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Developing Mathematical Thinking in High Schoolers Through "Curricular Thinking Tasks"

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Through “Curricular Thinking Tasks”

by

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ABSTRACT

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A review of the literature and reflection on personal experience show a need to rethink the way we teach and learn mathematics. A traditional math class relies on direct instruction and emphasizes decontextualized procedures and algorithms. This type of instruction does not work well for the majority of students, nor does it prescribe to the goals of math education in our modern world. We don't need "human computers" like we did before the internet. Instead, the primary goal for most high school math students is to develop the mathematical thinking skills needed to tackle new problems. Inspired by Peter Liljedahl's 2020 book *Building Thinking Classrooms in Mathematics,* the unit plans designed for this capstone project are rooted in curricular thinking tasks - open-ended problems meant to help students experience the curricular content and gain conceptual understanding before formalizing mathematical procedures. The goal is that these unit plans will be a resource for other teachers to aid in promoting students' mathematical thinking and engagement.

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CHAPTER ONE

Introduction

Thirty or so students file into their high school math classroom, surveying the poster that displays the Quadratic Formula and the number line that spans across the top of the front wall. The desks are in neatly designed rows, each facing the whiteboard which currently presents a warm-up problem. Students go through the familiar motion of finding their assigned seats, and some begin working on the warm-up. Meanwhile, others pretend to attempt the problem, while actually just waiting for the teacher to eventually go over it. Others do not even try to hide that they are on their phone. This pattern continues throughout the period as students copy down teacher-led, worked-out examples of the skill of the day, then try a similar example on their own or with the person seated next to them. Finally, students complete all the odd-numbered problems from the textbook to drill this skill into their brains. This is my memory of math class. Maybe it sounds familiar to you, too.

Research Question

Five years ago, I began teaching high school math, eager to make math class more engaging than this memory. During these first few years, my frustration with the traditional system for teaching math only grew while continuing to witness many students who believe that math is difficult and irrelevant, boring and formulaic. So, the traditional methods of math education obviously make math inauthentic and unapproachable, when math underlies so much of how we understand and live in this world. Through my undergraduate coursework, student teaching, and first few years in the classroom, I developed a passion for collaborative problem solving and student discourse. These are at the core of my educational philosophy. However, although my current students sit in groups, I still fall back into the boring, formulaic, “I do an example, you do an example” traditional lesson structure all too frequently. In response to these observations, my research question for this capstone project is: *How can teachers use “curricular thinking tasks” to enhance students’ mathematical thinking, engagement, and math attitudes?*

Personal Background

Growing up, I was always good at math, but did not necessarily enjoy it. I remember the praise after finishing each daily “Mad Minute” multiplication quiz perfectly in elementary school. I remember desks in rows, lectures, note packets, and practice problems upon practice problems. I remember looking forward to taking AP Calculus in high school, but mostly for the prestige, not for practicality or passion. I tested well and could learn algorithms quickly. However, like many of my current high school students, my motivation to learn math was mostly extrinsic. I did not understand math as a powerful tool for understanding the world, but merely as a set of rules to learn, a gatekeeper for admission to college, and a marker for individual intellect.

I liked math class because math class liked me, but also knew full well that many of my classmates dreaded math class. Thirty-four percent of respondents cited math as their least favorite school subject in a recent national survey of high school students. Further, math seems to be the overwhelming choice for least favorite subject. The second least favorite subject reported, English Language Arts, is disliked by a much smaller percentage: only 14% of students (Shaw, Dougherty, & Wohlstetter, 2017). Traditional math instruction, which emphasizes formulas and procedures, just does not work well for many students.

Entering college, my original plan involved a career in medicine or engineering. However, I quickly grew passionate about math education, after taking math classes that showed me how it could be a creative and collaborative endeavor. In each of my college math classes, we sat in small groups rather than the familiar rows. We were eagerly encouraged to form study groups and share our knowledge with each other to maximize our understanding. We were assigned group projects, partner quizzes, and take-home tests. Suddenly, the type of math teaching and learning I had grown accustomed to was completely abandoned. I could no longer simply copy down examples to then complete a set of practice problems that were merely these examples in disguise. Instead, I gained an appreciation for the problem-solving process through our collective struggle to complete difficult problems together. During the spring of my junior year of college, my math education methods course continued to poke holes in my conceptualization of a good math student and my definition of math in general. Math became more about the thinking process and logical reasoning than solely about the outcomes, and I hoped my future students would also come to see math this way.

My first math teaching experience took place the summer before my senior year of college. I was able to put the theory I had been learning into practice during a month-long summer math course for high school students. Together with another undergraduate student as my co-teacher, and with the guidance of an experienced mentor teacher who observed and debriefed each class with us daily, we created and facilitated a curriculum which was anchored in collaborative problem-solving. Students of varying abilities and grade levels, hailing from different schools within the city, actively worked on rich problems in small groups. They would display their thinking on posters, which served as the basis for our class discussions. In exit surveys, students recounted how much they enjoyed the class and this way of learning.

While eager to replicate this structure and learning environment in my future classroom, I fell short in my first five years of teaching. I felt constrained by the amount of curriculum that needed to be covered in a short amount of time. I also didn't have a rich task that would link to every curricular standard and was not sure where to find one, nor had the time. So, I often reverted to teacher-centered, direct instruction for the sake of time and ease.

Gradually, I added techniques for student collaboration to my repertoire as our district emphasized cooperative learning in our math department professional development. Specifically, Kagan Cooperative Learning structures were added to my teaching toolbox. These include activities such as Rally Coach, in which pairs of students take turns solving problems while explaining their thinking aloud, or Round Table, in which groups of students solve problems together by writing one step in the process before passing it on to the next person until the group agrees they have arrived at the correct conclusion (Kagan & Kagan, 2009). Using these strategies helped make instruction more student-centered. However, these instructional tools taken alone do not necessarily promote effective group engagement or positive math attitudes. Students still get off-task or perceive other group member's ideas to be more valuable than their own, or vice versa, and disengage. They also do not necessarily promote deep mathematical thinking, but are rather a framework for practicing procedures. While convinced that collaborative learning is the way to go, I remain interested in learning more about how to make it work most effectively in a traditional school setting.

Rationale

Reading Peter Liljedahl's 2020 book *Building Thinking Classrooms in Mathematics: 14 Teaching Practices for Enhancing Learning Grades K-12* inspired this capstone project. His research provides concrete changes to my current practice that would make my classroom truly ascribe to my educational philosophy. Since all fourteen practices Liljedahl (2020) describes would be difficult to implement at one time, he suggests starting with the first three:

1. *Begin each class with a "thinking task."* Instead of beginning lessons with teacher-led examples of a mathematical procedure, thinking tasks ask students to grapple with a problem and gain conceptual understanding before formalizing those procedures .
2. *Form visibly random groups (VRG).* Instead of teacher-selected or student-selected groups, students work on these thinking tasks in randomly-selected groups. Further, the randomization is made visible to the students, either using technology or cards.
3. *Use vertical non-permanent surfaces (VNPS).* Instead of working on the thinking tasks sitting down at desks, students work standing up at whiteboards.

