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Using Group Work in Mathematics to Introduce Social Justice Concepts: Curriculum to Help Students Learn About Each Other Through Mathematics

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USING GROUP WORK IN MATHEMATICS TO INTRODUCE SOCIAL JUSTICE CONCEPTS: CURRICULUM TO HELP STUDENTS LEARN ABOUT EACH OTHER THROUGH MATHEMATICS

By

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CHAPTER ONE

Introduction

Summarized Project Goal

My project is a curriculum based around best practices for social justice math education, including discussions about social justice issues like race, class, and gender. This curriculum includes sample activities and lessons for a precalculus level math course. The research question guiding my project is, "What are the best practices for engaging with social justice topics and goals in a modern, constructivism-based mathematics course?"

Introduction and Background

To introduce my project, I will first expand upon my own history with mathematics education and social justice topics. This will help provide some rationale for why I'm interested in creating a curriculum that is themed around these issues.

My Time in Mathematics Education

For the extent of my K-12 education, mathematics was presented to me as a marker of individual success. One of my earliest school memories is of competing against a classmate to see who could finish a worksheet of additional problems in the shortest amount of time. I do not remember if I won or lost; instead, I just remember feeling isolated. Math was like this for years: occasionally check your answers with a peer, but largely do the legwork yourself.

When I was a junior studying mathematics at university, I realized that I could not achieve my goals without outside help. At that point in my math career, I had to take the dreaded Analysis I, a course devoted to rigorous (and often tedious) mathematical proofs. For the uninitiated, mathematical proofs are effectively persuasive essays. However, the tools to argue that a mathematical theorem is *exactly* correct are a bit different than the tools one would use in another discipline; us mathematicians are mostly stuck with logos, or logic. To make matters worse, my regular professor, the one who led me through Calculus III, Linear Algebra, Modern Geometry, and was ostensibly going to lead me through Analysis, had to take a medical leave. The adjunct professor who took his place did a serviceable job, but the connection just wasn't there for me. I started to feel more isolated than ever before. The material kept getting harder and harder and, soon enough, I found that I was failing the class.

Luckily for me, the math department at my school had a plan for situations like this. The Quantitative Research Center (QRC) was a place for math students to work together on difficult problems, tutor underclassmen, and get to know each other better. I had rarely used the room during my first two years at Hamline. That changed during the fall of my junior year. The QRC helped me engage with the material more deeply while also helping me form new connections with my peers. Ultimately, I was able to pass the class. More importantly, though, I walked away with a new understanding of the important facet of education: we cannot make it through without support.

Speaking with other math teachers about their experience in college, most have confided that they “made it through” college mathematics the same way. After a lifetime of thinking about mathematics as a metric of individual success, we had to turn to collective understanding to succeed. When we came together, we were able to become more than just the sum of our individual strengths and weaknesses. With enough teamwork, it really felt like we could accomplish anything.

My Journey in Social Justice

Simultaneous to my mathematical journey was my journey as a white, straight, cisgender man in the United States. Growing up in a rural and mostly racially homogenous state in the upper Northwest part of the United States, I was not really exposed to diversity in any meaningful way in the classroom. Instead, my understanding of the experience of people who did not look like me largely came from the internet. And while the internet did begin the process of teaching me about my own social position, especially on social media sites where I was able to hear from people who did not look like me, it was ultimately always just theoretical.

This changed when I enrolled at a college in a more urban environment for my undergraduate education. Suddenly I was surrounded by people who did not look like me. My own K12 education had failed to teach me any meaningful things about how racism, sexism, and classism continued to the present day; instead, I was presented with the view that racism ended with desegregation, gender inequality ended with the 19th amendment, current class inequality was the result of people not working hard enough. Even the social media sites that I started using in my later high school years could not fully erase and replace the lessons that the educational system had taught me.

To learn the truth about these topics, I had to actually witness them first-hand. I learned the truth about the current state of racism by seeing it happen to my peers. I learned the truth about modern gender inequality by discussing experiences with women and nonbinary people in college courses and clubs. I learned the truth about the current class divide by taking to the streets alongside protest movements like Black Lives Matter and the Fight for 15. It was never an individual path towards education that created meaningful change for me; instead, the pursuit of knowledge always showed itself as collectivist.

Project Goal

From the conversations that I have had with my colleagues, I am left with the impression that many teachers of mathematics experienced a shift from individual to collective understanding in college. With this in mind, why is mathematics so often considered a solitary science? Why is it taught in ways that emphasize individual success and achievement instead of collaborative success? More importantly, how can a shift towards collective understanding at an earlier level than college impact student understanding and achievement in math courses?

Additionally, I have always been told that mathematics is a “race neutral” field. Yet I worry that applying a race/class/gender-neutral approach to this topic will result in a curriculum that does not adequately serve all of my diverse students. Therefore, I’m looking to incorporate techniques that address race/class/gender distinctions in the classroom. I am also going to incorporate student voice into the curriculum that allows students to tackle these social justice issues with each other.

My goal with this project is to create a sample curriculum that addresses these two concerns. This curriculum will be built upon the basics of constructivism and lean heavily into social justice topics and group work, utilizing best practices that I find over the course of my research. In doing so, I hope to create materials that are useful for teaching mathematics to the diverse student body that makes up my classroom.

Project Rationale

In my experience as both a teacher and a student, I have noticed that the individualization of mathematics negatively impacts student experiences in math classes. In many classes, the main expectation for students is to sit, listen to a lecture, and take notes. After a few teacher-led examples, they are left to their own devices to do related homework problems. At this point, students who are adept at this learning style complete the problems with minimal difficulty, while students who don't catch on to the idea presented that day struggle. These students walk away with the impression that they are not "good" at math. Some may reach out to others for help, but many don't for fear of coming off as unintelligent or lazy. This was exactly the type of problem that I faced in my Analysis class. Luckily for me, I had access to a group of students who would not consider me unintelligent or lazy. Many students do not have the resources to access the same level of extracurricular support.

Similarly, I have noticed that students who are not white tend to have more difficulty excelling in mathematics courses. While mathematics could be thought of as a "race-neutral" field (Shah, 2017), in that it deals with a concept that could be separated from the larger systems of oppression in racism, I would argue that a lot of math is simply coded as "white." My math department now has a saying: "A naked problem in mathematics [a problem with just numbers and equations] reads as white."

