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# Designing A Professional Development Series For Educators On The Benefits Of Mathematical Manipulatives For Elementary Students' Understanding And Mindset

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# DESIGNING A PROFESSIONAL DEVELOPMENT SERIES FOR EDUCATORS ON THE BENEFITS OF MATHEMATICAL MANIPULATIVES FOR ELEMENTARY STUDENTS' UNDERSTANDING AND MINDSET

by

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To my family, friends and advisors, who were there every step of the way.

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# CHAPTER ONE

## Introduction

Throughout a typical school day, many subjects are covered, mathematics being one that elementary students often dislike. There are numerous reasons why students develop an aversion to math, but I believe lack of understanding and engagement tend to be at the forefront. Age can also be a factor, as love of math fades as elementary students get older. Many mathematics curricula are implemented all over the world, with various strategies and techniques. Knowing what the best practices are can be difficult and challenging. Educators need to have the right tools in order to encourage and promote positive mathematical attitudes and thinking.

Providing a high-quality math education to all of my students is my top priority each year. I tend to work with students who struggle with mathematics and they do not always have a growth mindset when it comes to this subject. The term "growth mindset," comes from the book, *Mindset: The New Psychology of Success* (Dweck, 2006). A growth mindset gives students a more positive outlook on challenges in their life and how to work through them. I want to use the tools and resources that will best assist them in their learning, then share this information with my fellow educators. This leads me to my topic, *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement.*

In this chapter, I will describe my personal journey, why this topic is essential to me and my students, and how it could affect the world of education. Mathematics is vital

in everyday life, and should be made the best experience possible. According to Boaler (2012), 60% of jobs need people to be comfortable working with numbers and understand mathematical reasoning. I am hopeful that this project will have a positive impact on the way we view teaching and learning mathematics.

**My Mathematical Journey**

Growing up, math was not my favorite; I struggled through it. I was not interested in completing workbook pages and was never really sure why I had to learn about it in the first place. I would get extremely frustrated when I did not understand a topic and thought there would never be an end in sight. It was my mother's help at home that guided me in the right direction. She is also an educator and took various mathematical courses while getting her undergraduate degree. She showed me how to represent numbers through hands-on materials and, as a result, I was engaged more in the content. Using manipulatives helped me better understand mathematical topics and become a problem solver while also helping make math more enjoyable.

It was not until I went to college that I was excited about teaching mathematics to others. I was a tutor at an elementary school nearby my college and assisted students who were struggling with various topics. As time went on, I noticed how I kept gravitating towards the math time slots. I think this was due to the fact that I saw myself in the struggling math students and wanted to help them just as my mother had helped me. At the time, the students and I were using more pictorial and abstract representations of numbers, but I wondered if there would have been a greater understanding and impact with manipulatives.

My love for math grew even more when I took my math methods course while pursuing my undergraduate degree in Elementary Education. It changed my perspective on how a person could learn and teach math. I grew up learning math at school through memorization or through one standard strategy. My professor showed me that "One of the key myths that hold students back is the idea that some people are 'math people' and that struggle is a sign you are not a 'math person' " (Boaler & Anderson, 2017, p. 1). A strategy that works for one student might not work for another. They progress at different paces and may need more time to process through a concept. This is what leads me to my position today, an Elementary Math Specialist.

When I first started out as an educator I had a strong set of teaching strategies and techniques to implement, but I still was not confident I knew best practices. After being in this position for the last five years, I have seen the power of manipulatives in the classroom, particularly for student understanding and mindset in math. The concrete and tactile experience students are able to partake in truly does provide a solid foundation for their skills to develop and grow. I want to share the positive effect I have noticed with my fellow educators. Students should have the opportunity to reach the highest levels of math and, with manipulatives, that is possible.

**Students' Stories**

As I mentioned, many of the students I work with struggle with math like I did as a child. I have seen how powerful using manipulatives can be for them. In one particular instance, one of my first grade students was having a difficult time understanding addition. They were working on adding numbers to 10 and stopped every time they saw

an addition symbol (+). As we were reviewing what addition was, I took out a handful of counting bears for them to use. After a few attempts, they started to truly understand addition and how it worked. Not only was their understanding of the topic stronger, but the counting bears were able to represent a real life situation for the student. They were able to see the importance of addition in their everyday life.

Manipulatives have not only benefited younger students, but upper elementary grades (3-5) as well. One of my fifth grade students was having a challenging time adding and subtracting fractions with unlike and like denominators. More specifically, they were struggling with how to get unlike denominators to be the same and why that was even important. Explaining the concept to them with numerals was not making sense. We took out some fraction circles and started combining two unlike fractions together. They were then able to see that depending on the denominator, the circle could be cut into different size pieces and it would not make sense to add or subtract them. The expression on their face was priceless, and is one every student should experience. Using manipulatives took their understanding to a whole new level.