Reflecting on my own experiences and desire to implement these three Thinking Classroom practices, it seems the most daunting endeavor is adapting the current curriculum into "thinking tasks". Liljedahl's book offers many "non-curricular thinking tasks", which promote mathematical thinking and engagement, but are not connected to state content standards. However, he only offers a few examples of "curricular thinking tasks" that directly relate to concepts we are required to teach. Additionally, while many teachers have shared glimpses into their initial implementation of the Thinking Classroom framework on Twitter or personal blogs, I have not come across a full unit plan to use as an example. For this capstone project, I will develop two unit plans for advanced algebra that include “curricular thinking tasks.” Additionally, they will provide teacher tips for implementation to promote students’ mathematical thinking, engagement, and math attitudes. This will ultimately serve as a reference for colleagues who are curious about Building a Thinking Classroom, but may be overwhelmed by the thought of reconfiguring the curriculum into thinking tasks.

Summary and Preview

My personal experiences as a math student and educator motivate my passion for researching the effects of beginning class with “thinking tasks,” using visibly random groups (VRG), and vertical non-permanent surfaces (VNPS) on students’ mathematical thinking, engagement, and math attitudes. Although I achieved success in the traditional math classroom, too many students do not. Despite this knowledge, in my first few years in the classroom, I struggled to fully deviate from these traditional teaching methods. Liljedahl’s (2020) *Building Thinking Classrooms* illuminates a framework to continue developing my own high school math classroom into one where more students are engaged in mathematical thinking and believe in their abilities to be successful.

In Chapter Two, I will review the current literature surrounding the research question: *How can teachers use “curricular thinking tasks” to enhance students’ mathematical thinking, engagement, and math attitudes?* This analysis will provide the history behind math instruction as well as current research about best practices. It will also further discuss and define the first three components of Building Thinking

Classrooms and how they relate to mathematical thinking and math attitudes. Overall, this literature review will place my research question in a broader context and offer more insight into why this capstone project matters.

Looking forward, Chapter Three will outline the research-based project developed in response to the literature and my personal experiences: two unit plans for advanced algebra that are based around “curricular thinking tasks”. Finally, Chapter Four will discuss my personal reflections regarding the capstone project process and implications of this project for the future.

CHAPTER TWO

Literature Review

Introduction

This chapter explores the literature that informed the creation of this capstone project responding to the research question: *How can teachers use "curricular thinking tasks" to enhance students' mathematical thinking, engagement, and math attitudes?* This literature review bridges the practices in Peter Liljedahl's book, *Building Thinking Classrooms in Mathematics* (2020), with outside studies and contexts. Additionally, it will connect these to the current research surrounding mathematical thinking, student engagement, and math attitudes. It will also provide further rationale and direction for developing this capstone project: two unit plans for advanced algebra that include "curricular thinking tasks," which will serve as an example for teachers to reference.

The first section of this chapter will outline the history of math education in the United States, from the origins of traditional instructional methods and curricular standards to why these continue to persist today. This history highlights the need for "thinking tasks" and a shift in math teaching paradigms. The next section will further describe the first three practices of a thinking classroom: thinking tasks, visibly random groups, and vertical non-permanent surfaces. This section will also define mathematical thinking and connect these three components of Thinking Classrooms to current best practices, such as cooperative learning and growth mindset development. The last section will describe "curricular thinking tasks" and how they are best implemented to promote mathematical thinking, student engagement, and positive math attitudes. Also detailed in this section will be examples of curricular thinking tasks, ideas for how to create one's own curricular thinking tasks, and open-source resources to find existing curricular thinking tasks. Together, the themes and research examined in this literature review will inform the creation of this capstone project, which is described more fully in Chapter Three.

History of Math Education

In order to make the case for revamping traditional math instruction, it is helpful to understand the history of math education in the United States. Looking at this historical context shows how traditional methods of teaching were derived. It also sheds light on how these methods remained largely the same amidst calls for reform and vast changes in the social, economic, and technological landscape.

Origins of Traditional Math

The standard mathematics content taught in high schools across the United States today has consisted of algebra, geometry, and calculus dating all the way back to the 1800s. According to Waggener (1996), these courses align directly with changes in admissions requirements for Harvard and other leading universities. Even arithmetic was considered a difficult subject usually reserved for study by teenagers during this time period, but when Harvard added algebra as a requirement for admittance in 1820, then geometry in 1844, high schools across the United States followed suit. However, although more and more people were attending secondary schools, a small percentage went on to pursue higher education at college. Thus, an increasing number of people were learning math that they perceived as irrelevant to them, leading to "a revolt against the idea of mathematics as a subject worthy of study for its intellectual value," (Waggener, 1996). This was just the beginning of the debate surrounding which math concepts students need to know and why math learning was important.

According to Klein (2003), support for the elimination of the more abstract topics of algebra and geometry from the common high school math curriculum sustained through the early 1900s. It was during this time that the progressive education movement, spearheaded by John Dewey, took root. The main tenets of progressivism included experiential learning through content that was directly relevant to students' lives and interests. Dewey's contemporaries William Heard Kilpatrick and David Snedden, who were colleagues at Columbia University's Teachers College, supported the principles of progressivism with their view of math education. They believed math should only be taught for its immediate practical merit, rejecting the value of the thinking skills students gain through learning math. Further, Kilpatrick believed that math is "harmful rather than helpful to the kind of thinking necessary for ordinary living," and that "we have in the past taught algebra and geometry to too many, not too few," (Klein, 2003). Snedden agreed, stating that algebra "is a nonfunctional and nearly valueless subject for 90 percent of all boys and 99 percent of all girls," (Klein, 2003).

Of course, many also opposed these sentiments, maintaining that mathematical thinking was valuable for all people. This viewpoint prompted the creation of the National Council of Teachers of Mathematics (NCTM) in 1920. However, while instruction of algebra and geometry persisted in secondary schools, opinions similar to Kilpatrick's and Snedden's held dominance until the 1950s.

Origins of Reform Math

In 1957, the Soviet Union successfully launched *Sputnik* into space. This created widespread urgency for more robust math and science education in the United States, turning the previous narrative on its head. As Waggener (1996) put it, "not only was it a matter of pride, it was a matter of national defense." Legislators issued the National Defense Education Act, which funded the creation and implementation of a more competitive secondary math curricula that now included calculus and other "New Math" concepts for high schoolers, such as modular arithmetic and set theory. Along with New Math content came a greater pedagogical emphasis on developing mathematical thinking skills. However, by and large, teachers felt ill-prepared to teach this new content or to shift their methodology. Additionally, parents were ill-informed of the changes, so "by the early 1970s, New Math was dead," (Klein, 2003). Focus returned to memorization of algorithms and rote practice of computational skills.

