Spring 2022

Supplementing Math Instruction in the Elementary Classroom with Cognitively Guided Instruction (CGI)

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SUPPLEMENTING MATH INSTRUCTION IN THE ELEMENTARY CLASSROOM WITH COGNITIVELY GUIDED INSTRUCTION (CGI)

By

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A capstone project submitted in partial fulfillment of the requirements for the degree of

Master of Arts in Teaching

Hamline University

Saint Paul, Minnesota

May 2022

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CHAPTER ONE

Introduction

Ask a school-aged student what they think about math, and the answer will likely fall near the extremes of *I love math* or *I hate math*. If you were to follow up with the student and ask them why they answered what they did, some common responses would be *I'm just not good at math*, *I'm not good with numbers*, *Math isn't my strong subject*, or the detrimental *I'm not a math person*. Besides physical education, there is likely not a more polarizing school topic, from the point of view of students, than math. In most school subjects there are a variety of ways that the material can be learned and practiced and a variety of ways that students can tackle a problem. In writing, students are offered choices such as what they would like to write about, what they would like to include. These choices allow students to activate prior knowledge and to be able to fulfill the writing in a way that works for them. In art class students can decide what colors to use, sometimes even the types of material used for coloring, as well as what they would like to include in their art. Even during a traditional game of dodgeball in gym class a student can launch balls to get hits, catch the ball, or simply dodge, being able to select how they want to approach the activity. In math, traditionally it starts and ends with the standard written algorithm for doing mathematics.

When thinking about how math has been traditionally taught in public schools in Minnesota, students end up with very few choices in how to solve math problems. This shortcoming is why I chose to explore the following capstone question: *How can cognitively guided instruction (CGI) in math be implemented in the elementary classroom?*

Cognitively guided instruction, or CGI, in math is essentially the concept that students have an inherent sense of math knowledge, of doing math. Students bring into the classroom their own understanding and strategies for solving math problems and situations, with emphasis on being able to directly model the math (Carpenter & Fennema 1996, p. 6). This chapter goes over my own personal experience with learning math in American grade school, including the use of the standard algorithm, as well as CGI math in graduate school.

The chapter describes what led me to want to learn more about CGI math, including what was observed in several classrooms at a charter school in St. Paul. The chapter also explores the effects of CGI math and its impact on students' math performance and attitude towards math on parents and educational leaders in regard to math curriculum and how it is implemented in the school.

Background

Although I was able to start kindergarten in America, I was actually born in a refugee camp in Thailand. I was almost four years old when my family moved to America, settling down in Minneapolis, which is where I attended kindergarten as well as most of my schooling up until high school. I knew very little English when I started school, but was able to achieve high marks and was exempt from having to attend English Learner (EL), at the time known as English as a Second Language (ESL), classes after about 2 years. I did not notice I was underperforming in any of the school subjects until sometime during the fourth-grade.

I can trace my negative attitudes toward math as well as my belief that I am just naturally not a *math person* to a math routine in the fourth-grade at a large elementary school in Minneapolis. Throughout the school year my fourth-grade teacher had pages of 100 single-digit math multiplication problems for students to complete. The worksheets were timed, and each worksheet revolved around one of the digits in our multiplication table. So, the first set of 100 multiplication problems would be 1 multiplied by the numbers 1-10, the second sheet would be the 2 multiplied by 1-10, and so on and so forth, up to a mix of all multiples up to 10. There was also a chart with all of the students' names on it, and ten columns next to the names. When a student was able to finish all the multiplication problems within the time limit, a sticker would be added next to that student's name under the multiplication set that the student completed. This math routine occurred every so many days. If a student did not get all the questions correct, the student repeated a similar sheet of the math problems the next time, while students who answered all questions correctly would have the next set of problems the next time.

The fourth-grade math exercise started well for me, but once the multiples of 6's and subsequently 7, 8, 9, and a mixture of them all were reached, I found myself frustrated, unable to complete the problems within the time limit. A friend of mine was able to finish up to multiples of ten well before I did, as I remember being stuck repeatedly attempting the multiples of 6 and my friend had quiet time during this math activity. Although I did eventually complete the math exercises up to multiples of 10, this was really the first time during school that I felt I would not be *good* at a specific subject. After learning about the math methods in CGI math and how it can assist students to better understand and grasp math concepts, I have a fairly good idea about why I had such a difficult time completing the math activity. The only way I knew how to multiply at the time was to count in multiples, or manually count and add one at a time when I was not able to count by a certain amount. As most students, and probably many adults as well know, it is fairly easy to count by 2, 3, and 4 to a lesser extent, and 5, fairly easily. Counting by 6 and up becomes more difficult. Looking back at the math activity completed for multiples of 6 and up, it would seem that I either had to outright memorize or associate what any of the two digits multiplied would result in, or manually count using fingers, which would have taken too long and therefore the set of problems would be incomplete. I was taught really just one way to solve the multiplication problems, which was to count up. The other strategy was to outright memorize them, which clearly was not utilized effectively by me at the time.

The math experience in fourth-grade was definitely one of the main reasons I persuaded myself that math was not a subject where I was going to excel. That same year I was the top points scorer in the Accelerated Reader program at the school, a nifty program where students read participating books and took quizzes on the books for points. I even had more points than the fifth graders, so I figured I was a *reading* and *language arts* person, and just not a *math person*.

Rationale

When Dr. Brickwedde, my professor in the Teaching Mathematics in the Elementary School course at Hamline, told me and the rest of the students in the class that many students, including middle school students, still relied on using their fingers to count for simple addition and subtraction math problems, I was in disbelief. I had considered myself to not be a *math person* toward the end of elementary school, but assumed many other students excelled at math and would not need to use fingers to count for simple adding and subtracting. Even though I did not consider math to be one of my stronger areas of learning I still thought the standard algorithm was an effective way of learning math, that I was just inadequate at utilizing the method. Learning about CGI math has shown me that there are different ways to approach math instruction in the classroom, at the very least it can be an addition or supplement to whatever method a school or district is currently using. At the clinical field experience for the Hamline mathematics course for K-6 licensure I saw third-graders who were able to effectively utilize CGI math much more effectively than I could at the time. Seeing third-graders effectively utilize CGI math methods led to my desire to explore CGI math more closely.

CGI Math

When I began working at a large charter school in Saint Paul I was able to see firsthand students really having difficulty working with numbers. There seemed to be a lack of number sense with students as well. Students were seen not only using their fingers for fairly simple math addition and subtraction problems, but also to write-out and set-up math problems that really did not need to be aligned. For example, a math story might have stated that a person had one dollar and spent twenty cents, with students asked to find out how much money remained. A CGI math approach could be to just quickly count back from one dollar by tens, then fives, whereas the standard algorithm for math would influence students to set up the problem vertically. The potential advantages of CGI math are many, but a few quick advantages here include the reduced amount of time spent on a question, as well as being able to utilize math theories such as number composition. In an eighth-grade algebra class, I saw a student have difficulty adding an addition and subtraction problem, for example, "-6 + 5 + 6 = ___." The student appeared to be following the well-known math method of adding and subtracting from left to right, and was having difficulty finding the solution. A CGI math approach here would have been to see that the negative six and positive six would negate each other's amount, resulting in five being the answer.

When there is only one way of math taught, a student may not excel at math if the student has difficulty with that method. Not only that, but the CGI math approach integrates more math concepts, builds on previous learnings, and requires students to use what they have learned and apply it to new concepts. For example, a CGI math approach for multiplying could show students that multiplying is really the same as adding groups of quantities together. The standard algorithm approach involves students crossing out place values and moving digits one place value to the left, for the tens, hundreds, and so on, with many students probably unable to explain what is happening to get their answer. I believe the CGI approach will help students understand math more, especially since it builds on what students intuitively know already. Understanding math more in-depth could also mean students can solve math problems quicker. With only one way of doing math, students may feel discouraged or at a roadblock if they are not able to master the standard written algorithm quickly. CGI could open up and promote different approaches and strategies to math solving, and perhaps students' math anxiety could be alleviated by a CGI approach. When I learned about the CGI math methods, math became more approachable to me and I no longer felt deficient at it. This capstone project seeks to supplement math instruction in classrooms that mainly utilize the standard written algorithm for math. The project would allow opportunities for students to utilize the learning and problem-solving strategies present in CGI, strategies that would especially benefit students who may have difficulty utilizing the standard written algorithm. It would also help answer my research question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?*

Alternatives to the Standard Written Algorithm

In addition to the positive effects, as well as the student-centered learning atmosphere CGI math offers, there are several other reasons for pursuing this capstone project. I was the schoolwide and statewide test administrator for a large charter school in Saint Paul with an emphasis on Hmong culture and language. When reports were generated with historical data, such as test scores, math scores were shown to be significantly lower than the state average. I spoke several times with a math teacher for the school at the time and had several conversations about how math was taught. The math teacher knew of the various methods of learning math, including methods present in CGI math as well as Singapore math. When asked why the other general education teachers did not seem to teach anything other than the standard algorithm for math, the explanation given was that the other teachers were not aware of how to do math other than the way they were taught, which is the standard written algorithm. One of the main concerns stated to me was the inability or discomfort of teachers not being able to follow students' math work, nor assist with students who were using those different methods.