## Obstacles for Teachers

The educators in my building are amazing individuals. They all bring strengths to our elementary team, and support one another when needed. However, there have been some deficiencies during their mathematical instruction when it comes to manipulative use. There are various reasons and explanations why that I hope to combat with the information I have gathered from my research.

### *Lack of Time*

One of the most prominent arguments is the lack of time. I do believe we should have a longer math block during the day, but right now that is not feasible with our schedule. I also understand the pressure to cram as many math topics in before state testing, but students need to be able to apply what they learn and not just memorize facts. There are many mathematical skills that are built on one another each year and if there is not a solid foundation, the students will continue to struggle and have gaps in their learning.

***Tools not Toys***

Another misconception among staff is that manipulatives cause too much trouble and are seen as toys. While I think it is beneficial to allow students to explore and play with manipulatives, they also need to know that they are math tools. During my math groups, we discuss and come up with expectations for our tools. Some of them include not playing with them during instructions and using them gently so others may as well. Students might need some reminders, but if you have high expectations for them, the benefits are endless.

***Age***

The age of students is another factor that affects manipulative use at my school. As students enter third grade and above the likelihood of manipulative use decreases. Some educators think that because of their age they should be able to think more abstractly, but that is definitely not the case for every student. If there are disparities in their mathematical knowledge these hands on tools can further their growth and hopefully get them to where they need to be.

### *Lack of Experience*

For others, it is due to the lack of experience. Some teachers are inexperienced with using manipulatives. They might not be opposed to the idea, but have not been informed of their benefits. I believe this to be the case with some of our newer staff members. Which is why this project is so essential. Teachers should be lifelong learners just like their students.

## Components of a Positive Math Experience

I want the students at my school to receive the best math education possible and for their teachers to have the knowledge and support to make that happen. In order to see that through, teachers need to help students develop a deep understanding, a strong mindset, and critical thinking skills. Teachers should also create engaging and differentiated activities where students are collaborating with classmates. Students need to be able to understand and engage in the content in order to think critically at any age. They might need the instruction to look a little different depending on the student, but when they are able to collaborate the learning will only extend further. These components are critical to be successful in math, but they are not enough. I know manipulatives can help build these essential skills when they are used in mathematical instruction.

## Chapter Summary

Math can be a difficult subject for some students. They may not have found a strategy that works for them yet or may not have the right mindset. It can depend on a couple of different factors and manipulatives can play a large role in changing that. With so many different curricula to choose from, having a solid set of tools is important. I

always want to provide the highest quality instruction possible for my students and I would like to share this information with others. That leads me to want to create discussions around *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement.*

I was a child who dreaded math time during the school day, it took my mother's guidance to appreciate and understand how I best learned math. Tutoring at an elementary school during my undergraduate years demonstrated to me how I could assist students who were also struggling and how I could be a crucial part of that. My math professor taught me how to teach math in a new way, filled with multiple strategies and critical thinking. I want my students to fully understand each concept, have a positive mindset towards math, stay engaged, use materials regardless of age, collaborate and learn in the way that best suits them. I want to encourage all educators to implement the best math practices, and in order to do that I need to share my findings, allow discussion and time to practice.

In Chapter Two, the literature on math manipulatives, understanding, mindset and age is reviewed. In Chapter Three, the methods of my research project will be discussed. During Chapter Four, I will share my concluding thoughts and highlights.

# CHAPTER TWO

## Literature Review

The purpose of this chapter is to analyze research done on the topics of manipulatives, understanding, mindset, and age in the learning of elementary school math. This review will highlight information on *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement.*

Students should be able to get the most out of their math experience, and I want to provide them and their teachers with the best education possible. In order to shed some more light on the subject, I will dig into research on math manipulatives to understand what has worked in the past and what has not. Research also notes the levels of understanding students go through and how it correlates to mathematics. They also look at various factors that affect understanding. The sources demonstrate how motivation, engagement and interest impact a student's mindset. Lastly, age is unpacked to note developmental stages in relation to learning mathematics. All of these topics are essential to truly understand what high-quality math instruction should look like.