The school where I learned how to become a math teacher featured a wealth of papers, perspectives, and examples of the *constructivist* model of education. Put simply, constructivism acknowledges that students learn best when they themselves are the ones making the meaning of a topic (Zain et. al., 2012). In a classroom that values the constructivist approach to education, students are spending class time working through problems and discussing them with their peers, rather than sitting and passively listening to a lecture. My college years provided me with many examples of ways to achieve this in a math classroom. One of the clearest ways to implement this in a math classroom is through the use of challenging and interesting problems that require students to think of multiple methods for solving them. Problems like this often have multiple entry points coupled with a more straightforward solution. Students then have more things to discuss with each other, which strengthens everyone's overall understanding.

In modern, non-homogeneous classrooms, selecting problems that are appropriately interesting and challenging for all students is difficult. Theoretically, having a classroom structure that emphasizes debate, problem solving, and collaboration can help alleviate this issue (Byrne & Prendeville, 2019). Students who are more naturally interested in a given problem can take the lead with the understanding that they won't be forced into that position forever. Meanwhile, students who have gaps in their previous mathematical knowledge can fill them in with their peers. In any case, it is clear that communication is key to achieving these goals (Chval & Pinnow, 2018). No student is an island.

What is less clear, though, are the best means for fostering communication about math in the classroom. Though an emphasis on challenging and interesting problems can help boost student engagement, the actual strategies for fostering communication at the level I am suggesting here are not totally defined. I am interested in looking into the realm of ESL (English as a Second Language), English, and Social Studies classrooms to see how they handle discussions. Once I have seen some examples from these other disciplines, I would like to compare their structures to existing research on math communication in the classroom. Then, over the next semester, I would like to systematically introduce these strategies in my classroom via my unit planning and track how student understanding (and, hopefully, achievement) changes.

Summary

Based on my experience in the world of mathematics education, the current model of individualistic expression and expectations is leaving students (and especially students of color) in the dust. I think that a shift towards communication-based math education is necessary to boost student understanding of mathematical concepts. Similarly, my experience as a cis, straight, white male gradually learning that many people experience the world differently has had a major impact on my educational goals. I think a shift towards social justice themed education can help all students feel at home in the class. Throughout this project, I intend to look at research inside and outside of my discipline, meld some best practices together, and see how this impacts student achievement. With this research in mind, I will create a sample curriculum that demonstrates how to meld these two concerns in a modern mathematics classroom.

In the next chapter, I will undertake a literature review to explore my research question, "What are the best practices for engaging with social justice topics and goals in a modern, constructivism-based mathematics course?"

CHAPTER TWO

Literature Review

Introduction

In order to tackle the question of "What are the best practices for engaging with social justice topics and goals in a modern, constructivism-based mathematics course?," the first section of this chapter details the constructivist theory of education. This is the foundational theory that my curriculum guide will be based on. Constructivism largely looks at the ways that people create knowledge, both through the interpretation of external stimuli and the preconceived notions that we carry around. As Canipe (2016) succinctly introduces it in the SAGE encyclopedia, "If a learner is engaged with the speaker, then he or she is actively trying to make sense out of what is being said, in other words, making meaning by connecting what is known to new thoughts being presented" (p. 248). I have included some history of constructivism as a theory alongside examples of it being applied in the classroom environment.

If constructivism is the foundation of my curriculum guide, then social justice is essentially the floorplan. In general, Social Justice is a field of study that analyzes any area of power differentials in society, alongside the effects of the oppression that result from said power differentials (King, 2017). It can also be applied to larger structural issues or smaller interpersonal situations. I explore some of the history of social justice as a whole, then break it down into three primary subtopics: racial justice, gender justice, and class justice. After some discussion of these three topics, I look at intersectional examples of social justice work in the classroom.

The aesthetics of my curriculum guide will be the mode of pedagogical delivery: group work. While this may seem like a general term, I think it's important to define it as any work that specifically involves two or more students. I will lay out the case for group work as an important component of the modern mathematics classroom, then look over examples of group work. Some of these examples will involve constructivist techniques and social justice themes, tying up the chapter nicely.

Radical Constructivism

Early Development of Constructivism

The theory of Constructivism developed in two factions: the Jean Piaget-led cognitive constructivism and the Lev Vygotsky-led social constructivism (Canipe, 2016). While they have since both branched off into further subdivisions, it is useful to explore these basic fields first to gain an understanding of the core tenets of constructivism.

Piaget's cognitive constructivism developed over the course of many years. Piaget, a psychologist, first attempted to categorize adolescent development into four stages: the sensorimotor stage, the preoperational stage, the concrete operational stage, and the formal operations stage (Canipe, 2016). Each stage was defined by the tasks that children were able to complete within it—for instance, kids in the sensorimotor stage are able to conceptualize the world only through what is immediately apparent, usually via their touch. Later, they learn that things still exist when they do not see them, conquering the idea of object permanence. Eventually they learn to use symbols to represent logic, which brings them into Piaget's preoperational stage. The stages continue from here, with kids advancing as they gain the ability to think and conceive of things in more complicated ways (van Geert, 1998). This early work of Piaget's is a useful contribution to constructivism, as it gives a framework for understanding how someone at a different age will react to specific stimuli.

Around the same time, Vygotsky applied similar ideas directly to the classroom environment (Canipe, 2016). Vygotsky's principles can be summed up by the idea that learning is inherently a collaborative experience because learning is scaffolded to students over time, both inside and outside of school. Since learning is social, it's not restricted to just a one-way relation between teacher and student; instead, students are both learning from and teaching their peers, parents, teachers, and neighbors. Vygotsky developed a model for learning called the "Zone of Proximal Development," which, like Piaget's stages of adolescent development, breaks tasks down into categories that kids can achieve.

For context, one of the other leading theories at the time was Behaviorism, broadly defined as the study of how the environment impacts a person's actions (Fernand, 2008). It relied heavily on the idea that external stimuli produce predictable results in human subjects. Early behaviorism focused on "classical conditioning," which is best known through Pavlov's work with dogs. He found that a subject could be trained to have a specific reaction (drooling) to a specific stimuli (bell ringing). Later, B. F. Skinner developed the concept of operant conditioning, which, while more complicated in practice (it examines longer-term reactions to stimuli, including reactions that may seem voluntary later), still comes down to the basic concept of an environmental stimuli causing a specific reaction. Unlike constructivism, behaviorism does not take into account a child's internal development stage. While later forms of behaviorism would end up considering a child's psychology, these internal drives were never considered to be foundational in student's behavior, but instead just another form of behavior influenced by prior exterior forces (Yi & Lejuez, 2017).

Radical Constructivism Emerges

Then, the term "radical constructivism" was coined by Charles Smock and Ernst Von Glasersfeld in their 1974 paper "The implications of radical constructivism for knowledge acquisition" (Von Glasersfeld, 1995). Despite the title, the concepts of radical constructivism are not necessarily radical to anyone who has stepped into a classroom in the past several decades. Von Glasersfeld simplifies the theory into "the assumption that knowledge, … is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience" (p. 1). An immediate application in the field of education might jump to mind: this means that students can only learn through their own lens, which is formed by their own history and experiences.