Research on the effects of CGI math on students could add to the conversation of implementing it into the current math curriculum, as well as add to the conversation of what math methods teachers should know and teach as well. The research could also contribute to efforts for improving student performance on high-stakes math tests. The National Assessment of Educational Progress (NAEP) test scores for math show that there remains a significant gap in math performance when comparing students from low socioeconomic status and high socioeconomic status (Lubienski and Lubienski, 2005, p. 698). There still exist gaps in learning opportunities all across the United States, so the research here will contribute to discussions on how math instruction can help improve student performance in math, especially at schools with a significant portion of students from low socioeconomic backgrounds.

Chapter Summary

This chapter went over why I chose to ask the question: *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?* I briefly went over my own experience with, and self-doubt in, math during my school years. I brought up the standard algorithm in mathematics and my learning about how the standard algorithm has several shortcomings when implementing it in math. These problems included an inability to mentally solve fairly simple math problems, as well as a seemingly ingrained tendency to strictly follow rote procedures in math, even for math problems that do not benefit from those procedures. Some reasons why educators may be reluctant to implement other math methods, including CGI math, were also explored, one of the findings being that teachers tend to teach what they know and how they know. I touched on the ramifications of not finding other ways to teach mathematics, relating it to test scores in mathematics at the school I worked at as well as national test scores where student household income was taken into account. Specifically, Lubienski and Lubienski (2005) found that in public schools, as well as private schools, the gap in NAEP test scores could be seen when students' socioeconomic statuses were factored. Students with higher socioeconomic status in public and private schools scored higher than lower socioeconomic students in public and private schools, respectively.

In chapter two a review of the literature is examined. The review gives an overview of the history of mathematics instruction, particularly the standard written algorithm, in American schools and includes discussion on past interventions for improving math performance. The literature review also includes a look at math test score data over the years. Singapore math, one of the other math methods taught in some United States school districts, is examined. Lastly, the literature review concludes with a detailing of what cognitively guided instruction (CGI) in math is, and how it relates to the other topics in the literature review. Chapter three goes over the project description, discussing the math learning material in the project, what instruction will be given, the timeline for the project, as well as formative and summative assessments that are part of the project. Chapter four discusses concluding thoughts about the project and the processes that went into the project, and also discusses ideas for further future developments and ideas with the project.

CHAPTER 2

Literature Review

Introduction

Literature pertaining to the capstone question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?* is reviewed here. The review starts by going over what math anxiety is, and then goes into looking at the standard written algorithm for math, looking to see the written algorithm's role in students' current math performance. A review of the literature on math education in the United States and other countries, including taking a look at international math test scores, follows. International math test scores reveal information regarding how other countries perform on math as well as how students in those high-performing countries view math. A review of the role parenting might play in students' math education and how that contributes to how a student might perform in math or how their attitude may be affected is also reviewed. CGI math is very student-centered and student-driven, so it may be helpful to see how students use CGI math methods in the absence of assistance or opinion from a parent. The math curriculum in the United States, as well as those in countries performing highly on international math achievement tests, is explored. Next, cognitively guided instruction (CGI), as it pertains to math instruction, is detailed and explored, reviewing how CGI math classrooms may affect students' performance of math and their deeper understanding of math concepts. The benefits of CGI math instruction are reviewed as well.

Math Anxiety

There are people who do not like to do math. However, math anxiety is more complex than simply not liking math, with liking or not liking math having little to do with actual math anxiety. Ashcraft and Kirk (2001) made the distinction that math anxiety is not the same as feeling anxious about math due to a person having poor math skills, but instead is when a person worries so much about doing a math task that the person's working memory resources become reduced, thus resulting in a decreased ability to do the math task (as cited in Beilock & Maloney, 2015, p. 5). Beilock and Maloney (2015) further elaborated and said that essentially the person experiencing math anxiety has to attend to their anxiety as well as the math task at hand, causing the reduction in math performance (p. 5). Lee (2009) found that math anxiety is associated with decreased math achievement (as cited in Beilock et al., 2015, p. 1480).

My capstone question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?,* seeks to introduce and implement CGI principles for math to utilize and increase students' math competency, and to reduce math anxiety students may have. I have seen in classroom sessions students who take one look at a math problem and not even attempt it, the student rather waiting to go over the answers as a whole class. Beilock et al. (2015) suggested that students who do less math, learn less math, and the result could be that students' math anxiety then increases (p. 1485). With CGI math an approach that is inviting to the students would be introduced, following CGI guidelines that work with the students' strengths in their inherent math understanding and abilities.

The Standard Written Algorithm For Mathematics

For many who grew up in the United States or spent some time in American schools, there is a good chance that math was learned using what is commonly referred to as the standard written algorithm. Vocabulary that may be familiar for many from this method include *borrowing*, *carrying*, and *crossing out*, all three things that may be utilized for a math problem such as *250 - 173 = ?.* The standard written algorithm is the way many students in American schools traditionally learned how to add, subtract, multiply, and divide, with heavy emphasis on following rote procedure or repeated steps. Many will remember working through worksheet upon worksheet of addition or subtraction problems, doing them enough that the standard written algorithm gets quickly applied for problems that did not really require it. For example, when a subtraction problem such as *61 - 58 = ?* is to be solved, the problem is written out, made sure to have the digits correctly lined up according to their place value, and worked from starting from the right side, crossing out numbers and borrowing or carrying other numbers left to right. The standard algorithm is reviewed here, including looking at its benefits and drawbacks.

Torbeyns & Verschaffel (2016), using subtraction as an example, defined the standard written algorithm as,

> fixed and well-defined step-by-step procedures for solving multi-digit subtractions, involving operations with digits rather than the real magnitude of the numbers in the problem, such as calculating the difference between 5 and 3 (rather than 50 and 30). (p. 101)

They contrasted their definition of the standard written algorithm by referencing Buy's (2001) definition of mental computation strategies, which are "strategies that require children to calculate *with* their head--using their knowledge of numbers and operations--rather than *in* their head--without paper and pencil" (p. 100). It is important to note here that number place value is essentially lost when subtracting using the standard written algorithm, an idea that will show up along with other concerns regarding the standard written algorithm.

At least in Minnesota schools, there seems to be a misconception that the standard algorithm is the required method to teach math for students. A closer look at the Minnesota Academic Standards in Mathematics show that teaching the standard algorithm as the only way or even as the primary way is incorrect. For example, consider this fourth-grade math standard for dividing numbers:

> Use strategies and algorithms based on knowledge of place value, equality and properties of operations to divide multi-digit whole numbers by one- or two-digit numbers. Strategies may include mental strategies, partial quotients, the commutative, associative, and distributive properties and repeated subtraction. (Minnesota K-12 Academic Standards in Mathematics 2007, p. 12)

The key is that although the standard algorithms can be included in the learning, it is not the sole or main method that teachers have to teach. A discussion of the positives and negatives of the standard algorithm will also allow a comparison of what CGI does differently from what is commonly taught in schools.

Analysis of the Standard Algorithm

Although there may be widespread teaching of the standard written algorithm for math, the algorithm has many drawbacks. Some of the drawbacks may be more trivial, such as requiring a little more time to solve an otherwise fairly simple math problem. Other drawbacks are more concerning, especially to students, and are discussed here.

Loss of Number Sense

Fischer et al. (2019) found and listed several drawbacks regarding the use of the standard algorithm for elementary students, as well as adults. A few of their findings are shared here. One drawback they found was a loss of number sense, and they give an example of adding two larger numbers, 57 + 99, where adults may line up the numbers in columns just to solve a problem that can be solved through mental computation much more quickly (p. 108). A student using the algorithm would just repeatedly add starting from the right side, such as the ones place, and repeat this method, which means the student would be adding a series of small numbers, one through ten, and not take into account the large number, and its place values, as a whole (p. 106). Adding 457 to 231 uses less number sense if a student views it through the standard written algorithm, lining the numbers up and solving from the right side to the left side, as seven plus one, five plus three, and four plus two. Another example included by the authors were second graders who added 72 + 37 and got 19 as an answer, with Fischer et al. (2019) attributing this error to students erroneously viewing 7 + 3 as 10 and writing a 1 in the tens column, when it really is 70 + 30 (p. 106).

Reliance on Rote Procedure

Newton (2007) also added to the discussion of drawbacks that in some ways can be attributed to strategies such as the standard algorithm. Newton (2007) noted that the math education in China relied more on the students knowing why they chose a specific approach to get their answer, and Newton contrasted this with noticing that American students and teachers are much more concerned with getting the correct answer and not necessarily how they got there. Recalling how the standard algorithm is very reliant on memorizing procedures and steps, along with the Fischer et al. (2019) findings that students arrived at fairly obvious incorrect answers when solving simple math problems, we further see how the algorithm can negatively affect students' overall understanding of number sense.