### Manipulatives

Belenky and Nokes (2009) define manipulatives in math as " Physical objects that are supposed to help the student concretize his or her knowledge by expressing concepts and performing problem-solving steps with them" (p. 103). There are many different kinds of math manipulatives, such as cubes, counting bears, pattern blocks, geoboards, dice, and fraction tiles. There are also virtual manipulatives--"Digital objects that resemble physical objects and can be manipulated with a mouse" (Cockett & Kilgour,

2015, p. 2). The sources in this section will provide insight into the beneficial effects of math manipulative use on various academic skills. They will also cover setbacks when using manipulatives. Lastly, they will provide effective ways to utilize manipulatives in a classroom setting.

***Benefits of Manipulatives***

Previous research has found that the use of manipulatives has increased students' academic performance (Battle, 2007; Boggan et al., 2010; Carbonneau et al., 2013; Liggett, 2017). They have helped educators introduce topics and even helped create common language between the teacher and students (Larbi & Mavis, 2016). It is also important to note that students were more successful with math manipulatives when they were told what the tool was used for in an organized way (Moyer, 2001). Students who correctly used manipulatives were excited and motivated to learn (Battle, 2007). If a student is motivated, they are more likely to perform well mathematically (Buehl & Alexander, 2005).

Even when motivated, students learn in many different ways. Abstract methods of teaching mathematics should not be the only ones implemented (Kablan, 2016). In fact math manipulatives can be used to help facilitate abstract thinking (Barmeyer, 2004). The more a student uses manipulatives early on, the more it will help them with their abstract thinking (McNeil & Jarvin, 2007). Some students need a more tactile experience, and it seems there are positives in doing this. Manipulatives have also shown to help students who are struggling in mathematics to catch up with their fellow peers (Boaler &

Anderson, 2017). The following studies have also shown the benefits of manipulative use in elementary classrooms.

Carbonneau et al. (2013) looked into the effectiveness of using math manipulatives to teach versus only using abstract symbols for elementary students. They wanted to find the average effect manipulatives had on instruction, if there were any relationships between manipulatives and learning outcomes, and if instructional practices had any influence on the outcomes. Fifty-five previous studies were identified and reviewed. Carbonneau et al. (2013) found that concrete manipulatives helped students problem solve and retain information. They also found that the level of instructional guidance affected the use of manipulatives. For example, if a teacher only incorporated manipulatives and did not explain them or provide time for understanding, the tools do not have a large impact on student learning.

A quantitative study on manipulatives also wanted to know if there was a significant increase in academic achievement for elementary students when they use manipulatives (Battle, 2007). The population sample was a group of first grade, low achieving students who were working on adding and subtracting whole numbers. They were split into two groups of eight, one who used manipulatives and one who did not. A pre and post-test were given to both groups and it was found that manipulatives did improve the understanding of those who used them. Liggett (2017) also noted promising results from his research on the effectiveness of mathematical manipulatives for elementary students. For this study, forty-three second grade students were divided into two random groups. Again, one group used manipulatives (experimental) and the other

group did not (control). Pre- and post- tests were used to collect data, and it was determined the experimental group scored higher on the post-test compared to their peers.

During Graham's (2013) math manipulative study, she interviewed and observed three 3-5 grade teachers for six weeks. They all agreed that using manipulatives is more beneficial for their students than not, especially when routines and expectations were put into place. Students utilized tools such as base ten blocks, cuisenaire rods and fraction tiles during their math instruction. The 3-5 grade teachers also noted that using these tools was important for their standardized test scores. The study did demonstrate the need for more teacher support when teaching with manipulatives. This telling data proves that the use of manipulatives positively impacts student understanding for grades 3-5.

Kablan (2016) took a different approach in his study on math manipulatives. The goal was to note the effect lecture style teaching and manipulative use had on various learning styles (diverging, assimilating, converging and accommodating). The study also focused on the length of time manipulatives were used to see if it had anything to do with achievement. Three environments were created. One of the groups learned through a lecture style, the second contained both lecture and manipulatives, and the final group only utilized manipulatives. All three had the same teacher. Results were found through post assessments and stated that manipulatives were beneficial, especially for the accommodating and diverging groups. They also found that increasing the amount of time used with manipulatives was helpful to students.

Another study that noted the benefits of manipulative use for elementary students was conducted by Dennis (2011). The purpose of this qualitative research study was to

note the effects of manipulatives on math understanding. There was a control group, taught with a lecture style of teaching, and an experimental group, who used manipulatives. 40% of the participants were male and 60% were female in each group. The effectiveness of the materials were measured by a pre- and post- test. Post-test scores were higher for the experimental group compared to the control group. There was a significant difference between female scores in the two groups, where that was not the case for males.