Von Glasersfeld (1995) originally developed this theory along two primary axioms:

- "Knowledge is not passively received but built up by the cognizing subject.
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality." (p. 18)

The first of these axioms supports the familiar application of constructivism. Since everyone is socialized by a unique set of factors, every person will interpret new knowledge through their own lens. However, the second axiom is denser, and might seem less applicable to the classroom environment. Von Glasersfeld includes this axiom as a gesture to the philosophical world. When he says the function of cognition is adaptive, he is simply suggesting that thinking is not rigid. We do not objectively take in and logically sort all of the information that we see in the world around us. As external stimuli is encountered, it is filtered through our preconceived notions, interpreted by way of our biases, and placed into a unique position in our larger understanding of the world. Therefore, we do not achieve a fixed, objective view of the world, but a constantly changing subjective one.

As Von Glasersfeld was developing these axioms, he was working alongside other educational psychologists and philosophers. Early study of radical constructivism in the classroom began with Leslie Steffe, who conducted interviews with children attempting math problems (Steffe & Johnson, 1971). Unlike other forms of research at the time, the process was non-standardized; researchers had to react to students' explanations of their thinking in real-time, asking appropriate questions to further gauge student understanding. These interviews were recorded and analyzed by the researchers later. This style of research should be familiar to any modern classroom teacher. Teachers are constantly adjusting questions for students on the fly, adapting their wording and sequencing to best bridge the gap between where the student is at and where the teacher wants them to be. In this way, the constructivist methodology of research reflects the current reality of mathematics classroom teaching (Sofroniou & Poutos, 2016). It's worth noting that this implicit understanding of constructivist concepts in the classroom may be the result of the larger acceptance of constructivist thought in the field of education.

Von Glasersfeld's (1995) research and thinking led to the suggestion of a few best practices for applying constructivist education. He argues that traditional forms of schooling emphasize the acquisition of knowledge as the passing down of objective facts from generation-to-generation. In this way, knowledge is constructed as impartial. Students are not allowed to ask questions like "Why is this important?" as the importance is already rubber-stamped by the fact of it being taught. Under a constructivist school of thought, Von Glasersfeld argues that students can interrogate *why* something matters, which also allows them to have a deeper understanding of material. Instead of learning discrete pieces of information that they will apply in a test and then promptly forget, they can connect their learning to their larger understanding of the world (Von Glasersfeld, 1995).

One of key elements of constructivist education that Von Glasersfeld (1995) suggests is a shift from *training* to *teaching*. The primary models of education, both at the time when he wrote his book and currently, are based around drilling techniques into students' heads. Von Glasersfeld (1995) believes that the emphasis must be placed on teaching students to think critically, ask questions of the material, and place it all into students' larger conceptions of the world. He suggests that teachers emphasize "fun" problem solving, so as to get students more interested in asking these questions. He wants teachers to help students solve problems for themselves, rather than teach them the methods beforehand. This process of discovery must be paired with an equally rigorous process of student reflection, where students can reinterpret their findings in the context of their other knowledge. Part of this reflection should include some type of social interaction, where students can share their insights with other students. All of these strategies are in the service of achieving a deeper understanding of the material.

Radical Constructivism Grows

Over time, Von Glasersfeld's axioms were adapted to fit new modes of education. Steffe and Thompson's 2000 anthology *Radical Constructivism in Action: Building on the Pioneering Work of Ernst von Glasersfeld* does exactly what the title suggests, expanding the original concept. In that book, Jan van den Brink (2000) connects constructivism to geometry, arguing that geometric concepts are easier to understand when connected to real-life examples. He looks at spherical geometry as an example. Airplanes do not fly directly between two locations, as seen on a two-dimensional map; instead, they follow the curve of the Earth. This scenario gives students a chance to connect their own conceptions of the world to mathematics—How has the mathematics they've been taught thus far failed to account for this? What other situations may mirror this property of air travel? Importantly, he also outlines a method for teaching students definitions in geometry: Rather than teach a singular definition at the beginning of the lesson, create a class definition from student input. This way, students can compare and contrast their personal perspectives, creating a definition that more intuitively fits into their own understanding.

In the same book, Paul Cobb (2000) discusses the importance of a rigorous consideration of social context in the mathematics classroom. The bulk of his section is devoted to the idea that the social norms of the classroom itself can intersect with and influence the norms that students are bringing with them into the classroom. For instance, clearly laying out and encouraging classroom norms like requiring students to both explain their own reasoning and listen to other students' reasoning helped change students' overall conception of how mathematics works. In the case of one elementary school classroom where these norms were established, students saw growth in understanding of basic arithmetic concepts like counting and addition. Interestingly, these students also had broader conceptions of what counted as an acceptable solution—instead of only recognizing one form of counting as correct, they recognized multiple methods as being valid.

Further Developments and Applications

In more recent years, other mathematical studies have leaned heavily on constructivist foundations when suggesting rich activities and best practices for the classroom. In Hord and Yinn's 2015 article "Teaching Area and Volume to Students with Mild Intellectual Disability," constructivist strategies are applied to help students with mild intellectual disabilities gain deeper conceptual and procedural knowledge of area problems. This is primarily done through the use of CSA (concrete−semiconcrete−abstract) and COMPS (conceptual model-based problem solving) strategies. In simple terms, CSA strategies build understanding by moving from physical objects (the concrete stage) to representational drawings (the semiconcrete stage) to internalized knowledge (the abstract stage), while COMPS strategies introduce mathematical concepts through real world equivalents (Hord & Yinn, 2015). While the emphasis on teaching these strategies may seem to be more like training than teaching, the fact that they are strategies that encourage students to explain their reasoning, and not simply jump straight to the answer, implies a constructivist edge. Further, these strategies are effective in helping students find the correct answer, and they also helped at least one student feel more positively about their ability to solve mathematical problems.

In Begolli and Richland's (2016) "Teaching Mathematics by Comparison: Analog Visibility as a Double-Edged Sword," constructivism rears its head by way of students discussing multiple methods for solving problems. The "double-edged sword" of the article title refers to the positive and negative results that come from the discussion of misconceptions in method. The researchers suggest three primary methods of breaking down misconceptions with students: problems and work were "a) presented only orally, b) visible sequentially in the order they were described, or 3) all solutions were visible after being described throughout the discussion" (Begolli & Richland, 2016, p. 1). In all cases, the spoken portion of the problem was the same. The study found that students gained deeper conceptual understanding with option 3, perhaps because they were able to continually compare different methods of solving problems throughout the entire process. Option 2, which presented problems sequentially but did not allow for students to see the entire slate of methods at once, actually seemed to hurt students' understanding.