In 1983, the United States Department of Education published *A Nation at Risk,* once again calling for change in math and science education after studies reported that the U.S. was lagging behind other countries academically and economically. The return to traditional content and teaching methods in the 1970s had not produced competitive results. In response, the National Council for Teachers of Mathematics published *Curriculum and Evaluation Standards for School Mathematics* in 1989, which outlined national benchmarks for math education and proposed that national "goals for mathematics instruction had to be much broader than mere content mastery," (Schoenfield, 2004). The *Standards'* greater emphasis on developing transferable problem-solving skills and conceptual understanding over computation, however, was reminiscent of the New Math era and was again met with opposition. The controversy, known as the "math wars," prompted heated debate through the 1990s and 2000s. Then, in 2002, the No Child Left Behind law ultimately slowed "reform" efforts as schools focused on preparing students for standardized tests that were now tied to their funding, and which emphasized procedural knowledge (Schoenfield, 2007).

Current Perspectives

Today, while the average math class continues to employ mostly traditional methods, there is ample support and mounting evidence for "new math" methods and curricula to promote mathematical thinking and problem-solving skills. In 1970, computation was the second "most-valued" skill, only behind writing, according to Fortune 500. By 1999, teamwork and problem-solving were the top two, and computation had dropped to twelfth (Boaler & Dweck, 2016, p. 28-29). Now, twenty years later, problem-solving remains consistently among the most in-demand skills listed by professional networking and job search engines (Anderson, 2020; Bortz, 2021). In 1970, a single computer could take up an entire room, was difficult to operate, and was less efficient than doing hand calculations. Today, the majority of teens and adults in the United States own a smartphone and, thus, are able to bring a portable computer along with them anywhere they go. This is not to say that there is no place for procedural knowledge and computation in the math classroom. Rather, these are not dichotomous with conceptual understanding and thinking skills; all of these are important to students' mathematical proficiency (Ansari, 2015). Traditional methods, however, create an imbalance. They continue to emphasize procedures and computation, with little focus on developing mathematical thinking and problem-solving skills, despite their clear and increased importance in our modern world.

Thinking Classrooms

To address this imbalance in students' conceptual understanding and procedural fluency, Peter Liljedahl's (2020) *Building Thinking Classrooms for Mathematics* offers a framework for transforming the traditional math classroom to one with the primary goal of developing students' mathematical thinking. The following section further defines what is meant by mathematical thinking. It will also link the first three Thinking Classroom practices to other existing research into best practices for math teaching that honor students' thinking skills.

Mathematical Thinking

To better understand how teachers can develop mathematical thinking given its demonstrated importance, we must first clarify its definition. Mathematical thinking is multidimensional. Goos and Kaya (2019) emphasize "strategic competence" and "adaptive reasoning" as main themes in their review of the literature surrounding mathematical thinking. Strategic competence includes the "ability to formulate, represent, and solve mathematical problems" while adaptive reasoning includes the "capacity for logical thought, reflection, explanation, and justification," (Goos & Kaya, 2019, p. 10). Cuoco, Goldenberg, and Mark (2010) call these types of skills "mathematical habits of mind." They include finding and describing patterns, using representations and models, visualization, articulating in precise language, and generalizing from concrete examples in their definition of mathematical thinking. Moreover, Cuoco, Goldenberg, and Mark (2010) argue that not only should these thinking skills be part of any new math curriculum, they should be seen as even more important than the content standards themselves:

> Given the uncertain needs of the next generation of high school graduates, how do we decide what mathematics to teach? Should it be graph theory or solid geometry? Analytic geometry or fractal geometry? Modeling with algebra or modeling with spreadsheets?
> These are the wrong questions, and designing a new curriculum around answers to them is a bad idea. There is another way to think about it, and it involves turning the priorities around. Much more important than specific mathematical results are the habits of mind used by the people who create those results, and we envision a curriculum that elevates the methods by which mathematics is created, the techniques used by researchers, to a status equal to that enjoyed by the results of that research.While it is necessary to infuse courses and curricula with modern content, what's even more important is to give students the tools they'll need to use, understand, and even make mathematics that doesn't yet exist. (Cuoco et. al., 2010, p. 1)

The majority of high school students will likely not encounter concepts like polynomial division or complex numbers again after high school or college. It is not helpful to note that electrical engineers use imaginary numbers in a very real sense when the majority of students do not dream of being an engineer. While math concepts themselves might not be readily applicable to their lives after high school, all students will benefit from the mathematical thinking and persistence they build while learning these concepts.

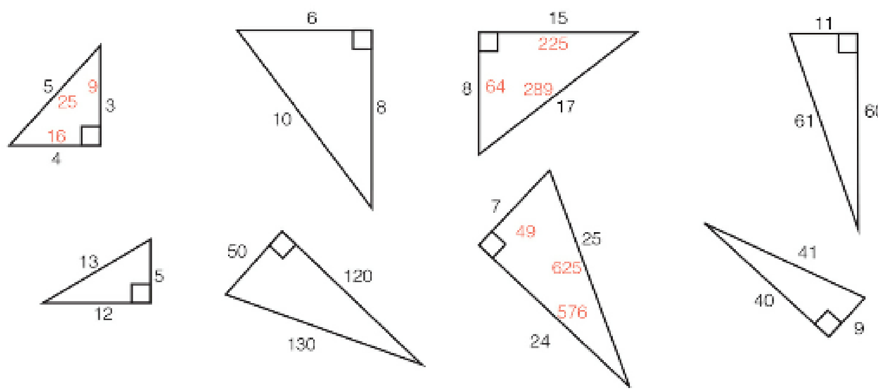
Thinking Tasks

The first practice in building a Thinking Classroom is beginning each class period with a "thinking task." In a traditional math class, students are first shown a mathematical formula or procedure and subsequently asked to apply it. For instance, a geometry lesson about finding the side lengths of a right triangle often begins by giving students the

Pythagorean Theorem , $a^2 + b^2 = c^2$,and defining sides *a, b,* and *c* on a diagram. The teacher shows a few examples of how to use the formula for different types of problems, then students practice using it on their own. A "thinking task" turns this sequence upside down. Students are first given a set of right triangles whose three side lengths and squared side lengths are already labeled, as shown in Figure 1, and the teacher asks what they notice about the picture or what patterns they can find (Liljedahl, 2020, p. 34). Students, in turn, think more deeply to develop the formula themselves instead of merely applying it in a vacuum.