***Barriers of Manipulative Use***

Although there are many benefits to using manipulatives during math instruction, some teachers disagree with the recommendation to use them. According to Gilakjani (2012), teachers tend to prefer the lecture style of teaching, because they think their students will be more successful. Some teachers are concerned that students can have a difficult time transitioning from concrete to abstract understanding while using manipulatives (Graham, 2013; McNeil & Jarvin, 2007) or that they should just complete worksheets to better their mathematical understanding (Moyer, 2001). Students may also interpret the manipulative differently than the teacher had originally introduced (Moyer, 2001). Moyer (2001) found that many teachers regard manipulatives as too "fun." She studied three different K-8 classrooms and found that teachers were using them as rewards instead of math tools. If the students behaved well, they got to use manipulatives during math on Fridays. In a way, this disconnects the correlation students make between what they are learning and the tools they are using; it can be hard for them to see manipulatives as a quantity versus a toy (McNeil & Jarvin, 2007). Other teachers just do

not feel comfortable or know the correct way to use manipulatives (Graham, 2013). This is where professional development would be crucial. If a teacher did not know how to correctly utilize the tools, further instruction is necessary, just as we would do for our own students.

### *Effective Teaching Strategies with Manipulatives*

Effectively teaching students with manipulatives can be harder than educators realize. Some believe if students just play with them, it will lead to understanding (Puchner et al., 2008). Unfortunately, that is usually not the case. Teachers will get a better idea of students' understanding when they are using manipulatives if they listen to their students talk about their thinking, observe students working independently and in cooperative groups, ask them "Why?" questions instead of yes or no questions, and have students write the whole answer sentence to their math problem (Hedden, 1997, as cited in Larbi & Mavis, 2016).

## Understanding

Tomlinson and McTighe (2006) argue that "Learning happens within students, not to them. Learning is a process of making meaning that happens one student at a time" (p. 22). For a student to understand mathematics, they need to be given tools to be successful. For teachers to provide the necessary tools, they need to recognize how students understand the material. The first part of this section provides a definition and covers various levels of understanding. Following that is an overview of factors that affect understanding, both positively and negatively .

### *Levels of Understanding*

Bloom et al. (1956) created a taxonomy of understanding and divided it into six categories. The categories are as follows: Knowledge, Comprehension, Application, Analysis, Synthesis and Evaluation. The updated version also includes the last category, Creation (Krathwohl, 2002). The levels are hierarchical and a student usually needs the prerequisite skill before they can attain a higher level. For example, before you can apply a concept you need to comprehend it. Before you can evaluate a concept, you have to be able to synthesize it. In the mathematical world, before a student can analyze place value, they need to be able to apply it, and before they can apply it, they must comprehend it.

***Positive Factors***

Fennema and Roberg (1999) wrote about the process of working on understanding that "[a]ny task will do as long as it is engaged in for the purpose of fostering understanding, not for the purposes of completing a task" (p. 24). If an assignment, assessment, or project is created for students to deepen their understanding, they will be better prepared to move up Bloom's taxonomy. According to Tomlinson and McTighe (2006), if an educator plans backwards, it should allow students to understand big ideas. First, an educator should identify the desired results. Next, determine acceptable evidence and then plan learning experiences and instruction.

The questions used in the classroom can also create conditions for better understanding (Boaler, 2016). Educators should start with questions concerning basic recall, and gradually work their way up to composing or relating new ideas (Bloom et al., 1956). Selecting the correct tools is also essential when teaching for understanding, such

as manipulatives (Fennema & Romberg, 1999). It is important for students to have multiple representations of math topics, due to their various learning styles and intelligences. By using math tools, students are able to add a whole new level of understanding (Boaler, 2016). “Inevitably, to grasp the key concepts and principles of any subject also helps us better understand ourselves, our lives and our world” (Tomlinson & McTighe, 2006, p. 38).

When students first begin to learn new concepts, such as fractions or telling time, they experience varying degrees of dissonance in finding a connection between prior knowledge and what they are currently learning (Piaget, 1926). Sometimes students struggle to understand new topics or even ones they have already gone over (Bloom et al., 1956). To combat this, students need to learn mathematics in a way that enables them to apply this knowledge and use it to solve new problems. When a student is able to apply what they are learning, this is evidence they truly understand a concept (Fennema & Romberg, 1999). Every teacher wants their student to reach the highest level (create) but there are difficulties that can get in the way.