Summary

At this point, constructivism has been making waves in the education field for several decades. It is especially beneficial in mathematics, where it represents a break from the traditional "drill-and-kill" model. While students under that model typically complete repetitive tasks that only ask them to apply surface level math techniques, the constructivist model opens up the potential for students to tackle more complicated problems. Now that the foundation of my sample curriculum has been placed, we can move on to the floorplan: Social Justice.

Social Justice

Social Justice is a difficult term to define, partially due to its long-standing and shifting nature. An extremely basic reading of the term would imply any system of justice that exists in the social sphere. Cheryl Simrell King (2017) traces the history of the term to attempt to define it. While the term first appeared in Italy in the 1840s, it didn't receive a proper definition until 1863, when John Stuart Mill described it as "a standard of equality toward which all institutions and virtuous citizens should treat people," as cited by King (2017, p. 1578). Some say that the concept of social justice has existed for far longer: Elliott Dorff and Danya Ruttenberg's (2009) *Jewish Choice, Jewish Voices: Social Justice*, argues that the tenets of social justice are established in the Torah. Similarly, Christianity and Islam both lay claim to similar arguments (Twietmeyer et al., 2019; Cifti, 2019). In general, it may be best to define social justice as encompassing any ideological movement that focuses on solving disparities in societal power dynamics.

Naturally, this means that social justice can take many forms, as power differentials are common in many of the social layers of society. This level of applicability may be useful for classroom applications; issues of race, class, and gender cut a wide swath of the study body. While it's often useful to lump these different groups together into a single term (social justice), it's worth exploring them individually, too, to see how and where they show up in the classroom environment, both in the power dynamics between us and our students and in the interventions we take to lessen them.

Race

Perhaps the most omnipotent identity factor in the United States is race. Discussion about race in the classroom often begins with the topic of segregation. Segregation, the process by which Black Americans were excluded from access to, among other things, the schools that white Americans attended, was ostensibly overturned by Brown v. The Board of Education in 1954; however, schools today remain widely segregated (Rosiek 2019), often due to zoning restrictions and white flight to suburban neighborhoods. The schools that aren't as segregated often have disparities in student achievement along racial lines (Yeung & Conley, 2008).

Much has been written on how to fix the achievement gap between Black and white American students. Robert P. Moses, a notable figure in the Civil Rights Movement of the 1960s, turned his attention towards math education in the later part of his life. His book *Radical Equations* (2001) spends equal amounts of time reminiscing on the Civil Rights Movement and applying its lessons to the specter of math education. As he describes in the book, Moses noticed that Black students in the South were not succeeding in math classes, which are also notoriously known as the classes that gatekeep college attendance. If Black students were given the tools to succeed in algebra, he reasoned, they would also be able to later succeed in the tests that gatekeep college admission. He developed an organization called the Algebra Project to get students learning algebra in middle school. One of the classroom suggestions that the Algebra Project makes to better assist Black students in the study of algebra is the "elimination of ability grouping" (Moses, 2001, p. 99).

Ability grouping, also known as tracking, is the process by which students are placed by perceived mathematical ability into different classes, usually in the form of an advanced class and a remedial class (Boliver & Capsada-Munsech, 2021). Students in the lower-level math classes are less likely than their peers in the higher-level math classes to enjoy the subject, which leads to lower engagement in the class and less subsequent interest in mathematics-related topics (Boliver & Capsada-Munsech, 2021). Moses (2001) noticed that, in American schools in the 1980s, this grouping often occurred along racial lines, leading to achievement gaps between white students and students of color. Switching to individual and small-group instruction helped lessen this problem (Moses, 2001).

While most of this literature review is focused on examples of overcoming inequality in classrooms in the United States, it is useful to look for ways that inequality has been addressed internationally. In South Africa, Nkopodi Nkopodi and Mogege Mosimege (2009) attempted to increase Black South African achievement by introducing morabaraba, a game that is part of the indigenous tradition in South Africa, into the classroom. Importantly, this game not only provides students with something familiar to learn from, but is also mathematically complex enough that it allows for a lot of discussion between students. Its effectiveness suggests that the usage of familiar cultural activities in the classroom can be a beneficial tool for student engagement and learning.

Gender

Aside from race, gender inequality in the classroom is arguably the biggest differentiating factor for students. Politically and socially, women in the United States have not always had the same access to power that men have. Similarly, women have not always had the same access to education that men have had (Eisenmann, 1998). Despite this, wide-ranging studies like "Gender Inequality in Education" by Claudia Buchmann et. al. (2008) have shown that women generally achieve higher academic success in school than men from similar backgrounds. This is not true in mathematics, where stereotypes about women being less capable remain rampant to this day (Hall & Suurtamm, 2018; Solomon, Lawson, & Croft, 2011).

Claude Steele's 2010 book *Whistling Vivaldi* offers up some solutions to the problem of gender inequality in the classroom. Steele is well-known as being the pioneer of the concept of "stereotype threat," the idea that even the knowledge of negative stereotypes can be enough to negatively impact a person's performance in a given situation. In this case, the idea is that girls who are aware of the stereotype that women are inferior at math will fall prey to the stereotype, performing worse on exams than their male counterparts. Among other things, Steele suggests that teachers emphasize that tests *do not* represent the mental abilities of their students, but instead are meant to check in on students' progress towards goals of understanding. In this way, the classroom is reformulated away from a model where achievement is linked with intelligence into a model where students are allowed to show continual growth.

Class

Another factor that impacts student performance in the classroom is poverty. Students who experience poverty, or are in any sense part of a low-income family, are also marginalized by the school system (Raffo et. al, 2009). While the actual circumstances that cause students in poverty to do poorly in school are varied, the data that shows the connection between poverty and poor performance is strong. For instance, Yeung and Conley found in their 2008 study “Black–White Achievement Gap and Family Wealth” that children from families with liquid assets like stocks and mutual funds attained higher test scores. This seems to be especially true in mathematics, as children from wealthier families scored better on math tests than children from poorer families, even if the wealthier families don’t have liquid assets.

In fact, as it currently stands, education may actually reinforce poverty over the long-term, as educational opportunities are often unfairly attributed to students based on their income accessibility (Connell, 1994). So those students from families without liquid assets tend to do worse on math tests, which can lead to them being placed into lower-level math classes through tracking (Ngo & Velasquez, 2020). This creates a pile-on effect, where students know less mathematics, and then tend to do worse in math classes, which further restricts them from gaining deeper math knowledge. It goes back to what Bob Moses was thinking about when he created the Algebra Project: if students do poorly on math tests, they tend to ultimately be denied access to a college education

(Moses & Cobb, 2001). Without a college education, many high-paying jobs are out-of-reach, and the cycle of poverty continues.