Figure 1

Thinking Task Example



Productive Struggle. Thinking tasks are open-ended and force students to employ their *mathematical habits of mind*. Additionally, they ask students to push themselves to persist through sticking points - a process known in the math education world as *productive struggle*. In his TED Talk, Dan Meyer (2010) suggests that teachers are too nice for their own good. They cannot stand seeing a student sit in struggle and grapple with a problem for too long. Consequently, they decide they have to step in and save the day, directly guiding the student through the problem, perhaps even taking their pencil and writing it out for them. But by helping too much, teachers actually hurt students in the long run. Valuing and providing productive struggle by letting students work through problems promotes a culture of perseverance. Notably, “productive” is the key term here. The call to “be less helpful” certainly does not translate to “don’t help at all”; that would be preposterous. But the notion that there is a niche in which students can experience, grapple with, and struggle through a problem on their own or alongside peers makes learning more substantive and lasting. By jumping in as super-helpers, we also may inadvertently send the message that we do not believe that they can learn. This toys with students’ confidence, motivation and willingness to try new things, and effort through challenges. Students may even learn to feel helpless when presented with a problem if they always know that a teacher will be there to over-eagerly help out, creating a culture of reliance rather than of independent thinkers.

Another piece of evidence of this is seen in a study which compared the learning of two groups - one which received direct instruction and one in which students solved a complex problem with little teacher intervention. While the latter group did not come up with a correct answer, the ideas they discussed led to a deeper conceptual understanding of the topic. The teacher used the students’ initial thinking to explain the algebraic procedure, which students then had the opportunity to practice. In a post-test, this group “significantly outperformed” the first group who had only experienced teacher-led examples (Kapur & Bielaczyc, 2012).

Visibly Random Grouping

The second practice for building a Thinking Classroom is forming visibly random groups of three students to work together on the daily “thinking task.” While many teachers facilitate small group work, Liljedahl (2020) argues that the optimal groups must be randomly-selected. Students should not choose their groups, nor should teachers pre-determine them. Moreover, the randomization should be made visible. This might mean giving each student a card from a shuffled deck or using technology such as Flippity.net. Flippity.net is a website that allows teachers to upload class lists and create random groups of any size easily. The “demo” interface is shown in Figure 2 below. Teachers can choose to make random groups based on the group size, or based on the number of groups they wish to have using the tabs at the top of the page.

Figure 2

Retrieved from: https://flippity.net/rp.php?c=Katharine,Daniel,Meryl,Jack,Ingrid,Bette,Spencer,Marlon,Denzel,Cate,Robert,Jane,Dustin,Jessica,Gary,

The important part is that the randomness is completely transparent to the students. Furthermore, students should be assigned to a different, visibly random group every day.

Theoretical Perspective. Using visibly random groups makes thinking tasks more effective in developing mathematical thinking skills. This practice is rooted in cooperative learning theory, which has been studied and implemented for decades across grades and subject areas. Facilitating small group work is not a novel teaching practice. Contrary to more traditional paradigms which consist mainly of lecture while students sit quietly in rows and the teacher talks, cooperative learning allows ample opportunity for student-to-student discussion and is supported by developmental psychologist Lev Vygotsky's sociocultural learning theory. Vygotsky (1978) theorized that cognitive development and learning occur through social interactions. He also coined the concept of the Zone of Proximal Development (ZPD), which he defines as the space between what a student can do or understand on their own and what they cannot do. This in-between space is where the magic of learning happens best. Moreover, Vygotsky argues that learners are within their ZPD when they are interacting with peers, constructing knowledge and meaning together. Thus, to optimize learning, Vygotsky's theory suggests maximizing student discourse. When implementing "thinking tasks" using visibly random groupings, this is exactly how students are engaged. Specifically, when students are randomly placed with different students everyday, they benefit from the multitude of perspectives each of their classmates has to offer.

Many researchers have since shown how Vygotsky's sociocultural theory plays out in educational settings to increase student achievement. Robert Marzano, Deborah Pickering, and Jane Pollock (2001) included cooperative learning as one of their nine teaching strategies that increase student learning and achievement, citing positive effects seen in a number of previous studies. Kristina Whicker, Linda Bol, and John Nunnery (1997) added to this literature after noticing a gap. While there were many studies about the benefits of cooperative learning in the elementary and middle grades, there were fewer for secondary grades, and almost none specific to secondary math. In response to this lack of research, they developed a study to examine the effects of cooperative groups in precalculus courses. Compared to the test scores and surveys of a control class that learned independently, they too found greater improvements in achievement with the class using cooperative groups.

Studies have also shown that cooperative learning benefits students socially. Robert Slavin (1996), for instance, has devoted much of his career in educational research to these advantages. In one of his articles, “Cooperative Learning in Middle and Secondary Schools,” he cites not only increased academic achievement, but increased self-esteem and intergroup relationships as key assets of cooperative learning when it is implemented intentionally. Jo Boaler (2008) also concludes that heterogeneous, mixed-ability groupings increase relational equity, which she defines as “equity seen in relations that include students treating each other with respect and considering different viewpoints fairly” (p. 1). These studies are all sure to note that these benefits arise most effectively when cooperative groups are structured so that students are actually engaged in collaboration, not just learning independently while configured in groups. The Thinking Classroom practices of beginning class with a “thinking task” and using visibly random groups provide this sort of structure.

Vertical Non-Permanent Surfaces (VNPS)

The third practice for building a Thinking Classroom is using vertical non-permanent surfaces. Essentially, this means students work on their daily “thinking task” with their visibly random group while standing up, on an easily erasable material, most commonly a whiteboard. This is contrary to the typical classroom in which students are seated at a desk and working individually in a notebook. When used in conjunction with thinking tasks and visibly random grouping, VNPS help to further develop mathematical thinking, especially when considering how they relate to math attitudes and brain science.

Math Attitudes. Considering math has a deep-rooted reputation of being students' least favorite subject, positive math attitudes are pertinent to developing mathematical thinking. The term "math attitudes" pertains to students' self-perceptions, beliefs, and overall reactions related to math and math learning. This section outlines studies explaining how positive math attitudes are important to math achievement. It also aims to uncover the origins of students' negative math attitudes, including how to combat them to develop positive math attitudes, and how Thinking Classroom practices can play a role in this effort.

Students' attitudes toward math learning are strongly correlated with their math achievement (Ma, 1997). Positive attitudes yield positive math outcomes. Recent studies have sought to disentangle the underlying reasons for this association. One such study out of the Stanford University Medical Center (2018) deduced a neuroscientific link. Researchers found that elementary students who displayed a positive attitude towards math activated their hippocampus, the part of the brain related to working memory, more often while solving math problems. This was counter to the researchers' hypothesis that positive math attitudes were related to the "reward centers" of the brain, such as the amygdala. Furthermore, the study concluded that "math performance correlated with a positive attitude toward math even after statistically controlling for IQ, math anxiety, general anxiety, and general attitude toward academics," (Stanford University Medical Center, 2018).