### *Negative Factors*

Math anxiety and lack of confidence can negatively affect a student’s math understanding (Kaskens et al., 2020). Math anxiety can be caused by a number of things, such as lack of number sense, gender stereotypes, anxiety within teachers and parents, and not enough parent involvement (Szczygieł, 2020). Unfortunately, math is the subject which most people have a fixed mindset about giftedness (Boaler, 2016). Scientific evidence suggests the difference between those who succeed and do not are the messages

they are receiving and the opportunities they have to learn (Boaler, 2016). If students are not being encouraged at home or school, their likelihood of understanding a math topic will decrease.

## Mindset

Research shows that motivation, engagement, and interest play a role in each student's mathematical mindset, both positively and negatively. Dweck stated that "Math is the subject in most need of a mindset makeover" (as cited in Boaler, 2016, p. x). Math can be a stressful time for students, and it can often cause anxiety (Ashcraft & Moore, 2009). This section will first address what research says about what motivates students intrinsically, extrinsically and socially in a school setting. Next, it will address the role of engagement and how proactive classroom management strategies and specific math-focused instructional strategies can increase it. Last will be research highlighting the role of interest and how it affects academic outcomes for students.

### *Motivation in Mathematics*

Motivation in math strongly correlates with success in the subject. This is formed early in elementary school (Orosco, 2016). Gottfried et al. (2007) wrote that math motivation "captures the extent to which individuals embrace math challenges, value the importance of math abilities, and are motivated to perform well in math" (p. 317). The more motivating and less anxiety-ridden a math lesson tends to be, the more students will get out of it (Wang et al., 2018). To be a motivating learning environment, students need to feel respected, feel prideful of their accomplishments, and work collaboratively with their peers (Bowman, 2007).

Both intrinsic and extrinsic factors can positively affect motivation in the classroom (Bowman, 2007). In order to promote both intrinsic and extrinsic motivation in students, teachers should have positive expectations, provide positive feedback, revisit values, and provide new perspectives (Bowman, 2007). When a student is motivated, it can provide them with a positive experience and affect how they interpret new skills and knowledge (Dweck, 1986). This success then shows the students that if they work hard and try, their understanding of math topics can grow. This also allows them to associate math in a useful way and could lead them to look forward to the next lesson.

### *Impact of Engagement*

While motivation plays a large factor in students' mathematical performance, engagement is also a key for successful learning experiences. Nagro et al., (2018) pointed out the immense task teachers face daily of "engaging diverse populations of students with varied individual needs to sustain learning and promote positive student outcomes" (para. 4). It can be difficult to engage students, especially those who have challenging behaviors (Baker, 2005). Implementing whole group responses is a strategy that can lead to higher engagement (Nagro et al., 2018). Choral responses and using dry erase boards are just a few examples of what whole group responses could look like. Including movement can also increase engagement, whether it be dancing or using gestures (Nagro et al., 2018). Student choice is another factor that contributes to higher student engagement (Shevin & Klein, 2004) and offers students some autonomy of their own learning. Engaging students in their learning leads them to make connections between the

content and their life. The more students see the purpose of math, the more they will realize its importance and have a better attitude about it.

### *Importance of Interests*

Just like student choice holds power in increasing engagement, student interest does as well, and involving students' interests will likely enhance a lesson. At the early elementary level, when the activity is interesting to the student, it is more meaningful (Johns, 2015). Incorporating interest can also increase effort, persistence, and engagement (Phan et al., 2018). Building interests into math lessons has also been found effective for students with developmental disabilities (Ozen & Ergenekon, 2011). In a math class, interest can be found when a student makes a personal connection to the word problem they are working on, or when their favorite sport is involved in the lesson. In contrast, an activity with a lack of interest to a student can be demotivating or lead to unwanted behaviors (Phan et al., 2018). If done poorly, however, involving perceived student interests in the classroom can be seen by some teachers and students as irrelevant (Bergin, 1999). Using time to color, for example, or extending a lesson for interest purposes when it is not needed, can be seen as a waste of time and resources (Bergin, 1999).

## Age and Development

It is important to consider age in this study, because children go through various stages of development as they mature in their mathematical understanding. I also want my project to reach teachers and students from Kindergarten to fifth grade. This section will focus on development in relation to mathematical skills, such as the work of Piaget,

whose “ work on children’s quantitative development has provided mathematics educators with crucial insights into how children learn mathematical concepts and ideas” (Ojose, 2008, p. 26).

***Relation to Mathematical Development***

Piaget (1953) said, “When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with mental growth” (p. 74). A student's reaction to any mathematical concept will depend on age and it can be distinguished into 4 developmental stages (Piaget, 1953). Each stage can occur for months or years, without skipping or going out of order (Ojose, 2008). In the Sensorimotor Stage, which occurs from birth until the formation of language, children will be able to link numbers to objects (Piaget, 1977). Educators in this stage should provide children with various counting activities to increase their number development (Ojose, 2008). The counting could occur with their fingers, or other physical objects.