Intersectionality and Applications

Beyond these specific classifications, it's useful to consider applications in the classroom that are *intersectional,* or are involving more than one of these categories. Climate change is an example of an intersectional issue, as Julia Hathaway (2019) establishes in her article "Climate Change, the Intersectional Imperative, and the Opportunity of the Green New Deal." Climate change impacts everyone, but the effects will disproportionately affect marginalized people; while she specifically discusses the impact of climate change on women, she also acknowledges that "gender, combined with ethnicity, age, and socio-economic background, determine to a large extent people's resilience in the face of climate change" (Hathaway, 2019, p. 5). Richard Barwell (2013) argues in "The mathematical formatting of climate change: Critical mathematics education and post-normal science" that climate change is "post-normal science," science where the facts are in dispute (in this case due to political fragmentation), but the issues are urgent and widely encompassing. As such, Barwell claims that studying climate change in the classroom is important. He specifically points out that, since math deals in "describing, predicting, and communicating climate change" (Barwell, 2013, p. 6) scientifically, a math classroom could be an ideal setting to help students build a better understanding of the issue.

Students who are learning English as a second language also fit under the definition of intersectionality. These students come from a variety of different racial, gender, and class groups, and have varying levels of English proficiency. Halliday and Neumann explore the intricacies of creating responsive lessons for this group in their 2012 article "Connecting reading and mathematical strategies." They emphasize a few key strategies: Having students make predictions about what will happen, having students determine what information in a problem is important, and having students make connections between their results and the rest of the world. As these strategies are also being taught to ESL students in their English classes (and, depending on their level of proficiency, their ESL-support class), students are better able to engage with the mathematic material, rather than just the terminology around it (Halliday & Neumann, 2012).

Others have agreed with the assessment that intersectional issues require interdisciplinary responses. In their 2020 text "Mathematical mirrors, windows, and sliding glass doors: Young adult texts as sites for identifying with mathematics," Rezvi et. al. (2020) establish that young adult (YA) literature often displays mathematics in a negative light. YA literature also tends to replicate existing stereotypes of math achievement, such as only portraying white men as high achievers in math classrooms. Rezvi et. al argue that students (and especially students who are otherwise marginalized by their race, gender, or class) use YA literature as a lens to see and understand the world; as such, the portrayal of math as a difficult subject for students can contribute to the normalization of under-achievement in math classes. They propose that this problem can be lessened with the introduction of reading material that positively portrays mathematics and emphasizes that math success can be achieved by anybody. They mention texts such as *Hidden Figures: Young Readers' Edition* by Margot Lee Shetterly (2016) and Talitha Williams's *Power in Numbers: The Rebel Women of Mathematics* (2018) as examples of positive portrayals of non-white people achieving success in mathematics (Rezvi et. al., 2020, p. 594).

Summary

As established above, it is clear that social justice intersects with the interests of math education. Like most areas of education, the effectiveness of any type of social justice lessons in math classrooms will ultimately come down to implementation. In the next subtropic, I will examine group work in mathematics in order to better structure my sample curriculum.

Group Work

How can teachers implement constructivist and social justice strategies into their classrooms? While there may be a range of different pedagogical strategies that teachers could adopt, the effective implementation of group work seems to be one of the more promising ones (Sofroniou & Poutos, 2016; Byrne & Prendeville, 2019). In general, group work can be defined as any class work that involves two or more students working together to complete a task. Group work, applied correctly, is constructivist in nature, and "plays an essential role in students' question acquisition and in criticizing constructively" (Sofroniou & Poutos, 2016, p. 2). Of course, not all forms of group work are designed the same way, and thus the effectiveness depends on the implementation.

Best Practices

Sofroniou and Poutos (2016) collected a wide-range of studies involving group work and noted some positive and negative examples. In general, group work where students were able to have meaningful discussions about the material with their collaborators was beneficial in building their mathematical knowledge; groups with nonresponsive members tended to lead to frustration and no increase in mathematical knowledge. Sofroniou and Poutos (2016) noted that the largest benefits came from situations where group discussions were followed up with a) opportunities to apply their knowledge to a novel situation or b) a large class discussion where the small group conversations were synthesized. Further, these small group discussions ran more smoothly and led to better knowledge gain when students were taught *how* to work in a group together.

Ryve et. al. study the topic of effective small group work in their 2012 article "Analyzing effective communication in mathematics group work: The role of visual mediators and technical terms." They specifically look at how students use two modes of discussion: visual mediators and technical terms. Visual mediators are preexisting "symbolic artifacts such as algebraic expressions, tables, graphs" (Ryve et. al., 2012, p. 500), while technical terms are simply words that have special meanings in a given subject. Visual mediators are interesting because, though they are unaffected by student input in their construction, they can be (and often are) viewed differently from student to student. A visual representation of a sequence of numbers, for instance, can often be interpreted in dozens of ways; this is where the concept of a number talk comes from (Boaler, 2015). Ryve et. al. (2012) find that effective group work often involves students who eventually come to a consensus on a common understanding of a visual mediator, despite perhaps initially having different interpretations of the visual mediator. They also find that students need to have some base-level access to the technical terms of the mathematics at hand to be able to effectively communicate their understanding in a group. However, they do suggest that some activities may be structured in a way that allow students to learn technical terms through small-group exploration followed by full class discussion.

How can these groups achieve effective discussion patterns in the first place? Kathryn Chval and Rachel Pinnow's 2018 article "A Path to Discourse-Rich Communities" suggest a few strategies to implement in the classroom. While they are primarily concerned with applying strategies that are effective for bilingual students in the third grade, these strategies can also prove useful for helping older students learn mathematics. Among other things, they suggest that teachers allow students to have individual work time to explore a concept (and clear up any misconceptions that may arise from unfamiliarity with specific words or cultural artifacts) before they are placed in a situation where they must discuss it with their peers (Chval & Pinnow, 2018). The culture of the classroom is also important; in the classroom that Chval and Pinnow look at, students do not feel silenced because of unfamiliarity with language or terminology. Instead, students are supported and accepted as they are. Discourse-rich communities require this sense of safety from students. Otherwise, discussions will be stilted, awkward, and limited.