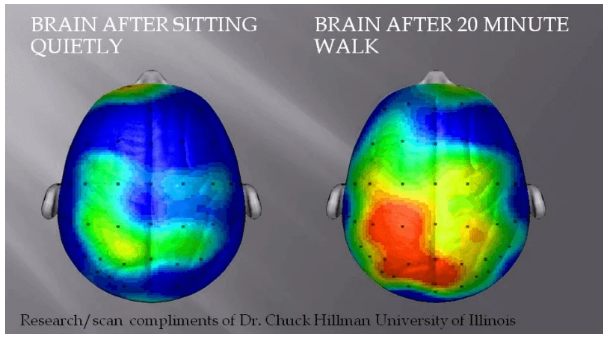
Additionally, there has been much research into the psychological explanation underscoring the link between math attitudes and achievement, especially rooted in the work of Carol Dweck (2014), surrounding growth and fixed mindsets. In her research, Dweck has shown a link between students with a growth mindset, who believe that their intelligence is malleable and can improve, having better learning outcomes than those with a fixed mindset, who believe that their intelligence is limited. Jo Boaler and Robin Anderson (2018) have supported and narrowed Dweck's theory to address how mindsets affect mathematics learning and achievement more specifically. They note that a big component of negative math attitudes is the belief that one is simply born a "math person" or not - that math requires a natural ability. Additionally, traditional systems of tracking students by ability and the attitudes and mindsets of parents and teachers can contribute to students' learned attitudes and hinder math achievement.

Since positive math attitudes are strongly linked to achievement, it is imperative for teachers to understand any strategies to sustain these attitudes or reverse negative ones. Boaler, Anderson, and Dweck have been working to spread the word about the power of a growth mindset, creating resources for teachers to use to dispel the myth of being born a "math person" and to encourage students who have developed a fixed mindset to flip the script. They both suggest that intentionally teaching students research about brain plasticity can promote the development of a growth mindset in secondary students. Recent brain science studies have shown that the brain is capable of growing and creating new synaptic pathways at any time. This means that all students have the potential to learn math at a high level, not just students who seem to have some innate mathematical ability.

Another strategy for developing a growth mindset, and therefore more positive math attitudes, is to help students to reframe mistakes as learning opportunities. Teachers must create an environment where students feel free to share their ideas without the pressure of being correct most of the time, because these little failures are where learning happens (Boaler & Anderson, 2018). Engaging in a Thinking Classroom can help develop a growth mindset in this way because the framework encourages students to rethink their definition of mathematics and can help foster the community and security for students to take mathematical risks. Instead of a set of problems to solve with one correct solution and one narrow pathway to get to that solution, during a thinking task, students are conversing with one another, making predictions, and asking questions. When math is so often perceived as rigid and formulaic, thinking tasks challenge students to understand math as a tool to help explain and model rich problems, which are not always straightforward. Furthermore, the non-permanence of a whiteboard helps students to forget about mistakes and understand that they are a part of the mathematical process. If they make a mistake, they can simply learn from it, erase it, and move on.

Neuroscientific Perspective. Another way that vertical non-permanent surfaces promote mathematical thinking development is seen in studies about how the brain functions when standing versus sitting down. While much has been said about the impact of sitting too much on physical health conditions such as diabetes and heart disease, it can also impact our thinking. In a study of adult office workers, simply standing up for 10 minutes every hour improved participants' scores on a "validated cognitive test battery", compared with staying seated the entire hour (Reynolds, 2017). This effect has been seen and studied in humans from elementary school students to adulthood (Hillman et. al., 2009). Across age groups, Hillman et. al. found increased P3 amplitude in the parietal lobe when performing cognitive tasks after exercise, as seen in Figure 3.

Figure 3



Retrieved from: https://standdesk.co/blogs/news/why-working-at-a-standing-desk-makes-your-employees-more-productive

This "reflects the allocation of attentional resources towards a particular aspect of the stimulus environment," (Hillman, et. al., 2009) - they were able to pay better attention to the task. Both adults and students in their studies performed better on the Eriksen flanker task, which tests "neurocognitive function," after they had gone on a twenty minute treadmill walk versus after they had been seated for twenty minutes. Moreover, elementary students also scored higher on standardized reading, spelling, and math tests after walking. While the increase was least pronounced on the math test, Hillman et. al. (2009) are quick to point out that they gave the math test last after the reading and spelling tests, and therefore was less likely to show the benefits of physical activity. They also cite two other studies which do show a significant positive correlation between physical activity and math achievement.

Curricular Thinking Tasks

The discussion so far in this literature review has bridged classical theory and current research about best practices for math teaching. It has shown that the traditional structures do not sufficiently develop the critical thinking skills necessary for our modern world. It has also shown that in order to foster mathematical thinking, students must be given opportunities to *productively struggle* through rich problems, as they do in a Thinking Classroom framework. There are ample problems freely available which provide these types of opportunities, but are not directly tied to the curricular standards teachers must cover. It is more difficult to find rich tasks that help illuminate each content standard required by the state. This is what is meant by the term "curricular thinking tasks." The following are some examples of open-source resources for "curricular thinking tasks" found through this research:

- *Illustrative Mathematics* - a website with tasks, searchable by content standard or topic. The curriculum writers describe it as both "problem-based" and "designed to address content and practice standards," (Math Curriculum, 2021).
- *Math Medic* - a website curated initially by two current teachers, Lindsay Gallus and Luke Wilcox, at the "most diverse high school in Michigan." The website includes lessons that follow an "experience first, formalize later" philosophy for a range of high school courses.
- *Open Middle* - a website with tasks searchable by topic and grade level. Open middle problems are described as having the same beginning and end result, but "multiple ways to approach and ultimately solve the problem," (Johnson & Kaplinsky, 2019).
- *nRich* - another website that has problems which are "ideal for developing subject knowledge, mathematical thinking, problem-solving skills and good mathematical habits." (University of Cambridge, 2021)

- *Desmos Activities* - an online tool, especially for graphing. Many teachers have created and shared digital activities that harness the power of technology to "open up a world of possibilities for students to explore concepts more deeply, collaborate with their peers on problem-solving, and apply knowledge creatively as mathematicians." (About Us, 2021).

Adapting Current Curriculum

However, when unable to find a rich, thinking task for a certain content standard using resources such as these, it is possible to adapt existing, traditional curriculum into thinking tasks. One way Liljedahl suggests doing this is dubbed "thin slicing," which is probably best described through an example. In Figure 4 below, teacher Lam Nguyen (2021) shows how he gets his students to discover how to solve a quadratic equation by completing the square.