The second stage, Preoperational, should involve some problem solving with blocks while also having the student explain their thinking (Ojose, 2008). It is difficult for students at this stage to reverse operations. If a student knows 3 + 4 = 7, they will not correlate it to 7- 3 = 4. Concrete Operations is the third stage, and this is where students' basic skills grow rapidly (Ojose, 2008). To develop their skills more during this stage, students should experience hands-on activities (Burns & Sibley, 2000). Hands-on activities are very influential in this stage, and provide a link from concrete to abstract representation (Ojose, 2008). In the last stage, Formal Operations, students are able to take educated guesses and predict possible outcomes. Students begin to develop abstract

thinking through symbols and are able to apply their knowledge learned in new situations (Ojose, 2008).

There are critics of the developmental stages that Piaget has created. In the concrete Operational Stage, for example, research has found that students can show one to one correspondence earlier than Piaget determined (Bryant & Trabasso, 1971; Cowan & Daniels, 1989). Additionally, Eggen and Kauckak (2000) found that Piaget overestimated childrens' math abilities at the Formal Operations stage. Despite possible shortcomings, Piaget's cognitive theory does provide insight on childrens' mathematical development at various ages, and encounters with math manipulatives will also shed light on how age matters.

**Rationale for the Literature Review**

In order to fully comprehend what the highest quality instruction for teaching math is, I need to thoroughly understand *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement.* This is also essential because I want to be able to share my findings with my fellow colleagues. Being an Elementary Math Specialist means other teachers come to me with questions and concerns for math. I want to be able to support them--and, more importantly, their students--in the best way possible. Knowing previous research on manipulative use, understanding, mindset, and development are essential to shape my approach to professional development.

**Chapter Summary**

While reviewing all of the literature on manipulatives, understanding, mindset and age, I have a better interpretation of what experts have found in the past. Manipulatives are found to be effective when teachers explain their use as tools for learning; the manipulatives motivate students to learn and help with abstract thinking. Math anxiety can undermine understanding, so teachers need to help students establish a growth mindset around math. Creating an environment that positively reinforces students and that involves movement, whole group strategies, student choice, and student interests will be motivating for learning. Finally, learning approaches need to be age-appropriate.

In Chapter Three, I will describe the design of my professional development series, the setting, audience and timeline. This is all in an effort to explain *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement.*

# CHAPTER THREE

## Project Description

In the previous chapter, the literature that was reviewed covered topics on manipulatives, student understanding, mindset, and age in order to further investigate the best math instruction and learning practices for elementary students. Boaler (2016) stated that, "If students are given the right math materials and receive positive messages about their potential and ability, they can make tremendous growth" (p. 4). Boaler's quote led me to want to share with my fellow educators *the benefits of using manipulatives to help foster elementary students' understanding and engagement.*

I conducted research on this topic because not only do I want to provide the highest quality instruction for my own students, but for students of other educators as well. As an Elementary Math Specialist, I want to be able to give data-driven advice to my colleagues. This chapter will describe my project, the setting, audience, and timeline. With this professional development I hope to further inspire others to utilize manipulatives.

### Project Overview

The goal of my project is to provide a professional development series for my colleagues around the idea of *the benefits of using manipulatives to help foster student understanding and engagement.* Research has shown that utilizing manipulatives benefits students in their mathematical understanding and engagement. I have also experienced and seen this firsthand with my math students, but there are obstacles standing in the way. Some teachers do not have sufficient time to incorporate manipulatives into their lessons.

Others see them as toys versus tools, and some do not see the need to use them with upper elementary students (3-5). The last barrier is lack of use, some educators need time to learn to incorporate manipulatives into their everyday lessons. The professional development I have created will hopefully counteract any negative experiences and provide an opportunity to practice and grow.

