highlighting or circling key features on each graph. The Resource Manager will get the graph and the worksheet. They will also pull up Desmos to help answer questions (Desmos, 2021). The Facilitator will write the answers to the questions down. The Recorder/Reporter will share their group's findings to the class. This should take 10-15 minutes.
- Hybrid Learning:
  - **Area of Sectors Pizza:** In this activity students are divided into groups of three. They are asked to answer the question, which yields more pizza, doubling the radius or doubling the angle? This activity should be done before sector area and angles are taught. This could be a fun introduction to the unit. Tell students your job for the day is not to teach but to listen. They should take the lead in their groups. This will be facilitated over a jamboard and google meet with students in the room grouped with students at home. This should take up to 30 minutes.
- Distance Learning:
  - **How Do You See it - Circles:** Give students various circle questions where they are solving for the missing arc or angle. This could be central angles, inscribed angles, chords, semicircles, etc. When they have an answer, have them use the raise hand emoji on google meets. Ask for an answer explosion in the chat where every student types in their answer and on the count of three they hit enter. When they do this you will see answers flood in. Then ask for volunteers to share out how they got their answer. Questions here need to have multiple strategies that could be used. This should take 5 minutes.

**Materials needed:**

- Circles on Graphs: Graphs with circles on them and worksheet
- Area of Sectors Pizza: Paper, pencils, online geometry jamboard to interact with people at home, and protractor.
- How do You See it - Circles: Various Circle Questions

**Sources:**

Desmos Graphing Calculator. https://www.desmos.com/calculator.

Minnesota Dept. of Education. (2008). Minnesota Academic Standards: Mathematics K-12.

# REFERENCES

Bowen, R. (2017). *Understanding by Design*. Vanderbilt University Center for Teaching. https://cft.vanderbilt.edu/guides-sub-pages/understanding-by-design/.

Minnesota Dept. of Education. (2008). *Minnesota Academic Standards: Mathematics K-12*.

Wiggins, G., & McTighe, J. (1998). *Understanding by Design.* Association for Supervision and Curriculum Development