Effect on Student-developed Math Strategies

Although the standard written algorithm is able to be used to find the correct answer, Torbeyns and Verschaffel (2016) detailed how it could be detrimental to students who learn it early on in school. Torbeyns and Verschaffel (2016) cited studies by Carpenter et al. (1998), Hiebert and Warne (1996), and Thompson (2000), which showed that students who were exposed to other strategies for computing multi-digit problems "developed a rich diversity of insightful and clever mental computation strategies that were mastered well" (p. 102). The studies cited included a large amount of instruction in the properties of the base-10 number system, as well as math concepts appearing in multi-digit numbers. The findings from the studies cited by Torbeyns and Verschaffel (2016) showed that students who learned the standard algorithm too early on would develop and use incorrect variations of the standard algorithm (p. 102)

Why Standard Written Algorithm Use May be Selected by Students

Fischer et al. (2019, as cited in Torbeyns and Verschaffel, 2016) stated one reason students chose to use the standard algorithm over mental computation methods at a significantly higher rate was that the standard algorithm allowed students to immediately use what they already know, as well as the standard algorithm being attractive to students since it helps limit the options that students have to choose from to solve problems (p. 108). Beilock et al. (2016) found that first and second-grade students in a study had math anxiety when learning more advanced math strategies, but that students who did use the standard algorithm experienced less math anxiety (as cited in Fischer et al. 2019, p. 109).

Why the Standard Algorithm Continues to be Taught

There are different ways that math can be taught, with the CGI approach being one of them. However, the standard algorithm is still being taught in schools today even with the disadvantages that have been discussed previously.

Baker (2014) discussed why teachers, specifically pre-service teachers, may have difficulty teaching math so that it more closely aligns with the problem solving emphasis that the Common Core Standards promote. M. Baker (2004) noted (as cited in C. Baker 2014, p. 6) “the beliefs and experiences individuals bring with them as they enter the classroom for the first time are resilient and difficult to change.”

CGI presents a different way to tackle math problems, and its strength comes from the idea that children intuitively have some knowledge of how to solve math problems. The capstone question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?* seeks to introduce CGI math principles to students who have difficulty with the limited math strategies currently being taught in schools, and to present to students opportunities to solve math in ways that makes sense from the student’s point of view.

Math Education in the United States and Other Countries

There have been many attempts to address the lagging mathematics performance in the United States. One of the most well-known of these attempts included the passage of laws such as No Child Left Behind (Rhodes, 2012), which required much standardized testing and sanctions for schools for failure to meet achievement marks (p. 163). There are definitely similarities that exist between the education system in the United States and other similarly-developed countries, as well as differences in how math is taught in countries with international math achievement scores at or near the top of those tests. Although it may be relatively easy to see how a country’s economy or how much they invest into education can influence the quality of the education received by students from those respective countries, there are also different factors such as parental involvement in students’ education, time dedicated specifically to math education, as well as how the math itself is taught.

Strictly looking at only the math achievement scores on the international achievement tests, such as the *Programme for International Student Achievement* (PISA) where the top five highest math scores for reporting year 2018 are China, Singapore, Hong Kong, Japan, and South Korea (OECD, 2019), one might assume students from these East Asian countries have a natural propensity for high mathematics achievement. Stevenson’s (1993) longitudinal comparative study found no general differences in students’ cognitive functioning as it relates to math (as cited in Zhao & Qiu, 2009, p. 341), supporting the idea that a student's race alone does not contribute to math ability. However, the focus here is to discuss how math is taught and learned in these countries, focusing on objective differences between the United States and East Asian countries, and not on any racial or stereotyping connotations. These differences in countries that perform highly on international math tests, the scores, and the literature behind the exploration of those results are discussed here. A specific focus is on countries that perform at or near the top in international math achievement, which happen to be East Asian countries.

Math Performance on International Tests

The United States has the largest gross domestic product in the world (World Bank, 2020). It would reason that the United States would also rank near the top in the various fields of education. However, this is not the case. The *Programme for International Student Assessment* (PISA) is a two-hour test that is administered to thousands of 15-year olds every three years, beginning in 2000. This international test measures three areas: reading, math, and science. The test is administered by the *Organisation for Economic Co-operation and Development* (OECD), an organization of 38 member countries that works together on policy-making, including administering and collecting international test data (OECD, 2021). According to OECD (2019), reporting on the 2018 PISA results, "Mathematics literacy is defined as students' capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (p. 27). The definition shows that it is not simply solving math problems, but includes aspects of math such as reasoning and explanation. Importantly, the results also make a note that test questions for math are a mixture of multiple-choice questions as well as questions requiring constructed response, which invariably requires deeper math knowledge (p. 27). The 2018 PISA results, as reported in the PISA (2019) publication, showed that for math the mean score was 489 points, with China, composed of the cities of Beijing, Shanghai, Jiangsu, and Zhejiang, scoring 591, followed by Singapore at 569, Macao/China with 558, Hong Kong at 551, Taipei with 531, Japan with 527, and Korea scoring 526, while the United States scored 478 (pp. 43, 59). For reference, an explanation of the scaled scores showed mean scores being around 500 (PISA, 2019, p. 56).

Taking a look at the Trends in International Mathematics and Science Study (TIMSS), findings are similar to those found on the PISA. The TIMSS is a test of mathematics and science achievement, administered every four years beginning in 1995, with the most recent data coming from the 2019 report. The data is composed of scores from fourth-graders. The report for 2019 (TIMSS, 2019) showed Singapore with the highest scaled score for math at 625, followed by Hong Kong with 602, Korea with 600, Taipei with 599, Japan with 593, and the United States with 535. When looking at TIMSS test results from previous years, similar trends emerge. The TIMSS 2007 International Mathematics Report (Mullis et al., 2007) showed that among countries that participated, the highest scale score came from Hong Kong with 607, followed by Singapore with 599, Taipei with 576, Japan with 568, while the United States scored 529 (p. 34).

A study by Stevenson (1987) confirmed these statistical differences have persisted for many decades now. In Stevenson's study of about 3,000 first-graders and 3,500 fifth-graders who were equally divided among the cities of Sendai, Japan, Beijing, Taipei, and Chicago, the findings were that in the top five percent of student scores in first grade there were only three students from the American city. Stevenson further stated that had all cities performed equally well, there would be over 40 American students in the top five percent (Stevenson, 1987, p. 5). In the following sections, the math scores on international achievement tests are examined.

Differences in Language and Mathematics Performance

Dowker et al. (2008) presented a helpful definition of the transparency of the number-naming system as it relates to the base-10 system as "the regularity of the spoken number system: the degree to which it gives a clear and consistent representation of the base system (usually base 10) used in the language" (p. 526). A recent study explored how children whose native counting language uses a base-10 system, such as saying *ten-five* instead of *fifteen*, may affect their math performance (Paik et al., 2011). The math abilities of not only United States students and students from another Asian country (Taiwan) were examined, the authors also included two more groups of students, Peruvian and Dutch, in the study. The study was completed on preschoolers, which included test administrators who spoke the native language of students in the study. The authors found that although Peruvian, Dutch, and American pre-schoolers did not perform significantly differently from each other, the Taiwanese students scored significantly higher than the other three groups. Fuson and Kwon (1992) found that language that uses a base-10 system can also affect early learning of numbers, at least in the very early grades. An example given in the study was that an American second-grade student counted on from *ten* to *fourteen* to get the solution to *ten plus four*, whereas in Korean, as well as several other East Asian languages, *ten plus four* could be stated quickly as *ten four* (p. 504).

Le and Noel (2020) discussed the differences between counting in Vietnamese and counting in French, particularly during the teen numbers, where Vietnamese keeps a consistent structure, such as *ten-one*, *ten-two*, with the French number counting structure less consistent (p. 5). Le and Noel (2020) reviewed the math performance of 3 ½ to 5 ½ French as well as Vietnamese children and found that of the eight math counting tasks presented to the students, the only task that Vietnamese children performed better at was for simple rote counting (p. 16), contributing this finding, which they noted as consistent with previous studies by other researchers, to the more regular Vietnamese number counting system. Le and Noel (2020) interestingly noted that the other areas of math that the children were tested on did not show significant differences between the two groups of students. One example the authors gave was the task of having students start counting from a number other than one, with no group of students appearing to have an advantage over the other (p. 17).

The studies mentioned, such as Paik et al. (2011), compared how students from Asian countries might be at a mathematical advantage due to the number-naming conventions in their native language. As Le and Noel's research suggested, the advantages may be more trivial than people may believe. Mark and Dowker (2015) conducted a study that also echoed Le and Noel's (2020) findings. Mark and Dowker's study compared young students who spoke Chinese with students from the United Kingdom who spoke English. The study found that the Chinese number counting system, similar to the Vietnamese system mentioned above, could not account for the general higher math performance in Chinese students when compared to students from other countries. Specifically, the study found that Chinese students could count backward from 30 more easily (p. 7), attributing it to the more regular structure of the Chinese counting system versus the English system which requires many numbers to have their own word. The study also found that the Chinese counting system helped younger students understand place value, but for older Chinese students there was no advantage in understanding place value (Mark & Dowker, 2015, p. 6). It would appear that the language differences cannot account for the vast differences in math performance between students from Asian countries with a more regular number counting system. Other areas will have to be explored, including differences in parental involvement in their children's math learning, and math instruction differences.