Implementing Writing Strategies

Hope Walter's (2018) "Beyond Turn and Talk: Creating Discourse" offers up some practices to deeper discourse in the math classroom. She notes that the turn-and-talk method, where students discuss a problem or concept with their neighbor, is limited in scope; it does not reach true mathematical discourse on its own. Instead, she wants teachers to invest in meaningful discourse, which can "reveal misunderstandings, support learning, deepen meaning, encourage language development, boost memory, and aid in the development of social skills" (Walter, 2018, p. 181). This requires an actual conversation between students, where they are dynamically engaging with what each other is saying, instead of simply passively listening while waiting to say their own opinion. Walter argues that turn-and-talks in isolation do not give students the opportunity to ask deeper questions of each other, instead turning into a simple linear movement of ideas. To remedy this, she suggests providing students with processing and writing time before they enter into a discussion with another student, allowing them to engage with the material individually before they're put on the spot to discuss it with another student. Teachers can further improve their turn-and-talk implementation by giving students sentence-starters, which signals to students what the conversation they have with their peers should sound like.

The idea of allowing students to explore their own understandings through writing has also been explored by Kathleen Kostos and Eui-kyung Shin in their 2010 article "Using math journals to enhance second graders' communication of mathematical thinking." Kostos and Shin had 2nd grade students writing in a math journal three times a week for five weeks, answering conceptual questions about the material they were learning in class. These questions required students to thoroughly explain the process they used to solve a math problem. After the five weeks were up, the students showed higher levels of conceptual understanding of the material on a standardized exam. The journals also served as a useful look for the instructor into student understanding during the 5 weeks. Perhaps most importantly, students also showed an increase in usage of mathematical vocabulary in their writing. If students are completing math journals at this stage, the transition to math-writing with turn-and-talk strategies in higher-level grades will be smoother, too.

Grouping Considerations

On the other end of the spectrum, how can groups be designed to best help students who don’t have the same level of English vocabulary? Miwa Takeuchi’s 2016 study “Friendships and group work in linguistically diverse mathematics classrooms” asks this very question. Takeuchi compares the group work dynamics of ESL students when they are placed in teacher-assigned groups and self-chosen (friend-based) groups. She noted that ESL students in both types of groups are often placed into roles where they request help understanding the material; however, ESL students who are working with friends are more likely to also be asked for help, allowing them access to valuable learning opportunities.

Another example of how group work can be used for intersectional social justice work: Claude Steele argues in *Whistling Vivaldi* that effective group work can be a factor to increase Black student achievement in the classroom (Steele, 2010). Though this has clear social justice implications, the constructivist connections are also interesting. Steele cites an earlier study by Philip Treisman, who found that a shift towards collaborative study groups, where students can compare their understandings of the material with each other, lead to more success within mathematics than students studying on their own. In Treisman's studies, Black students self-reported working primarily independently on their math work outside of class, while Asian students primarily worked in groups. In class, Black students were underperforming compared to Asian students. After intervening with workshops where Black students practiced communal study habits, their performance in the class improved. While Treisman conducted this study with undergraduate students, this type of cultural shift could apply to a K-12 setting, too.

Summary

In general, the literature above suggests that group work allows for students to form stronger connections with the material and with their peers. This works well for the goals of both constructivism (deeper understanding of the material) and social justice (lessening of power differentials by building mutual respect). For these reasons, group work will be an important method of instruction in my sample curriculum.

Summary

Constructivism, social justice, and group work will be the three primary components of the curriculum that I create for this project. The literature review above has explored the reasons why these are the foundational tenets of this curriculum project.

The next chapter will provide a detailed description of the project itself, including a timeline for completion, an introduction to the theoretical framework that will guide the project, and context for both the audience of the project and the school demographics where it will be used.

CHAPTER THREE

Project Description

Project Goals

My research question is “What are the best practices for engaging with social justice topics and goals in a modern, constructivism-based mathematics course?” To answer this question, it is my goal to build a curriculum in the form of a unit for my precalculus course that I can implement next year. Where possible, I am aiming to prioritize the sample lessons with guidelines set out by the International Baccalaureate (IB) math program. In this chapter, I will provide more specifics for exactly how I will be designing this unit, including the ideological basis and philosophies that I will be using and the setting that I will be designing the unit for.

Theoretical Framework

The main ideological theories that will guide my curriculum are constructivism, social justice, and an emphasis on group work. While I go into detail about the importance of all three of these topics in Chapter 2, it is worthwhile for me to summarize why and how they will be used in this curriculum. Constructivism is currently popular in mathematics education, so general best practices have been pushing towards constructivist thinking already (Sofroniou & Poutos, 2016). If I want my curriculum to be in-line with modern standards, it must align with constructivist values. Similarly, there has been a push towards teaching social justice in schools, especially during the resurgence of the Black Lives Matter movement in the middle of 2020 (Holst, 2020). As I will cover later, the intended setting for this curriculum guide is a diverse school, so teaching social justice topics in class feels especially important and applicable for my students. I have decided to emphasize group work as a means of delivering these topics to students. On the one hand, this is part of a larger personal goal for me, as I want to investigate and improve my own grouping techniques; however, as I've established in Chapter 2, group work is also a powerful tool for helping improve student understanding of mathematical concepts (Takeuchi 2016; Sofroniou & Poutos 2016).

In terms of best practices, I will look at two primary sources: *Mathematics Lessons to Explore, Understand, and Respond to Social Injustice*, a text published by National Council of Teachers of Mathematics (NCTM) in 2020, and the Open Up curriculum framework that Minneapolis Public Schools has adopted for the 2024-2025 school year.

Additionally, Berry III et. al (2020) suggest six different essential elements for teaching social justice in the classroom:

- "Equitable mathematical teaching practices,
- Authentic, challenging social/mathematical questions/concerns,
- allowing for clear social and mathematical understanding through the lesson,
- giving students the opportunity to view their social and mathematical identities positively,
- allowing for students to engage in social and mathematical reflection,

- including a step where students can take some type of real world action to address a social problem" (p. 58-60).

In terms of sample lessons (and lesson-sequence) models, I will be primarily working with backwards-design philosophies (Wiggins and McTighe, 2005) and IB unit planning recommendations (International Baccalaureate, 2019). I will be basing the sequencing of the course off of recommendations from *Mathematics: Analysis and approaches: Standard level course companion,* the textbook that my district uses for teaching my section of IB math. In terms of lesson planning, I will follow the Open Up (2024) structure: Jump Start, Launch, Explore, and Discuss.