## Project Framework

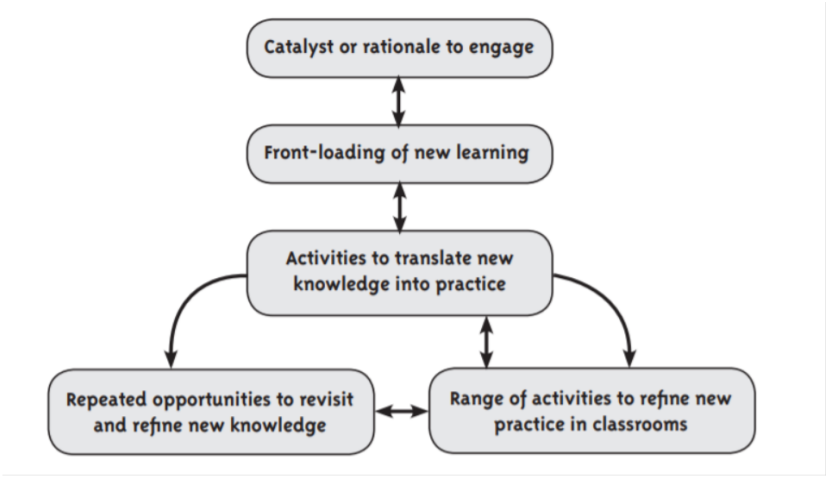
### *Professional Development*

In order for teachers to be lifelong learners and provide the best education possible for their students, effective professional development needs to be involved. The term "professional development" means a comprehensive, sustained, and intensive approach to improving teachers' and principals' effectiveness in raising student achievement" (Wei et al., 2010, p. 4). Professional development can be in the form of a course, workshop, networking event, or conference (Wei et al., 2010). It is valuable for both new and veteran teachers throughout the school year (Rogers et al., 2007). Professional development is useful as long as it is facilitated by well-prepared school principals, professional development coaches, mentors, master teachers, or other teacher leaders (Wei et al., 2010). Research shows that there are four essential elements needed for successful professional development to take place; adequate time for learning opportunities, collaboration with colleagues, feedback and observation through coaching, and active learning (Archibald et al., 2011; Bayar, 2014; Darling-Hammond et al., 2017; Wei et al., 2010).

**Adequate Amount of Time.** Long term professional development is more powerful than short term (Bayar, 2014; Darling-Hammond et al., 2017). Long term sessions allow teachers to grow deeper in their understanding of a certain concept by applying what they have learned. There is a small chance of educators using any of the information they learned during a one-time professional development (Hunzicker, 2010). Figure 1 (below) provides educators and administrators with a guideline that embraces the idea of long term professional development (Timperley et al., 2007). After teachers have considered the information in depth, and had the opportunity to apply and practice it, it is important that they meet again to share their successes and difficulties.

**Figure 1**

*Typical Sequence of Professional Learning Opportunities*




**Collaboration with Colleagues.** Compared to other countries, the United States has much less time for teachers to communicate and support one another (Wei et al., 2010). Yet more than 90% of educators agree that collaborating with their colleagues and school leaders would have a major impact on student achievement (Wei et al., 2010).

Archibald et al (2011) noted that "Teachers develop expertise not as isolated individuals but as members of collaborative, interdisciplinary teams with common goals for student learning" (p. 5). Professional development offers a space and time for teachers to participate in this critical experience. Through this joint effort, communities of teachers can positively change and impact one another's instruction and classroom culture (Darling-Hammond et al., 2017). In best case scenarios, professional development can also offer frequent opportunities for ongoing communication with their fellow educators who are also looking to make a similar change (Archibald et al., 2011).

**Coaching.** Coaching involves the sharing of expertise on content, specific to an individual's needs (Darling-Hammond et al., 2017). Numerous teachers have had positive experiences with support provided through coaching (Murray, 2008). In a recent study, 95% of teachers with a coach were more likely to transfer their training to the classroom (Murray, 2008). Having a mentor teacher allows for critique and feedback on lesson plans and instruction in a professional yet safe environment. This is especially helpful when implementing new strategies learned from professional development (Darling-Hammond et al., 2017). Ideas, strategies and techniques can be shared, refined and practiced throughout the school year. Not only can it help educators, but it has shown to help student achievement as well (Murray, 2008).

**Active Learning.** Involving discussion, problem solving, and role play is a vital component of any professional development. Providing educators with these opportunities allows them to delve deeper into the specific content area (Hunzicker, 2011). Active learning allows teachers to get hands-on experience practicing new

teaching strategies. It enables them to turn theory into practice. They often participate in the same type of learning format they are designing for their students, with real life examples of student work and curriculum (Darling-Hammond et al., 2017). Educators are then able to take it right back to their classroom, with little to change or alter. This type of activity may take longer than a traditional workshop lecture, but the benefits outweigh the cost.

### *Belief Change*

Incorporating effective professional development in my project is important, as well as incorporating things that change beliefs. According to Dweck (2008), "People's beliefs include their mental representations of the nature and workings of the self, of their relationships, and of their world" (p. 391). Beliefs can be flexible, but teachers can have a difficult time moving away from traditional math practices. Educators are more likely to change their thinking if they participate in instruction based on reform, such as professional development (Leder et. al., 2002). It is also essential that the said instruction take place over a period of time, with hands-on learning and a chance to experience the new strategies first hand in the classroom (Leder et. al., 2002). Teachers need strong examples and visuals to understand a new topic or idea, just like students.