Parental Involvement in Math Learning

A student does not only learn at school, but also at home. The home environment could have an effect on students' ability to learn, including learning math. Stevenson's (1987) work, again, provides some useful information about how parental involvement in countries that perform highly in math assessments may differ from American households. Although Stevenson's particular research referenced here may be from a few decades back, some of its points may still be relevant. Stevenson found at the time that American parents on average believed their students performed at above average, with only 7 percent of the mothers considering that their child's academic potential was at average or lower (p. 6). Stevenson (1987) mentioned 75 percent of the American fifth-graders believing they would be among the best students (p. 5). Important to note here is that Stevenson's takeaway is not that students and parents should not view their abilities and learning potential very positively, but instead it is that with this erroneous viewpoint it could lead to a tendency for the student or parents to not believe that higher math achievement is needed (p. 6). Furthermore, Stevenson (1987) makes a quick additional follow-up point about how American parents view reading as more important than mathematics instruction in elementary school (p. 6).

A study by Hunt and Hu (2011) further explored the relationship of parental involvement in students' mathematical learning and achievement. The study differed from Stevenson (1987) in that Hunt and Hu examined parental involvement and attitudes between American mothers and Chinese-born American mothers, with the students in the study all attending American schools, specifically in Florida. One point Hunt and Hu (2011) mentioned from prior research, citing Hess, Chih-Mei, and McDevitt, (1986) and Stevenson et al., (1990) that still has an overarching reach today is that "American parents tend to attribute success in mathematics to those who possess a special talent as opposed to those who worked hard and practiced" (p. 122). This is a common belief that many people hold, which is that they are just naturally good or not good at something. Hunt and Hu's findings suggested that locus of control is important to how parents go about supporting their children in learning math (2011). Although it was 1987 when Stevenson pointed out that parents viewed mathematics as less important than reading, Hunt and Hu (2011, citing Cannon and Ginsberg, 2008), mentioned a more recent finding that also found parents still do not believe mathematics to be as important as other school subjects, such as reading (p. 122).

The results from Hunt and Hu's (2011) findings revealed interesting information about different parental attitudes and beliefs regarding the parents' children and learning math. One of the findings from American-born parents was that math was learned as a series of steps, the parent's response being "I remember learning math primarily as the teacher was writing on a chalkboard. And they would show you, um, the different steps...the different steps to get to the answer" (p. 131). Adding to that parental response, the authors also noted that American-born parents overall expressed a general dislike of math, due in part to their inability to comprehend some of the math, and also again indicating math was not of importance for them (p. 131). Some direct quotes included "can't grasp it," (p. 132) and "I have better things to do than think about numbers" (p. 132). The innate or natural ability versus effort views that parents have was brought up by the Hunt and Hu (2011) study, reflecting in very different responses from the parents. A Chinese-born Mother stated, "I think you need to increase your interest in learning mathematics...Regardless of learning styles, practice is critical if you want to be good at mathematics" (p. 132), contrasted with an American-born Mother who said, "I think that you have to be somewhat number oriented...and not get a mental block about it. I think it happens a lot, where kids don't understand, so they push it out" (p. 132). Other findings from the study included Chinese-born parents viewing their knowledge of math as adequate to help their children if needed, whereas American-born parents expressed more concern and a feeling of not being prepared to help their children in math (Hunt & Hu, 2011, p. 134).

The negative attitudes toward math is also shown in Beilock and Maloney's research on math anxiety. Beilock et al. (2015) found that parents who experienced math anxiety would also negatively affect their children's math anxiety when the parents helped their children with math homework, but that children whose parents had high math anxiety did not automatically experience the math anxiety their parents did. The parents' math anxiety seemingly can be communicated to their children during these homework-help sessions. Beilock et al. (2015) summed up their findings on the transmission of math anxiety from parents to their children with this alarming statement: "When parents have a poor relation with math and frequently help their children with their homework, their children learn less math" (p. 1485).

Gibbs et al. (2017) conducted a study around Asian American children and their parents, focusing on the parents' role in their child's education. Gibbs et al. (2017) sought to answer the *why* of the Asian American advantage, citing Sun's (2011) finding that "the Asian American advantage emerges in force between ages two and four, but it raises additional questions about *why* the gap emerges" (p. 318). One important finding from the Gibbs et al. study is that Asian American parents were about a fifth standard deviation units above white parents in placing the importance of their child to learn the alphabet and ability to count (p. 324). However, in this study, it was mostly just the Asian American parents' view of the importance of education for their child that seemed to have an effect on their child's initial greater academic achievement. Their study did not fully attribute any particular behavior that a parent did to an increase in the young child's academic achievement:

> When we measured parents' tangible behaviors (e.g., read to child, maternal warmth, etc.), we had little success identifying the precise behaviors Asian American parents employ that promote their children's cognitive advantage. (pp. 331-332)

Examining these two studies show that language differences cannot account for the significant differences between the math test scores on international achievement tests between American students and students from Asian countries. The advantages that an Asian student may have over other students in math achievement is minimal and only present at the very young ages. There does not appear to be much success singling out any one factor that Asian parents have or do that contributes to their child's math ability or success.

Differences in Math Instruction and Curriculum

It is generally understood that schools in many Asian countries tend to be very teacher-centered, where the teacher is the source of the knowledge. In many Asian cultures older people are generally looked-up to more, and with more reverence and respect than perhaps Western countries, including the United States. The differences between math instruction and curriculum in other countries, particularly those who perform well on international achievement tests, and the United States, are explored and examined next.

Zhao's (2005) study found that East Asian countries are much more centralized in their curriculum. For many of those countries high-stakes national tests are much more important than even the United States, and therefore schools, teachers, and students all strive to learn the specific curriculum required for those tests (p. 221). Zhao also mentioned that in China subject-matter teachers will normally have college majors in the subjects they teach (p. 221). However, Zhao pointed out that a rigorous and systematic curriculum, coupled with essentially teaching to the test contributes to "stifling creativity" (Zhao, 2005, p. 220).

As mentioned throughout this literature review, Singapore has scored at the top or near the top in several international tests of math achievement, including the PISA and the TIMSS, for many years now. A math curriculum, Math in Focus, modeled after the math taught in Singapore, has gained some traction in the United

States and is used in several hundred school districts in the United States (Jaciw et al., 2016, p. 474). Subsequently, it is discussed more in depth here, including its advantages and disadvantages.

Leinwand and Ginsberg (2007) gave a fairly thorough overview of what Singapore math entails. The math curriculum referred to as Singapore math in the United States has a framework that connects five important aspects critical to mathematical problem solving, and includes concepts, skills, attitudes, metacognition, and process (p. 33). The authors described how the segments of Singapore math are aligned with each other, whereas the math curriculum in the United States were more spread out and disconnected, as evidenced by each state requiring their own academic standards (p. 34). A typical feature of math problems faced by students in Singapore math is that they require more steps and higher-order thinking. An example the authors gave showed what a pie chart in a Singapore math problem may look like in comparison to one typically found in a United States textbook (p. 36). The example for a Singapore math problem showed two slices of the pie across from each other, as well as other slices of the pie with various money amounts on them, with a right-angle indicator shown on them. The authors explained that a student would have to think about how two right-angles would equal half of the pie chart, and then work their way from that point to find an amount missing from the pie chart. In comparison the authors showed what a similar math problem would look like in a typical American textbook, which has just slices of the pie with money amounts on it, showing that a student would simply just have to add or subtract a number to find a missing amount (Leinwand & Ginsberg, 2007, p. 36).

The American Institutes for Research (Ginsburg et al., 2005) study of Singapore's mathematics system, of which one of the authors is also from the Leinwand and Ginsberg (2007) work mentioned, further detailed how the Singapore math way of teaching math differs from the United State's. One of the contrasting details from this report is that students using Singapore math learned math concepts to mastery (p. ix). The standard written algorithm that was mentioned earlier is an example of not learning the concept of multiplication to mastery, as students tend to follow rote memorization of the steps to multiply and can forget place value when multiplying. Learning to mastery is able to be completed more in Singapore math in part due to the lower number of topics per grade, with more time being able to be assigned to the mastery (p. x). Whereas the average number of math topics in a grade in Singapore is 15, New Jersey averages 28 math topics per grade. One explanation the authors gave for this difference is that textbooks must cover several states worth of math standards, and the result is that "individual topic coverage in the U.S. textbooks is much shorter and less comprehensive than what is found in Singaporean texts" (Ginsburg et al., 2005, p. xii). The common analogy of shallow learning, "a mile wide, an inch deep," would seemingly apply here.

In many math tests in American schools, including grade school and also on college entrance exams such as the ACT or SAT, multiple choice questions make up the majority of the questions presented to test takers.. The American Institutes for Research's (2005) examination of Singapore's high-stakes grade 6 test, the Primary School Leaving Examination (PSLE), showed that Singapore's test contained nearly twice as many constructed-response questions (p. xii). It is apparent that constructed-response questions require students to demonstrate their mastery of the math concepts, whereas with multiple choice questions the ability to see how a student solves a problem is minimized.