Figure 4

Thin Slicing Example

$$3x = -5$$
$$5x + 12 = 3x - 6$$
$$2x + 18 = 0$$

$$x^2 = 16$$
$$x^2 - 9 = 0$$

$$(x-2)^2 = 9$$
$$(x-2)^2 = -9$$
$$(x-3)^2 + 5 = 21$$

$$x^2 + 2x + 1 = 25$$

$$x^2 + 4x + 1 = 25$$
$$x^2 - 7x + 1 = 0$$
$$3x^2 + 9x + 5 = 0$$
$$-\frac{1}{2}x^2 + 6x - 1 = 0$$

Retrieved from: https://drive.google.com/file/d/1gwjso-tt_95qCUOsiy0wjRQqk3R5lSuM/view

In the first three equations, students solve basic linear equations. In the next two equations, students solve simple quadratic equations. In the next three equations, students solve quadratic equations that are slightly different than the one before. Finally, they get to the ninth equation, and they are motivated to try and make the equation look like the three directly before it to be able to solve. This is exactly what they do when they complete the square. Nguyen (2021) says to “treat the formula like the end of a movie - don’t spoil the ending!” Each successive problem changes only slightly. As they work through the problems, students are able to generalize the procedure to find the formula themselves. Nguyen points out that this string of problems looks like a typical textbook. So, when used intentionally, this makes course textbooks a good starting point for developing thinking tasks through thin slicing.

Summary

This chapter reviewed literature relevant to the research question: *How can teachers use “curricular thinking tasks” to enhance students’ mathematical thinking, engagement, and math attitudes?* It outlined the history of math education and traditional math teaching methods and discussed how current math students benefit from the mathematical thinking skills they develop in high school math more readily than much of the content itself. Peter Liljedahl (2020) provides a classroom model based on developing these thinking skills. He suggests starting each class with a rich task, using visibly random groups, and vertical non-permanent surfaces. This chapter included descriptions and examples of each of these practices. It also linked them to other research concerning productive struggle, cooperative learning, and math attitudes.

This literature review provides a foundation for the development of this capstone project. The capstone project consists of two unit plans for Advanced Algebra rooted in “curricular thinking tasks.” These unit plans provide detailed lesson plans for implementation in a Thinking Classroom framework to promote mathematical thinking, student engagement, and positive math attitudes. This will be described in more detail next, in Chapter Three.

CHAPTER THREE

Project Description

Introduction

This chapter will take the personal experiences and rationale from Chapter One and the guiding research discussed in Chapter Two to outline the project designed to answer the capstone question: *How can teachers use "curricular thinking tasks" to enhance students' mathematical thinking, engagement, and math attitudes?*

The first section will describe the project itself, which consists of two unit plans for advanced algebra that emphasize mathematical thinking development. The description begins with the rationale for designing this curriculum. The unit plans are based on both Liljedahl's (2020) *Building Thinking Classrooms for Mathematics* and Tomlinson and McTighe's (2006) *Integrating Differentiated Instruction and Understanding by Design,* which is discussed further in the first section. The second section identifies the setting and audience for which the project is intended. The next section discusses how the project's effectiveness will be assessed. Finally, an outline of the timeline and action steps for completion will be provided.

Project Description

Rationale

Both personal experience and the literature indicate a need to rethink the way we teach and learn math. While I found success in a traditional math class with almost entirely direct instruction and an emphasis on decontextualized procedures and algorithms, this does not work well for the majority of students. It also does not prescribe to the goals of math education in our current world. We do not need "human computers" like we did before the internet. Instead, the primary goal for most high school math students is to develop the thinking skills needed to tackle new problems. This capstone project consists of two unit plans for advanced algebra that aim to enhance students' mathematical thinking. The unit plans include "curricular thinking tasks" to be used alongside visibly random grouping and vertical non-permanent surfaces. The unit plans each span two to three weeks and include detailed individual lesson plans based on curricular thinking tasks.

Understanding By Design

Along with the Thinking Classroom framework (Liljedahl, 2020) discussed previously, I used Tomlinson and McTighe's (2006) Understanding by Design (UbD) framework to design these unit plans. Using UbD involves three stages. First, I identified the "desired results" of each unit. This includes the content standards, standards for mathematical practice, and other learning outcomes for the unit. Second, I decided how students would show evidence of their learning, including both formative and summative assessments. Tomlinson and McTighe (2006) use a photo album as an analogy for assessment (p. 63). For these unit plans, students will show their understanding in a variety of ways, through the daily thinking tasks, written tests, informal conversations with teachers and peers, and participation in review games, for example. Lastly, I developed the lesson plans themselves.

Specifically, the two unit plans address the following math standards for Minnesota (Minnesota Department of Education, 2008):

- **Standard (9.2.1.#):** Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.

- **Standard (9.2.2.#):** Recognize linear and other common functions in real-world and mathematical situations. Represent these functions with tables, verbal descriptions, symbols, and graphs. Solve problems involving these functions, and explain results in the original context.
- **Standard (9.2.3.#):** Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.
- **Standard (9.2.4.#):** Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential, and n^{th} root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.

They also address the Common Core's Standards for Mathematical Practice (SMPs). Most notably, the unit plans aim to develop the following:

- **SMP #1:** Make sense of problems and persevere in solving them.
- **SMP #2:** Reason abstractly and quantitatively.
- **SMP #3:** Construct viable arguments and critique the reasoning of others.
- **SMP #4:** Model with mathematics.
- **SMP #8:** Look for and express regularity in repeated reasoning.

Because the UbD framework begins with the end in mind, it is often called "backwards design." However, Tomlinson and McTighe (2006) believe that this model is effective because it helps teachers to prioritize content by importance and to remain learning-oriented as opposed to "activity-oriented." When starting the unit planning process with each lesson plan, followed by creating assessments, followed by determining learning goals, it is easy to fall into the trap of basing your classroom on activities, which “are like cotton candy - pleasant enough in the moment, but lacking in long-term substance,” (Tomlinson & McTighe, 2006, p. 29) rather than learning and conceptual understanding.

Each lesson plan details approximate timing, notes and materials, and “what teachers/students are doing” at each point during the lesson. For the most part, the lessons follow a launch - explore - summarize format. The teacher begins the class by posing the thinking task problem statement, then students spend the bulk of the time in their random groups at a vertical whiteboard discussing the problem and documenting their thinking. Finally, the vertical whiteboard work serves as the basis for whole class discussion to summarize and formalize the math behind the thinking task. After this, students have time to check their understanding through individual practice problems. The core of each lesson plan is a “curricular thinking task,” which was either created from scratch or found and adapted online.