Wiggins and McTighe (2005) suggest a lot of different strategies for designing curriculum in their book *Understanding by design.* Perhaps most crucially for my curriculum, they include and elucidate on the concept of "backwards design." In general, this concept can be simplified down to the idea of planning lessons by first conceptualizing the final student outcome. They break this into three stages: identifying the desired results, determining the acceptable evidence, and planning the learning experiences and instruction (pp. 17-19). For my curriculum, I will be implementing constructivist, social-justice, group work-themed lessons; these core tenets work well with a backwards-design themed curriculum. The desired results will include a mathematical goal alongside a social justice goal. What I view as acceptable evidence will have to be determined alongside the understanding that students will be working in groups often. The lesson experiences themselves must be based in constructivist thinking, and will have to deal heavily in the aforementioned group work. The framework that my school has developed for unit planning already utilizes backwards-design philosophy; with the desire to create a curriculum that can be immediately applied to a classroom setting, I will be exploring my ability to use or adapt those same templates.

The IB Curriculum Guide (2019) contains a lot of information that will be relevant to my curriculum. Like in other IB courses, one of the larger goals of my course is to "develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world" (IBO, 2019, p. 5). To do this in a math classroom, IB suggests emphasizing several key components: conceptual understanding, mathematical inquiry, mathematical modeling, proof, and the use of technology (pp. 19-25). My activities will also emphasize these components, sometimes more than one at once. IB further goes on to establish 6 "assessment objectives," which the final IB exam tests. These objectives are as follows: Knowledge and understanding (the ability to use mathematics in different situations), problem solving (choosing appropriate mathematics in abstract and concrete situations), communication and interpretation (turning real-life situations into mathematical notation, communicating with others through and about mathematical notation), technology (applying technology to solve problems), reasoning (construct logical proofs), and inquiry approaches (solve problems that are in unfamiliar contexts) (p. 29). My curriculum will specify which of these types of assessments are relevant in a given activity.

Similarly, Wiggins and McTighe classify understanding into 6 different "facets": Explanation, Interpretation, Application, Perspective, Empathy, and Self-Knowledge (pp.

85-103). Though these are more general conceptions of the structure of knowledge, it's useful to think about how they intersect with the suggestions of the IB guide. While the first three of them are effectively the same (or at least come from a similar perspective) as the IB assessment objectives, the concepts of perspective, empathy, and self-knowledge are importantly a little different. While we're discussing social justice issues in class, I'm going to need to intentionally call upon students' understanding of their own unique perspectives and identities. Students will need to have empathy for one another; developing this empathy through group work will be vital to the success of this curriculum. Finally, allowing students to gain a better sense of self-knowledge through reflection on individual and group work will be important, both for the students' social growth and their growth as mathematicians.

My school district has recently adopted the Open Up (2024) mathematics curriculum. Open Up structures lessons in a predictable format: Jump Start, Launch, Explore, and Discuss. As my incoming precalculus students will be used to this structure, I am going to plan my sample lessons around this framework. It is my goal to synthesize the social justice frameworks with the Open Up structure when creating the suggested practices and sample lessons for my classroom.

The framework I will use for my lesson plans will be the *Planning for Instruction Protocol* that is used in the Hamline School of Education (Basford et. al, 2024). The framework includes information like student learning objectives, academic language integration, assessment types, and differentiation prompts. This framework also includes a step-by-step script for the lesson, which uses some specific color-coding that is outlined by Figure 1.

Figure 1

Learning Sequence Color-Coding Guide

LEARNING SEQUENCE		
Time	**In the sections below (LESSON LAUNCH, MAIN LESSON, and CLOSURE/WRAP UP):** ● **Describe the learning activity** ● **Script out what the teacher will do and say, including** ■ **information/examples/models** ■ **directions** (including time cues, getting materials, moving to new places, getting out new materials, assigning groups, designating roles, tasks to be completed, etc.) ■ **questions you plan to ask** ● **Indicate slide numbers and/or examples/notes/images/props** to be shown. ● **Describe what students are doing and if working as class, small group, partners, or individually** ● **Include any additional information a substitute teacher would need** to carry out the lesson. Text color above indicates how you should color-code your writing in the 3 sections below	**Rationale: Why are you doing what you are doing?** For each learning experience, explain - what learning you want to result from the activity - why you selected the instructional strategy, grouping strategy, question or questions, materials, assessment strategy, transition strategy, etc.). Name the specific strategies, authors, or theories from coursework, when possible, in your reasons.

From *Planning for Instruction Protocol* by Basford., L., Benegas, M., Brickwedde, J., Hick, S., Lewis, J., & Mabbot, A. (2024).

The general sequence for the class will follow this order: Trigonometry, Sequences and series, Quadratics, Functions, Transformation of Functions, Exponentials, Logarithms, The Binomial Theorem, and Probability. The aforementioned IB textbook offers up a lot of practice IB exam questions; these can be adapted to fit group work parameters or presented as is to students to simulate exam-style scenarios. Alongside my constructivist and social justice-themed group work activities, it is still also my responsibility to prepare students for the many standardized tests (such as the ACT, SAT, and IB exams) that await them before their entry to college. As such, my unit will leave space for the inclusion of this type of material.

School Context

The demographics of the school are as follows: The setting is a public high school with around 850 total students per year. The IB class itself typically has around 34 students studying pre-calculus. Student demographics for the 2023-2024 year were: 41.7% Black, 18.8% Latinx, 27.5% White, 4.1% Asian, 4.7% American Indian, and 3.8% biracial. Around 73% of students qualified for free or reduced lunch programs. About 51% of educators are “experienced” (having more than 3 years of experience) and 88% of the teachers at the school are white (Minnesota Report Card).

The IB Precalculus course itself tends to be whiter than other math courses at the school. It is typically taken by 11th grade students, but some 10th grade students are enrolled (usually about 5 per class) and some 12th grade students are also enrolled (typically another 5 per class).

It is important to note, though, that while the class is whiter than other math classes, it is still generally a diverse student body racially, and has a diverse mix of genders and students from different class backgrounds. Typically, classroom size limitations means that students are seated in groups of 4, 5, or 6; as such, group sizes tend to be around that size, though they may have been broken into smaller subsets within these tables.

The curriculum that I create for this project will be designed with this setting in mind. However, it is my intent to create something that any high school math teacher could use (or take inspiration from) in their own classroom, whether they work in a school with a diverse set of students or not.

Project Timeline and Summary

The curriculum development will be done over the course of a summer class. My goal is to create one unit plan for a precalculus class, several complete lesson plans, and all of the relevant materials for those lessons. The unit I create is intended to be the first of the year, which will allow me to immediately apply these lessons in the classroom in the fall semester. The next chapter will detail my reflection on the process of creating the curriculum, where I will expound about these considerations more.