A close look at how multiplication in the third grade is taught in Singapore math and in the United States stands out in the American Institutes for Research study (2005). The study found that the Singapore math book taught multiplication facts of the numbers 6 to 9 as individuals, focusing more time on them, while the *Everyday Mathematics* textbook covered the multiplication facts in four lessons (p. 48). The study noted here that the Singapore math method used more visuals, including an array or row and column visual that activated multiplication facts that students would have learned from previous lessons. The American Institutes for Research study (2005) summarized the differences in the multiplication lessons as:

> The Singapore materials employ a much broader range of representations (arrays, grids, strips, bundles, number sentences, and related facts) and problem situations...both to develop conceptual understanding and to give students multiple models for practicing and memorizing facts. (p. 49)

A Review of Cognitively Guided Instruction (CGI)

Carpenter et al. (1996) summarized cognitively guided instruction, or CGI, as a development program that focuses on students' understanding of certain mathematical concepts (p. 4). Specifically, their main thesis in their 1996 work was that children already have a way to work out some of the common math problems they will encounter in the primary grades, including addition, subtraction, multiplication, and division (p. 6). The CGI framework is in stark contrast to the traditional teacher-centered instructional practices that many schools in the United States have used or are still using. Important to note here is that unlike the *Everyday Mathematics* or the Singapore math curriculum, CGI is not a math curriculum. Instead, it is a framework that focuses on getting teachers to see more closely how their students are understanding math problems, and how those students are subsequently going about solving them (p. 5). Moscardini (2014) added to the description of CGI as not being a teaching technique to be learned, but instead it is a recognition of "mathematical learning as a sense-making activity" (p. 71). The main ideas of CGI are discussed here, including examining what CGI looks like in a classroom, the developing metacognitive abilities of the students, and how CGI relates to or compares with the other math instruction discussed earlier in this paper.

CGI is a student-centered learning approach. It can be considered leaning toward a constructivist learning approach. Tobias and Duffy (2009) noted that although constructivism in learning may sometimes be difficult to define exactly, it is essentially learning in which the learner has an active role in how something is learned, a contrast from the traditional model where the teacher acts as the source and giver of knowledge (p.4). Moscardini (2014) explained that in a CGI math classroom or session students are given math problems to solve, with a major difference from traditional math learning being that the teacher pays attention not just to the answer but to how the students arrived at the answer (Moscardini, 2014, p. 71).

Carpenter et al.'s (1996) research described what kinds of problems are presented in a CGI math classroom, as well as what materials are used. The use of manipulatives is encouraged, especially during the start of the student's math learning. These manipulatives include counters, as well as individual blocks that can be connected. Problems that are solved include joining action, separating action, part-part-whole relations, and comparison situation (p. 6). An example of adding is detailed here. The authors gave an important example detailing how children's intuitive knowledge of math can allow them to get to the correct answer even though they may use a different operation that most adults would. In the example the student is told that a child has 7 dolls, and that if the child wants to have 11 dolls, how many more dolls would need to be bought (p. 6). This looks like a question most adults would use subtraction for, but in the study the child took out 7 counters, counted up from 7 to 11 by adding individual counters to get 4. Furthermore, another important detail that emerged from this child's strategy was that the child solved it by directly modeling or going word-for-word what the math question was asking (p. 7). Students eventually do not need to have the manipulatives counted, and instead can progress and transfer the concrete modeling to a more abstract form. An example given is a student having a group of 54 blocks and then adding 48 blocks to it and counting all together, to then just starting counting from 54 and then counting the 48 blocks to it, and then eventually to starting at 54 and counting using each finger as a tens amount, then as a ones amount (Carpenter et al., 1996, p. 11). In this way, the progression of CGI math is similar to Jaciw's (2016) description of Singapore math's *Concrete to Pictorial to Abstract* approach, where students concretely model a math problem, then visually represent the math learning and concepts before moving on to using more abstract symbols (p. 475).

Perhaps the most obvious difference between traditional math instruction in American classrooms and CGI is that in CGI math students are asked to, and are able to, explain their thought processes behind how they got their solution, instead of teachers directing the learning. This focus on metacognition is a significant aspect of CGI. As previously mentioned CGI leans toward a constructivist approach, with students experiencing the learning and finding the solutions as it makes sense to them. Carpenter et al. (1996) summed up the student-centered aspect of CGI, and how it differs from traditional teacher-centered learning,

> the emphasis shifts from teachers finding ways of representing mathematical knowledge for students to students constructing their own representations based on their intuitive problem-solving strategies. The

teacher is not perceived as the source of knowledge and does not provide ready-made explanations and representations. (p. 14)

Carpenter et al.'s (1996) study cited Schulman's (1986) term *pedagogical content knowledge*, defined as a teacher's knowledge of representing and explaining a subject to make it comprehensible as well as knowledge of students' conceptions, preconceptions, and misconceptions. Carpenter et al. argued that if teachers knew about their students' math thinking, it could help develop teachers' knowledge about how better instruction could be provided (p. 4).

Benefits of Cognitively Guided Instruction (CGI)

Current math instruction in America, especially in the primary grades, seems focused primarily on whether students arrived at the correct answer or not. As discussed earlier, while American standardized tests such as the ACT and SAT rely on many multiple-choice questions, the American Institutes for Research's (2005) report contrasted a Singapore standardized test for 6th-graders that required almost twice as many more constructed-responses (p. xii). In order to get detailed responses about their math reasoning from students, Carpenter et al. (2015) stressed the importance of thinking closely about the math problems that should be presented to students, with some of these ideas including using problems that can be directly modeled and in a context that students are familiar with (p. 135), and having students also restate the problem (p. 138). The metacognition aspect of CGI involves the teacher's careful and intentional eliciting of student thinking, asking things such as "Can you tell me how you solved that?," "What did you do?," and "Tell me about your strategy." (p. 140). The authors stressed asking questions directly relating to the response the student gave, and showed that with strategic questions a teacher could begin to see how far the student was advancing in their strategies (Carpenter et al., 2015, pp. 141-142).

The benefits of CGI math could potentially go beyond just learning and doing math a different way. Black (2015) mentioned one of these additional benefits of CGI, which is that since students are required to talk through their math thinking and processes students get more practice with utilizing verbal skills (p. 70). Black's

2015 study also found that teachers who taught CGI during the study noticed students learning from each other through the diverse ways the math problems were solved by students(p. 71). Black further noticed that some teachers encouraged students to find different ways to solve the same math problem (p. 71), a stark contrast from the standard written algorithm's one-and-done approach. Medrano's (2012) study also found that teachers reported students learned from each other through exposure to students' different solutions to a math problem (p. 119). Not only did students learn from other students, teachers in Medrano's study noted seeing students being more confident in solving the math problems (p. 119).

Chapter Summary

The literature review here looked at many aspects of math learning, and helped to guide how the capstone question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?* could be addressed.

The literature showed that a benefit of the standard written algorithm for math could be due to the fact that those familiar with it experience less math anxiety when they use it, compared to other methods introduced to them later on. However, the drawbacks included a lack of use of number place value, as well as being too procedure-driven. Along these lines, the phenomena of math anxiety was defined, as well as the ways math anxiety could be *passed down* from a parent with high math anxiety to their child through the child's exposure to math anxiety exhibited by said parents.

Math education in the United States and other countries was also examined, including test scores on international achievement tests. It was found that many of the highest-performing countries were East Asian countries, such as China, Singapore, Japan, Taiwan, and Korea. Although there is research to support the idea that Asian cultures with a more standard number counting method score higher in math tests in the early elementary grades, the literature showed that the initial gains were minimal and that they did not extend to the later grade levels.

Parental involvement and attitudes about their children's learning of math between American parents and parents with an East Asian background showed that parents of East Asian background viewed math as not requiring an innate ability, with Chinese parents overall stating they felt they were knowledgeable in math to help their students.

Many of the differences between the standard written algorithm for math, the Singapore math curriculum, and CGI math were discussed. In reviewing the literature it was found that the standard written algorithm for math minimized the student's use of place value, while in Singapore math many of the problems start out as concrete modeling before gradually becoming more abstract. In some of the East Asian countries, such as Singapore, the kinds of math questions posed to students require more steps and higher-level thinking to solve them. In CGI math, students in a sense self-direct themselves to find the correct answer, and through direct modeling and use of manipulatives, combined with familiar contexts or situations, students *think about how they think*, and teachers then have a better grasp of the student's math strategies. Several studies reviewed showed that potential benefits of CGI included increased student confidence in solving math problems, students learning from each other, and increased opportunities for students to utilize verbal skills.

Overall, the literature found that there are not as many inherent advantages in math in regards to specific countries as a typical person may think. Knowing that the higher levels of math achievement in the top performing countries cannot be attributed to something exclusive to a country helps guide the use of CGI in my project, to see what effects it could have on student performance and attitude, and to be more confident in implementing the CGI approach. In Chapter Three, how CGI math may affect students' performance on math tasks is detailed, as well as whether their attitude about math is affected after using strategies found in CGI math. The project entails a combination of a questionnaire, as well as strategically-constructed math problems that students will be able to use various math strategies to solve.