Setting and Audience

I considered my own students as I designed this project. I teach in a large, suburban, public high school in Minnesota. The school serves around 2400 students and participates in Project Lead the Way, offering special programs for Biomedical Sciences and Engineering. The school is growing in both racial and socio-economic diversity in recent years. The participants of this action research, in particular, are students across my Honors Advanced Algebra courses. The students represent various grade levels, with 16% in 9th grade, 24% in 10th grade, 57% in 11th grade, and 3% in 12th grade. Roughly 43% identify as non-white and represent eleven different non-English home languages.

While our school is implementing a new, problem-based curriculum in our Intermediate Algebra and Geometry courses, our upper-level courses are still using traditional textbooks and curricula. This is why I decided to focus my efforts on designing curricular thinking tasks for Honors Advanced Algebra. The intended audience for this project is teachers within my own school and collaborative team (CT). I will share my unit plans and findings with them, and also ask for their feedback throughout the implementation process. Should the project prove to be effective, I could also share it with teachers at the other high schools in the district, who implement the same curriculum, and our Teaching and Learning Specialists (TaLS).

Assessment

To assess the effectiveness of these units, with special attention to students' mathematical thinking, engagement, and math attitudes, I will conduct periodic student surveys. These will be administered via Google Forms and will include both Likert scale questions and open-ended questions. I will elicit opinions of students more informally through verbal conversations as well. Additionally, I will keep a teacher journal in Google Docs to record observations about students' mathematical thinking and engagement during the curricular thinking tasks. "Journals… are more than a single data source - they are an ongoing attempt by teachers to systematically reflect on their practice by constructing a narrative that honors the unique and powerful voice of the teachers' language. Regardless of your specific area of focus, journaling is recommended as a way to keep track of not only observations but also feelings associated with the action research process," (Mills, p. 127). I will ultimately codify and analyze any patterns that emerge. Additionally, I will ask for feedback from my colleagues, with whom I will be sharing my unit plans. During our weekly CT meeting, I will update them on my own implementation, assist them if they are also using the unit plans at the time, and honor any suggestions they have to make them more effective.

Timeline

The following list outlines the basic timeline for completing this capstone project.

1. Before school begins, create the first unit plan: Sequences & Series. Plan non-curricular tasks and class building for week one.
2. Administer initial survey during the first week of the trimester.
3. Begin implementing Thinking Classroom practices with non-curricular tasks the first week. Then, begin facilitating "curricular thinking tasks" as outlined in the unit plan.
4. Document observations about mathematical thinking and engagement in a teacher journal, in a timely fashion. Take photos to document student work and engagement at least weekly.
5. Administer semi-weekly check-in survey, including open-ended questions about math attitudes and the class structure.
6. Create the second unit plan: Rational Functions, keeping in mind observations and learnings from the first unit.
7. Administer final survey during the final week of the trimester.
8. Analyze the survey data, comparing results from initial survey and final survey. Codify any patterns that emerge.
9. Report findings and reflections in writing Chapter 4. Share work with colleagues and peers.

Summary

This chapter detailed the design of the capstone project responding to the research question: *How can teachers use "curricular thinking tasks" to enhance students' mathematical thinking, engagement, and math attitudes?* The project includes two advanced algebra unit plans developed using Liljedahl's Thinking Classroom framework and Tomlinson and McTighe's Understanding by Design unit planning model. They provide teachers with curricular thinking tasks and detailed outlines for implementing them.

The next and final chapter of this capstone paper offers my reflections on the capstone project process as a whole, referring back to the literature review and planning stages through the project development stages. Additionally, it will look to the future, providing implications and future directions for the project.

CHAPTER FOUR

Reflection

Introduction

Throughout my experiences as both a math learner and educator, my desire to make math meaningful and relevant has only grown. While much of the content my high school students learn in advanced algebra may not directly apply to their lives after graduation, the mathematical thinking skills they develop will certainly come in handy. Setting out to transform my classroom into one which emphasizes the development of these thinking skills, Peter Liljedahl's (2020) book *Building Thinking Classrooms* was truly inspiring. His insights, particularly the suggestion to start each class with what he calls a "thinking task," ignited the research question for this capstone project: *How can teachers use "curricular thinking tasks" to enhance students' mathematical thinking, engagement, and math attitudes?*

In Chapter One, I described in detail the motivation behind pursuing this capstone project based on my teaching and learning experiences. This set the stage for Chapter Two, which built on my personal experiences by connecting them to the existing literature concerning mathematical thinking development in the high school classroom. The literature review also made connections between Liljedahl's Thinking Classroom model, cooperative learning theory, and math attitudes. After gaining a foundational understanding of the current literature, I created my capstone project. The project consists of two unit plans for advanced algebra centered around "curricular thinking tasks," open-ended tasks meant to help students experience the curricular content and gain conceptual understanding before formalizing efficient mathematical procedures. Each lesson within the unit plans includes a thinking task tied to the lesson's content standard. The thinking tasks are meant to be explored by students in visibly random groups at vertical whiteboards. Also included in each lesson plan is guidance for launching and summarizing each thinking task.

This final chapter reports personal reflections and learning outcomes as a result of completing this capstone project. It will also return to the literature review to determine new connections after creating this project and further highlight the most influential research. The next section discusses the limitations of the project. Lastly, this chapter will discuss the future implications and potential benefits of this project to colleagues and the profession as a whole.

Major Learnings

The space to wonder, create, and read about developing students' mathematical thinking through completing this capstone project has helped me learn about myself as a researcher, writer, learner, and teacher. This is especially true amidst this specific era in education. After over a year of distance and hybrid learning due to Covid-19, I was at once eager to improve my teaching and also exhausted from the challenges of trying to teach math during a pandemic. This capstone process gave me focus and direction. It held me accountable to stay the course as a reflective teacher who continuously searches for ways to enhance my practice and students' learning. Writing this paper and developing this capstone project gave me the opportunity to be more intentional. I was able to step back and take the necessary time to plan how I could align my teaching practices to reflect the value of mathematical thinking skills. This has long been my desire and the capstone process was the catalyst to actually make it reality. One way this came to fruition in my project was re-familiarizing myself with the Standards for Mathematical Practice (SMPs), such as SMP 1: Make sense of problems and persevere in solving them. While my lessons previously included and communicated content learning goals to students, now they also have at least one SMP attached and communicated. This keeps me focused each day students' need to develop these mathematical thinking skills. It also clarifies to students that these types of skills are just as important as the content itself, if not more.