CHAPTER FOUR

Reflection

Introduction

I began this project with a research question that was very important to me: "What are the best practices for engaging with social justice topics and goals in a modern, constructivism-based mathematics course?" After conducting a literature review to help give me insight into the answers to this question, I attempted to gain a better understanding of these practices by utilizing them while designing curricular materials. In this chapter, I will reflect upon my experiences with the creation of this curriculum, including a description of my expanding understanding of the topics in my literature review, an honest assessment of the limitations and advantages of the work that I created, ideas for further connections that could be made to this curriculum, an explanation of why I feel that this work is valuable to my profession, and a conclusion that ties things together.

Major Learnings

For reasons that I will not go into here, there was a gap of several years between my completion of the initial Capstone Design course and my completion of the Capstone Project course. As I was teaching full time in the interim, I had the opportunity to test out some of the ideas at play in this project. For instance, I was able to implement many different group work strategies in class, which allowed me to select strategies for my curriculum that I had already found success with. Similarly, I had the opportunity to plan many lessons and units, often in collaboration with other math teachers, which generally gave me more experience to pull from when I actually constructed these curriculum materials.

Coming back to my initial project idea after several years also gave me something akin to a reality check, in that it reminded me of some of my initial teaching goals which had shifted over the years. That is not to say that social justice integration, an emphasis on group work, and constructivist ideology were things that I had thrown to the wayside, but rather that my attempts to integrate these topics into the classroom environment slowly became something a little more reminiscent of the "drill-and-kill" style I referenced in Chapter Two. I suspect that there may be a tendency for people to return to something easy and familiar in the face of something difficult, and doing the work of implementing these topics into the classroom was certainly not always easy. Returning to this project with more classroom experience gave me the space to think of methods of implementation that were both realistic and radical.

Some of my proudest accomplishments with this unit are my written protocols for group work, What's Going On in This Graph?, and team tests, as these have all been components of my classroom for years that I have never "codified" like this before. In general, I think that the unit is a good mix of ideas that I have refined over years of implementation and ideas that were developed directly from the results of my literature review.

Literature Review

Due to the gap in time between when I conducted the literature review and when I completed my capstone project, many of the concepts that I initially wrote about were condensed down to their core essence in my practice. For instance, Moses's ideas about the abolition of ability grouping simply became the standard in my classroom, even if the story behind Moses's decision to fight against ability grouping was not always at the forefront of my mind (Moses & Cobb, 2001). That commitment to ending ability grouping ultimately ended up being one of the key assumptions in the design of my unit.

Similarly, many of the ideas from the group work section of my literature became common practice in my own classroom, and thus worked their way into my curriculum. For example, Sofroniou and Poutos's (2016) ideas around small group discussions synthesizing into larger group discussions were foundational in my lesson plan design. Takeuchi's (2016) ideas around friend-based vs. teacher-chosen groups influenced my recommendations for group structures.

My school's adoption of the Open Up curriculum helped with my implementation of constructivist ideals in the classroom, as their curriculum is based in inquiry-based education, and requires the exact concepts that Von Glasersfeld first argued for: instruction that prompts critical thinking without sacrificing fun in the classroom (Cobb, 2000). Both due to my familiarity with it and its commitment to a similar set of constructivist goals, the Open Up framework seemed like a natural fit for my unit.

Implications

The creation of this project represented another step in the continued adoption of materials that meld social justice concepts, constructivist philosophy, and group work strategies in a math classroom. These concepts were applied at various levels of classroom instruction, and thus show that future attempts to meld these topics need not always be on the face of the curriculum, but can appear "behind the scenes" in terms of classroom instructional routines.

Limitations

Even though I created this curriculum with some tried-and-true methods that I utilized in my own classroom alongside some best practices as suggested by my literature review, the lessons themselves have not yet been enacted in a classroom setting. These are simply ideas at how a teacher could go about integrating social justice concepts in a modern mathematics classroom, constructed from research conducted in the literature review and in my own classroom. Teachers who attempt to use these lessons will likely need to adjust timing for certain activities, fit the lesson structure into whatever is expected of them at the school they teach at, and perform countless other adaptations for their specific student body. Additionally, the unit that I worked on was focused on trigonometry, which represents only a small subset of the math that students will encounter in high school.

The scope of my sample unit ballooned from my original vision once I entered the creation phase. While I could have arbitrarily limited the length of the unit for the purposes of this project, it made more sense to me to include the space for additional lessons, even there would not be time during this course to create all of the materials for them. I am satisfied with the final product and think that the included lesson plans were the most relevant for the stated goal of this project.

Future Research

Applying similar ideas to other concepts from a precalculus course would be useful for teachers who want to integrate social justice ideas into their class. Books like *High School Mathematics Lessons to Explore, Understand, and Respond to Social Injustice* offer many ideas for projects for Algebra I, Geometry, and Algebra II courses, but offer less ideas for post-Algebra II projects based around social justice concepts (NCTM, 2020). The curriculum I developed focused more on classroom routines and smaller projects that integrated social justice concepts, but these larger scale projects could serve as unique assessment opportunities.

Though my unit did feature some writing integration, primarily through the "What's Going On in This Graph?" activity, the IB SL Math: Analysis and Approaches course requires students to write a larger paper on mathematics called the Internal Assessment (IBO, 2019). Creating more writing assignments and projects that tie in social justice goals would be beneficial to the overall course sequence.

Use of Results

The unit that I designed will be implemented in my fall precalculus classes this year. Undoubtedly there will be elements that need to be adapted, depending on the student body in question, but the general framework is there to be followed. After the curriculum has been implemented, it will be tweaked, and the lessons learned from implementation will be applied to future units in the course and, ultimately, the same unit in the next year of the class.

Benefit to Profession

Aside from directly benefiting my own practice in the classroom, I believe that the unit plan will be useful for any math teacher who is interested in integrating social justice concepts into their classroom. In my experience, available materials centered around this goal are usually impressively constructed, but not always realistic. My approach to this unit plan, whereby I included both larger assignments that tackled social justice issues and day-by-day routines with social justice intentions, attempted to remedy this problem.

Conclusion

Over the course of this project, I had the opportunity to learn a lot about myself, my teaching practice, and how those two things intersected with the answers to my initial research question: “What are the best practices for engaging with social justice topics and goals in a modern, constructivism-based mathematics course?” I discovered that, though some of my focuses in the classroom had shifted over the course of the several years that the creation of this project took place, my commitment to the overall goals had not changed. I found new ways to work to achieve these goals in the classroom, both in my years of experience with classroom teaching and through a comprehensive literature review completed for this course. My final product reflected these years of work. I can only hope that, once implemented, it will be as effective at making my math classroom as collaborative and accessible for ALL students as the QRC at Hamline was for me.

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