CHAPTER 3

Project Description

Chapter 2, the literature review, showed that the standard written algorithm for math did not teach for deeper understanding. The analogy of learning sometimes being “a mile wide, an inch deep” (Leinwand & Ginsberg, 2007, p. 34) was used to show the algorithm’s short-comings, especially when compared to the math education in Singapore which covered less math topics per year, but covered them more thoroughly (p. 34). The standard written algorithm, though, was only one of the potential factors that could account for American students’ underperformance in math. Furthermore, other countries, specifically countries that outperformed the United States on international mathematics assessments, went about teaching math in different ways. The American Institutes for Research publication (2005) found that Indonesia, a country performing highly on math achievement assessments, spent more time on multiplication facts as well as more visuals. Hunt and Hu (2011) showed that Chinese-born parents viewed themselves as more math-capable and therefore more readily able to assist their children in learning math. Some factors in students’ learning cannot be easily addressed, but offering different approaches to learning math and more authentic contexts could benefit students. The capstone project addresses this issue of math being learned at the surface level, of being learned through rote memorization of facts and procedures, specifically regarding multiplication.

This chapter goes over how I will implement the project to address my capstone question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?*. I start with a project overview, followed by the rationale for the project. I then go over the curriculum framework, Understanding by Design (Wiggins and McTighe, 2011), and describe how I will implement the project from that design standpoint. The math standards that are covered by the project are discussed. Participants for the project, including ideal school settings, are then described in detail. I will then turn to discussing how assessment will work with the project, as well as the project timeline. Lastly, the procedure for how the project will be implemented is discussed.

Project Overview

My project implemented math teaching methods and strategies that were prevalently found in cognitively guided instruction (CGI) for math. Although I introduced different strategies to the participants in the capstone, I aimed to follow the CGI framework of letting the students guide the teachers' teaching, of teaching to what the students already intuitively know. This point is summarized in Carpenter and Fennema (1996), "for CGI teachers the goal is to work back from errors to find out what valid conceptions students do have so that instruction can help students build on their existing knowledge" (p. 14). For example, if during the project a student required drawings to model their solution, the student would do so, with the teacher scaffolding and eliciting student responses to better understand the student's thinking. Solving math problems in the project starts with what the students know and how they act on that knowledge. The curriculum guide displayed examples that students may commonly come up with, as well as common phrases that would help elicit student math thinking and responses. The guide included different ways multiplication problems may be understood or modeled from different students. Experience from my personal life as well as my professional life has shown that most adults seem to only know one way of solving math problems, and this includes schoolteachers who are my co-workers. Especially at my place of employment, there does not seem to be many alternative methods offered for students when learning certain math concepts. The project sought to introduce math methods and curriculum that may benefit those who are having difficulty learning math through the main method taught during the normal school day, which is the standard written algorithm. If a student is made aware that there are several ways to solve a math problem, or at least given free reign to solve the problem, math anxiety may be reduced as there would not be the feeling of hitting a brick wall or dead end. There would be other strategies for students to utilize in place of the standard written algorithm.

Rationale

Much of the rationale for this project format, including the afterschool setting and the standards addressed, came from personal experience during afterschool homework help sessions at the school where I work. There were a significant number of students with whom I worked who showed evidence of a lack of place value. In some cases they would have a multiplication problem such as a three-digit number multiplied by a two-digit number, and not realize that the answer they have is incorrect just based on the fact that their answer does not contain enough digits. Besides this, some of the students who use the standard written algorithm for math when multiplying have a difficult time lining up the numbers correctly, which in these cases tested students' ability to be organized and neat with the numbers, not their ability to multiply. Students understand what multiplication is, and with the project I hope to show students that what they already know about multiplication can be utilized and built on to solve problems that may appear overwhelming at first, to build confidence in their ability to unpack math problems and work through even math problems that may appear unfamiliar initially. A student may see a three-digit by three-digit multiplication problem and feel overwhelmed by how many steps or the amount of time it would take to utilize the standard written algorithm to solve the problem.

The CGI approach in the project allows students to utilize their strengths, such as their ability to do repeated adding, to solve multiplication problems that appear overwhelming at first. The focus of the math work would no longer be focused on whether the student can immediately solve the math problems, but rather the focus is on how the student will solve the problem. The project aimed to get students to not think of math as something they either can or cannot do immediately, but instead to think of *how* they can solve the problem. The unrestricting way math problems were presented to students include presenting story problems that were relevant and relatable to the students, not directing students to utilize just one specific procedure, and to consistently elicit students for verbal response to describe what the student's thought processes were when solving the problems.

The afterschool setting is ideal for several reasons. Many of the students who are required to attend afterschool programming are required to do so due to falling behind in their core classes. These students no doubt have a higher need for additional or instruction that takes a different approach. The afterschool setting is flexible in how material is taught. Whereas the core teachers during the daytime hours at the school may be more reluctant to supplement their current curriculum, with the afterschool setting there is much more leeway with how an afterschool instructor decides to teach the material. The smaller class sizes during the afterschool program ensures that there will be ample opportunity for the teacher to really elicit responses from students regarding students' thought processes when solving math problems. Teachers in Medrano's (2012) study somewhat agreed with the statement that higher level questioning should be used sparingly, since higher level questioning takes up time that could be used to cover more math content areas for testing purposes (p. 118). An afterschool setting where time is dedicated to improving students' learning would alleviate this all-too-real concern teachers may have.

Design

For the initial start of the project I borrowed from Taylor-Cox's (2009) pre-assessment for multiplication solving (p. 118). This pre-assessment lets teachers know what strategies students currently use to solve multi-digit multiplication problems, as well as assess where common errors occur when multiplying. The project introduces different ways of modeling math solving, which included drawings and use of manipulatives, and included opportunities for students to talk through their math solving processes. The math lessons in the project are story problems that students are able to unpack easily, and are made to supplement worksheets that are completed during the school day, worksheets that may be more of just numbers multiplied without context. A student who may have difficulty multiplying two-digit by two-digit problems would be presented with story problems set in contexts the student would be familiar with. Familiar multiplication contexts include milk cartons in cases, students on buses, and seating charts on bleachers. Lessons in context would allow students to talk through their math thinking, requiring students to utilize more language articulation skills, which is especially beneficial for school settings where there is a large population of English Learner students. The lessons offer students another way to continue and advance math thinking and abilities when the standard written algorithm falls short.

Teachers implementing the project have a written guide that clarifies CGI principles. Some things clarified include letting students initiate and take charge of the problem solving, as well as useful phrases that help guide a student's explanation of their math work. Ideas for implementing manipulatives into the lessons are provided as well.

Curriculum Framework

Wiggins and McTighe's (2011) Understanding by Design (UbD) framework is what guided the curriculum that was developed. This framework is especially useful in designing the math curriculum used here since UbD "is predicated on the idea that long-term achievement gains are more likely when teachers teach for understanding of transferable concepts and processes" (p. 4.) UbD notes that there is a distinction between regurgitating facts, and true understanding (p. 6). As it relates to multiplication, one could think of an example as a student who uses rote memory to recall that *twelve times twelve equals one hundred forty-four*, while a student who more deeply understands multiplication may view it possibly as *twelve groups, with twelve items in each group*.

The unique aspect of UbD is that it starts with the desired results first, and the design goes *backwards* from there. As summarized by Wiggins and McTighe (2011), UbD starts with the first step, identifying desired results, then determining acceptable evidence, and lastly what activities will be used to generate evidence of learning (p. 8). To avoid what the authors have labeled as one of the *twin-sins of typical unit planning,* which in the elementary grades is a tendency for teachers to only focus on creating kid-friendly or engaging activities, I will be implementing math problems that are relevant in its context to the participants (Wiggins & McTighe, 2011, p. 8). The UbD framework was used to work toward getting students to actually understand the repeated addition property of multiplication, as well as the various ways multiplication problems can be broken down, such as being able to use partial products to solve difficult multiplication problems.

The UbD framework is especially important in the planning of math learning. Too often, the goal in math is to be able to quickly recall facts, as evidenced by timed multiplication and division worksheets. The

UbD framework allows more time to be focused on the learning that occurs between the introduction of the lessons, and the solution at the conclusion of the math problems. The next section discusses the specific math standards that I used the UbD framework to create a unit of learning for.

Math Standards Addressed

The project addresses four standards from the Minnesota State Standards, and they are

- 4.1.1.1, *Demonstrate fluency with multiplication facts*
- 4.1.1.2, *Use an understanding of place value to multiply a number by 10, 100, and 1000*
- 4.1.1.3, *Multiply multi-digit numbers, using efficient and generalizable, based on knowledge of place value, including standard algorithms*

(Minnesota K-12 Academic Standards in Mathematics, 2008, p. 16). These three standards were selected due to evidence of students' difficulty with multiplication from personal observation in elementary classrooms, as well as from learning from the Teaching Mathematics in the Elementary School course at Hamline, and also for their importance in math as the students advance to middle school and beyond.