Another major, and somewhat surprising, finding was the amount and availability of free resources with ready-made rich, "thinking tasks" tied to curricular standards. I was usually able to find a curricular thinking task on one of these three websites used most frequently: Illustrative Math, Desmos, and Math Medic. The Open Middle website was also helpful, especially for extension problems. All of these websites were easy to navigate and searchable by topic. Moreover, I learned that many of the notes I had previously used during guided lessons were already "thin-sliced," as Peter Liljedahl (2020) puts it. This means they built slowly on concepts students already knew to make connections to a new topic. With zero or few edits to my previous notes, students could complete a set of problems in their visibly random groups with little guidance from me. From there, we could develop generalizations and efficient procedures. Additionally, if the notes were not easily "thin-sliced", the lesson could be introduced using "smudged math," in which certain parts of the process of a completed problem are "smudged out." Students then use their observations to fill in these missing pieces (Nguyen, 2021). Overall, I was happily surprised that one of these strategies could be applied to every lesson.

Through this capstone creation process, I also learned a lot about developing students' mathematical thinking skills and about math education more broadly. I valued the time to read and sift through the literature surrounding this topic, which has been on my mind for quite some time. As a practicing teacher, you get bits and pieces of the most recent educational studies and best practices in professional development sessions. However, returning to the time-consuming task of finding, reading, understanding, and synthesizing scholarly journal articles and writing an academic paper reminded me about the level of detail necessary within the research process. Over my teaching years, the term "research-based" has been overused and therefore has become rather convoluted. Reading and analyzing a variety of existing literature helped me regain appreciation for academic research.

Literature Review Revisited

In order for students to develop mathematical thinking skills and gain a deeper and more lasting understanding, they must experience productive struggle (Meyer, 2010; Kapur & Bielaczyc, 2012). They must grapple with interesting problems and persist through challenges. This was a theme that arose throughout the literature and was highly influential while developing this project. In creating, adapting, or finding the daily thinking tasks for these two unit plans, the goal was always to provide the correct level of challenge so that any struggle would indeed be *productive*. This also ties back to Vygotsky's (1978) Zone of Proximal Development. I was always assessing whether the thinking task would place the small groups in this sweet spot where they all could learn with the support of each other.

Productive struggle asks students to truly *think*, to employ mathematical habits of mind (Cuoco, Goldenberg, Mark, 2010) and not to merely mimic mathematical procedures. This is the impetus of Peter Liljedahl's 2020 *Building Thinking Classrooms in Mathematics,* which was the most influential research used for this project. I frequently revisited this book throughout the creation of these unit plans to maintain the spirit of the original research. I kept this quote in mind, in particular: "Problem solving is a messy, non-linear, and idiosyncratic process. Students will get stuck. They will think. And they will get unstuck. And when they do, they will learn—they will learn about mathematics, they will learn about themselves, and they will learn how to think," (Liljedahl, 2020, p. 20). Rereading this book after creating two unit plans based on the first three practices of starting class with thinking tasks, using visibly random groups, and using vertical non-permanent surfaces, garnered a whole new appreciation and understanding to the years of research he reports. My plan is to continue adding more of the remaining eleven practices to my toolbox after adding more curricular thinking tasks to my repertoire.

Researching the history of math education also had a lasting impact on the creation of this capstone project. Understanding that our current math curriculum and traditional teaching practices have remained stagnant through major societal changes, especially in technology, provided extra motivation to create and find these thinking tasks. Cuoco, Goldenberg, and Mark's (2010) work regarding mathematical habits of mind further emphasized this discrepancy, noting specifically that the world our students inherit will also be much different than today. Thus, the primary goal of education is to develop the thinking skills needed to solve "unknown" future problems. Mathematical thinking skills are as important, or more, than content knowledge itself. Again, this idea really stuck with me and bolstered my motivation to create these unit plans.

Limitations

These two unit plans just scratch the surface, as they only address four of eleven Minnesota Math Standards for high school. They are also linked to the learning targets that my school district emphasizes. While advanced algebra teachers outside of my school district may still benefit, they might have to rework the unit to meet the needs of their specific community and school expectations. Additionally, these units were planned with an accelerated advanced algebra course in mind, so teachers may need to revise the timeline to fit their students' needs.

Unfortunately, we are currently still in the midst of the Covid-19 pandemic. Some schools maintain social distancing guidelines and limitations on grouping students. These unit plans are predicated on small group work, ideally with random groups changed every single day and standing up at vertical whiteboards. While the thinking tasks may still be beneficial to students' mathematical thinking while sitting down in the same "pod" of students, visibly random groups and vertical non-permanent surfaces have shown to be highly effective for increasing engagement and thinking (Liljedahl, 2020). Along these lines, another consideration is the cost or access to vertical whiteboards and the space to use them, which may not be accommodated at all schools.

Future Research

There is much room for improvement and further work related to this project. These unit plans only address four of the Minnesota Math Standards, and there are eleven total standards in high school math. Each standard also includes around 6-8 more specific benchmarks. These also do not include curriculum after advanced algebra, such as precalculus and calculus. Clearly, there are more thinking tasks to be found, adapted, and created for other courses. After completing this capstone, I am determined to continue finding and creating curricular thinking tasks for future lessons.

Additionally, while creating these unit plans and reviewing the literature I was urged to rethink traditional paradigms for assessment. Individual, paper-pencil tests seem inadequate in assessing students' progress in a thinking classroom model. For the sake of practicality in assigning traditional grades, written tests all too often emphasize procedural fluency and speed over conceptual understanding and depth of knowledge. Future work will include streamlining ways to better communicate and document students' work on thinking tasks to both parents and students. This also means finding ways to use this information in reporting a required letter grade.

Communication and Benefit to the Profession

Apart from bettering my own practice in developing students' mathematical thinking skills, the hope is that these unit plans will aid other teachers, particularly those looking to increase student engagement and autonomy. I plan to share these unit plans with my colleagues teaching advanced algebra in our collaborative team planning time. After continuing to develop a repertoire of curricular thinking tasks across other courses, I will share those with colleagues teaching those classes as well. While these unit plans are meant to include enough guidance to implement on their own, it may also help to offer the opportunity for teachers to observe me using this methodology. Perhaps, it will be helpful to host a professional development session or short series of sessions to model lesson implementation, inclusive of other content areas as well.

Conclusion

Overall, the entire capstone project process has been a valuable experience with major personal and professional learnings related to the research question: *How can teachers use "curricular thinking tasks" to enhance students' mathematical thinking, engagement, and math attitudes?* Being afforded the time and space to review the current literature provided new understandings about developing mathematical thinking and improving student engagement and math attitudes. This context aided the creation of two unit plans using Liljedahl's (2020) Thinking Classroom framework, with "curricular thinking tasks" embedded in each lesson. Moving forward, these ideas will continue to inform future unit plans for more courses to address more math standards.

Math class in the modern world needs to develop the thinking skills that students can use to solve a variety of problems. These are the skills they can employ when they encounter the types of problems that do not even exist currently. Ideally, the unit plans created for this capstone project, which ask students to persevere and think through rich curricular problems, are a step in that direction.

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