Being able to quickly calculate multiplication facts is fairly self-explanatory, but understanding and using place value, as well as multiplying multi-digit numbers, are where students seem to only have surface knowledge. Students have been observed multiplying the non-zero digits in problems such as *3,000 x 5,000 = ?* and then simply counting and adding the zeroes. Multi-digit multiplication, as done by using the standard written algorithm previously discussed, is essentially a series of single-digit multiplication facts, lost in the process a focus on place value. Additionally, students are not able to identify where in the multiplication process they may have erred, usually having to ask the teacher to look over their work. Lastly, as students progress to middle school and beyond, distributing and factoring becomes much more common, with the standard algorithm really becoming obsolete. These standards are all a part of the fourth-grade standards, which lead to the next section on the participants that will be part of the project.

Participants and Location

The participants will be fourth-graders at a large charter school, located in a large urban city. The charter school currently contains grades pre-kindergarten through eleventh-grade, though the high school grades have significantly fewer students. Although the school is affiliated with a specific language and culture, any student can be enrolled at the school. The vast majority of the students in the school are of Asian descent, specifically Hmong, with a few students who are white, Black, Hispanic, or a mix of two or more races. Based on school-wide, as well as state-wide assessments, the student population is performing below average in both reading and mathematics when compared to state averages. Additionally, there is a large EL population, the largest group being in the lower grades, and then decreasing in the higher grades.

Procedure and Implementation

This project will take place during the after school program. There are several factors that make the after school program an ideal setting for the project. First of all, as mentioned earlier, many teachers at the school seem reluctant to teach math strategies that they are not familiar with, but with the after school program there is more flexibility for after school teachers to modify lesson plans. The students spend some time on finishing up their homework, but for most of the after school time the after school teachers can modify plans as needed.

Students who are in the after school program are usually selected due to their needing extra help with learning, particularly with reading and math. For many of these students, finding alternative ways to complete their schoolwork will benefit them. Some students I have observed were in a similar situation I was in when I was their age. Specifically, there were students who are unable to recall multiplication facts through twelve, and as a result rely heavily on a multiplication chart. For some students, following rote procedures seems to pose a great challenge, especially when having to multiply multi-digit numbers that require *bringing down the zero* to get the product.

Specific to the after school program during the spring semester, there is emphasis from the school to teach skills that will help students on the statewide standardized tests, in this case the MCAs. There will probably be more support from school personnel, in terms of allowing for modification of after school lesson plans, during the second half of the school year. The program would be anticipated to take around four weeks, with after school programming meeting only Mondays through Thursdays, with about fifty minutes of instruction time allotted.

The project heavily uses math problems in context. Several of the worksheets I have seen students work on during the current after school program usually involve numbers in isolation. Multiplication problems, for example, consist of a four-digit number multiplied by a 2-digit number, with the directions emphasizing students keep numbers properly aligned. For the project, I started out by checking for students' current math-facts skill level, using flash cards . There will be students who will not be able to quickly recall facts such as *9 x 7 = ?*, and for these students I would work on a deeper understanding of multiplication as repeated addition, as well as the ability to add partial products to get the full product. The use of context may be difficult for students whose first language may not be English. However, Carpenter et al. (2015) provides several suggestions to address this concern, including focusing on story comprehension and supporting each student's participation in the unpacking (p. 140). A significant difference between the math problems in the project compared to math problems students traditionally are given is the quantity of the math problems, as well as the quality of the math problems. The students are told to model the problems, to talk through the process, utilizing verbal skills, resulting in higher quality understanding. The quantity of the math problems are not the focus of the math tasks, and students are told the objective is not to complete the math problems as quickly as possible, but thoroughly instead.

To stimulate student interest and understanding, aspects of CGI as well as UbD regarding the importance of context in the math problems will be intentionally focused on and included in the project. Problems build upon each other, and follow a theme. For example, in a week students will solve multiplication problems regarding total costs for students to attend field trips, number of students that are present if a certain number of buses are filled, and amount of food to pack for a number of students attending the field trip. Context was intentionally limited to things that students have experience, or exposure to, during a typical school day.

The project is meant to introduce CGI concepts to students, as well as teachers, who have had very little exposure to CGI methods. The project is a way to present to students different approaches to learning math concepts, specifically multiplication. At the school where I am currently employed, which is an urban charter school with a majority of students of East Asian descent and EL needs, there is rigidness in the instruction. Students listen to the teacher disperse information and then set off to practice repeatedly and repetitively. Students in the afterschool program continue to struggle through solving multiplication problems using the standard written algorithm, a strategy some students do not fully comprehend but continue to struggle through. The success of this project is reflected in students' abilities to utilize the core principles of CGI. Teachers look to see if students are able to directly model the math problems, whether with manipulatives or through drawings, and teachers look for students' ability to solve a single math problem in different ways. Teachers look to see if students unpack and attempt the math problem instead of waiting for teacher instruction on what to do first. Lastly, teachers look for evidence of student learning of multiplication concepts through hearing students' explanations of the students' math processes.

Assessment

The majority of the assessments for this project involve formative as well as summative. For each individual lesson, the instructor will allow students to work out the math problems however the students see fit. The instructor's main role in the formative assessment aspect of the lessons is to continually check-in with how students are solving the problems. Some ways instructors can formally assess students include asking students to unpack the questions, describe any modeling used in the students' solutions, as well as asking students to confirm that their math work results in the solution. Summative assessments include problems posed to students once the lessons have been completed. The instructor would encourage students to use whatever space they need to solve the problems, as well as to keep whatever work the students had in solving the math problems. Each lesson in the project includes potential summative assessments for the instructor to implement. Being able to see and follow a student's work is an important part of the summative assessment aspect of CGI.

Project Timeline

The lessons in the project took about a month to complete. I started by having a rough outline of each lesson. Some of the lessons required more time than one after school session would permit, so I split a few of the lessons into two parts, spread over the same number of days. The lessons came out to be 10 lessons spread over three weeks, with one extra lesson with more challenging math problems on it. Ideally, the lessons would run two and a half weeks of after school programming, which in some schools is Monday through Thursday. A couple of extra days in the course of the three weeks allows the instructor to review a lesson students may have missed, as well as having room to revisit lessons that students may have had more difficulty with. Students would have an opportunity to use the math skills they learned during these lessons on other math work they may have throughout the rest of the semester's after school program. When I next teach after school I am looking to select a classroom of fourth-graders to implement the lesson plans I created.

Conclusion

Chapter three went over the logistics of implementing the capstone project, including which students will participate, the framework that curriculum was designed by, and the learning standards that were used as learning targets. The lessons are problems that require deeper understanding as they are unpacked and worked through, with a guide for teachers to use to elicit response to student work as well as to effectively scaffold student learning. The next chapter is a reflection of the project, including its impact on me, as well as possible future developments with the project.

CHAPTER 4

Conclusion

When I was thinking about how I could create a project that would answer my capstone question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?,* I was not really sure about several aspects of the project. I knew I wanted to create a project that would address some of the personal roadblocks I experienced with learning math in elementary school. I had an educational setting in mind that I would implement my project in, which is an after school setting, but some details of the project took me a longer time to finalize. Some of these details included things such as the scope of the project, including how many math standards to include, the amount of previous math content that would be reviewed, and how much time to allocate to certain lessons in the project. I felt the feelings of uncertainty with the math project initially stemmed from my past negative experience with learning math in school, but after completing the literature review, including a closer look at several cognitively guided instruction resources, I felt more comfortable with the topic of the project.

Chapter 4 goes over several sections, leading to a conclusion of the chapter. Chapter 4 starts with examining the major learnings I have had through completing the capstone project. Next, the literature review is revisited, looking particularly at sources that were especially helpful with the project. The implications and limitations of the project, as well as potential future research, is discussed. Ideas on how the results of the project will be communicated or used are in the next section. Before concluding the chapter, I discuss how the project is a benefit to the teaching profession, especially to the teaching of math in the profession.

Major Learnings

The greatest learning I had during the project was that of different perspectives. I had to keep in mind three unique perspectives, and find ways to create lessons in the project that would consider all three of the perspectives. First, my own perspective with the lessons in the project was that of a creator. Some of the language and descriptions initially used in the lessons in my project made sense to me, but I knew I had to continually revise them to ensure the lessons made sense to other instructors who may be using the lessons. There is a balance between assuming prior knowledge, especially math knowledge, from the potential instructors who may use the lessons, and not having the lessons be too wordy. Lastly, many of the lesson choices, such as activities and materials used, required me to take on the perspective of the younger students. I made the lesson plans for an after school setting and had to consider the limited amount of time students would have in that setting. For these reasons I had to revise the lessons to be really focused on narrower topics, and to spread the lessons out to a couple days when I felt it was needed. This having to see how students may see the lessons in the project was a little unexpected. I had assumed several times throughout the creation of the project that since it makes sense to me, the students will understand it as well. I had to change some of the wording to ensure that in addition to me, the instructors using the lessons will receive adequate guidance, and in turn be able to appropriately guide students to the completion of the lessons.

The project portion of the capstone taught me a lot about myself. I especially learned a lot about being out of my comfort zone, about addressing something that you may be passionate about, but initially uncomfortable with. The literature review went by fairly smoothly. In fact, there seemed to be more information than I really needed during that portion of the capstone, and I had to narrow my focus to just cognitively guided instruction in math. The project portion really took me out of my comfort zone. The project involved a topic that I was passionate about, which is learning math in ways that were not just the traditional written algorithm way. However, I was not sure if I had enough math background to develop a math unit for students. I referred to Carpenter et al's (2015) examples, as well as my Hamline math instructor's mathematics support website (Project for Elementary Mathematics).

Literature Review Revisited

The literature review process was a great source of learning for me. In a topic that is as number-oriented as math is, I was surprised to learn that there were so many components and factors involved in teaching math.

The piece of literature I used the most was Carpenter et al.'s (2015) work on cognitively guided instruction in mathematics. This textbook laid out the groundwork for CGI, including a breakdown of math concepts, as well as suggestions for how to implement CGI teaching in the classroom. Predictably, I zeroed in on the textbook's section on multiplication use by elementary students. Although the section on multiplication itself was fairly short, many of the ideas in that chapter were instrumental in the creation of the math lessons in my project. Some specific helpful bits of information included what prior math knowledge students should have grasped before approaching multiplication, potential ways students might solve certain math problems, and perhaps most importantly, examples of how to prompt students to describe their math thinking. With the standard written algorithm there would not be a need to ask students how they solved a math problem. With CGI the asking of students to describe or show how they understood a math problem and solved it is at the forefront.

The discussion by Baker (2014) was another piece of literature that I found very helpful, particularly with the creation of the lessons for the project. I had wondered why an instructor would not teach different methods for math, especially given the well-known sub-standard student achievement in math and science. Baker's discussion showed that teachers, especially those who were newer to the profession, had difficulty straying from what they themselves have learned growing up, in this case the use of the standard written algorithm to solve math problems. Furthermore, the other main takeaway from Baker's (2014) discussion was that many teachers felt the pressure to teach for success on standardized tests, which limited the amount of time teachers could use for teaching new math strategies. With this in mind, I made the lessons in my project more approachable. I kept the lessons detailed but succinct, making sure to not add too many multiplication concepts into one lesson. I planned my project for implementation in an after school program. From personal experience, after school programs for elementary students needing extra academic support are more lenient with how the time is used. With a more relaxed after school setting, instructors looking for alternative strategies or approaches to teaching math concepts can do so with more flexibility, as well as with time that does not feel reserved for teaching to meet standardized testing benchmarks.

The literature on student performance on international math achievement assessments, such as the *Programme for International Student Assessment* (PISA) and Trends in International Mathematics and Science Study (TIMSS), was literature that I felt helped present a strong case for the need for different approaches to teaching math. Fittingly, the math in these two international assessments showed that there is a need for improving math instruction in the United States. Although the numbers speak for themselves, it would appear from these two reports that the numbers have not been heard, and that different approaches to teaching and learning math should be implemented.

The literature I found and read during my capstone was overall very interesting and informative. Initially, I had wanted to explore the topic of math anxiety as well, but found that that was beyond the scope of my research question and project. Many of the other sources were very informational, I relied on these three the most during the creation of the project. I wanted to implement the CGI approach, to create lessons that were friendly toward newer instructors, which I myself am a part of, and to keep in mind that statistics from the international math achievement tests show that improvement in current math instruction is needed. The next section goes over the implications of my project.

Implications

The main implication for my project is that different, additional approaches to teaching math are needed in the elementary classroom. A student who has shown that they are unable to fully understand and utilize the standard written algorithm does not benefit from excessive repeated attempts with the standard written algorithm. In an after school setting it makes sense that students should be able to explore other strategies for solving math problems. Student-centered approaches are more involving for the student, and CGI math is something that can contribute to students taking more control of how they learn. Instead of assigning students extra work time to continue to fail at procedures that they never really understood to begin with, choices can be offered to students. Students are diverse in who they are and how they learn, including how they learn math. CGI math being implemented into a school's math curriculum, even if just for a short amount of time in an after school setting, is one way math learning and understanding in children can be improved. Next, limitations of the project are discussed.

Limitations

There are some limitations to my project. One of the most notable would be that it is mainly meant for use in a fourth-grade classroom. Students in lower grades may not be able to utilize the lesson plans, and students in higher grades may be more advanced at math and the lessons may not serve them particularly well. Another limitation that may be more prevalent is that the approach to teaching math in the lessons I created may be more different than what teachers and students are used to. Schools may not have additional math material that is similar to CGI math, so there may not be many opportunities for students and teachers to use CGI approaches outside of the project without further CGI resources. However, the lessons are created in a way that will allow instructors to easily duplicate then edit the math problems to create additional CGI learning opportunities for their students, as well as opportunities for the teachers to implement CGI strategies such as prompting for students to talk through students' math work. Potential research or projects are discussed next.

Future Research or Projects

CGI math is an approach that feels like an overarching mindset. Students are encouraged to solve problems how they see fit, with strategies that the students see fit. Having fourth-graders suddenly complete math problems the CGI way may be overwhelming, and these students may have difficulty with CGI math, particularly with the ideas of having free-reign to solve math problems, to verbally describe what they did to solve the problems, as well as increased visual modeling of math problems. To address these issues, future projects could involve implementing CGI math methods in other grades, particularly the earlier grades. If students start to utilize CGI math in kindergarten or first grade, for example, these students could more easily adapt to the CGI approaches in third and fourth grade. Students would not be the only ones requiring extra support with the CGI approach to math. From personal experience I know that when CGI methods are introduced, they can be overwhelming. For example, CGI may use significantly fewer math problems per math learning session since students are instructed to think extensively about the question, to model out the problem, and to discuss their strategy, as well as share with other students and the teachers. A future project that would be beneficial could include a professional development unit on CGI math for teachers. If teachers are aware of CGI and have practice with it, they will be more comfortable with letting students utilize CGI methods. The next section explores how I can communicate and use the results of the project.

Communicating and Using Results

I have a lot of faith in CGI math. Personally, it has shown me that success in math can be attainable, and that if you feel you have hit a brick wall, there are other ways to learn math. Many of the students I have worked with in an after school setting have shown that they really need math support. Some of the current math strategies being taught are not being learned by the students, and there is very little choice in strategies for these students. For example, I remember a student having significant difficulty multiplying a 2-digit by 2-digit number using the standard written algorithm, and then the student having to try to multiply 3-digits by 2-digits as well using the same strategy that has been difficult for the student. I plan to show other teachers or support staff that other ways of solving math problems may be one way to address the underachievement in math. Students who are in the after school program for extra academic support could potentially see improvement in their math solving abilities, and would be a great opportunity to show the benefits of CGI math instruction. Benefits to the profession will be explored next.

Benefits to The Profession

There are two significant benefits to the profession. I have heard many instructors say they are at a loss with how to teach a student something. For example, with the student I mentioned that had difficulty multiplying 2-digit by 2-digit numbers, then asked to further struggle with 3-digit by 2-digit problems, the instructor may not be inclined to offer other strategies if the instructor does not know of any other strategies.

This project would allow teachers to offer students additional strategies. Even students who are able to solve complex multiplication problems using the standard written algorithm may find that the CGI approaches make more sense to them, which leads to the next benefit to the profession: teachers can better explain math concepts to students.

Teachers who implement CGI methods, such as the ones used in my project, may find that they are able to better explain to students what is happening in math problems. Some teachers whose background is not in mathematics may have difficulty explaining to students certain math concepts. In a 3-digit by 3-digit multiplication problem that is solved using the standard written algorithm a teacher may not be able to communicate to students why the place value keeps getting shifted to left once each time you multiply to the tens, then hundreds place. Additionally, when a student errs on a multiplication problem such as the 3-digit by 3-digit multiplication problem instructors, myself included, have difficulty explaining to the student where they went wrong. With the CGI concepts in my project an instructor would be able to show numbers decomposed before multiplying, to show visually why a multiplication problem checks out, and to have more detailed written work to guide the students through how they arrived at their solution.

Summary of Chapter 4

Chapter 4 went over several important things I learned that stemmed from my research question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?* I first went over what I learned through having completed the capstone process as a researcher, with a main point being that I was taken out of my comfort zone. I then revisited some of the more important literature that found during the completion of my capstone, including literature describing why teachers are reluctant to learn and teach other math methods, the CGI textbook that laid the groundwork for the math lessons in my project, and literature outlining current and previous student achievement on international math assessments. The implications and limitations were discussed, including notifying instructors of potential wider implementation of CGI math, and the current project being intended for only fourth grade, respectively. Some future projects were discussed, specifically a professional development project to familiarize teachers with CGI principles, and to explore CGI learning projects for other elementary grades. Lastly, I explored how the use of the project in an after school setting, and the results of its use, could be shown to other teachers and support staff in order to garner more support for CGI math methods.

The capstone project has really taught me a lot about organizing not only my time, but my literature sources as well. In addition, the capstone process really took me out of my comfort zone to try to explore a research question that really interested me, but that I would have considered to not have a strong background in. The resulting project from my research question, *How can cognitively guided instruction (CGI) in math supplement current math in the elementary classroom?,* culminated in offering students another way to approach multiplication. It may only be one more way, but it is one more way than before.

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