## Setting and Audience

### *Setting*

I work at a K-5 public elementary school. My school is located in a suburban area of Minnesota with a wide variety of ethnicities and cultures. About 40% of our students are White, 18% Asian, 17% Black, 12% Hispanic and 13% did not identify their race.

There are two classes per grade level with about 50 students total per grade. Our student population is made up of 55% females and 45% males. There are also about 52% of our students enrolled in the Free and Reduced Lunch Program.

### *Audience*

The target audience for this professional development are teachers who work with students in Kindergarten through fifth grade. The professional development could also include and be beneficial for support staff, interventionists, and administrators to take part in. If they work with students during math--or will in the future--their participation would not only allow continuity, but also help create strong student mathematicians no matter who students are working with. Administrators and coaches should attend in order to provide constructive criticism to their mentees and staff.

## Project Description

This professional development is designed for elementary educators and is split into four sessions. The first two sessions will be an hour and a half to two hours long, and the last two sessions will be an hour and a half each. The first session, held in October, will lay the groundwork for the series. It will provide educators with an overview of the goals for the sessions and data that has been researched and collected. The following session, planned for November, will be allocated for review, planning, and practicing. In that session, teachers will create lesson plans that include the strategies shared during the first session. The session will also allow them to practice with their colleagues and connect with their coaches. Between the second and third sessions, teachers and their

coaches will set up a time for the coaches to observe the lesson in action. Coaches will record the lesson and take notes to bring to the following session.

During the third session, held in December, coaches would provide feedback about the lessons, successes and difficulties would be discussed with the group, and time would be provided for revision of the lessons. There would be one more observation set up by the teachers and coaches, followed by the fourth and final session in February. During this session, feedback would be given to teachers and time would be provided to allow for more discussion on the process. Finally, take-aways and next steps would be shared, including the possibility of meeting again.

**Chapter Summary**

This project is designed to share *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement*. This information and research should be able to assist K-5 educators with their math practices and lessons. This will be done through a series of professional development sessions. The sessions will take place over a four to five month period, because longer professional development is more effective (Bayar, 2014; Darling-Hammond et al., 2017). There will also be opportunities to actively learn, and collaborate with colleagues and coaches during the sessions, while creating their own lessons to implement in their classrooms. By including these important components, hopefully educators will be able to change their beliefs to better help their students.

In Chapter Four I will reflect on my overall capstone project experience. I will take another look at the literature review, note the implications and limitations of the project and what future research could entail.

# CHAPTER FOUR

## Conclusion

In order for mathematics education to be successful, teachers need to be aware of *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement*. Our students deserve the best education possible, and this professional development series can guide teachers in the right direction. Although providing that type of education is not an easy task, it is an important one. This is similar to my capstone experience: it has required a lot of time and hard work, but just as I tell my students, "Tasks that are challenging make our brains grow stronger." This process has shown me that I am a strong writer and as with most things, everything takes time and practice.

Throughout this chapter, I will reflect on my overall capstone experience. I will revisit my literature review to note the sources that were most influential in my work, as well as connections that were made. The implications and limitations of my project will also be discussed in detail. I will elaborate on my plans for continuing this project as well as what new topics I may research in the future. Lastly, I will reflect upon my own personal learning and what this project has meant to me.

### Literature Review Revisited

The main components of my literature review covered manipulatives, understanding, mindset, and age, all to further elaborate on the benefits of mathematical manipulatives for my professional development series. While all topics proved useful, the barriers that stand in the way of teachers using manipulatives played a major role. The

idea that manipulatives increase student understanding and motivation was also influential, especially when counteracting those barriers. Another component of the literature review that drove much of the professional development design was noting the levels of understanding and student development. Mindset was also useful, especially realizing its need for a makeover in the area of mathematics. Lastly, motivation and engagement techniques were implemented to not only allow for more participation from teachers, but also as a resource to use in their own classrooms. While Phan et al. (2018) recommended using interests to engage students in their learning, this did not play a large role in the series, nor did the age of students.

There are various barriers that get in the way of educators utilizing manipulatives to their full potential. For example, some see manipulatives as toys, others do not always feel comfortable using them, a handful believe that lectures are more effective, and some think students have trouble transitioning between strategies. Research further demonstrated that these obstacles needed to be addressed during the professional development series. Before talking more in depth about manipulatives, teachers will converse about their history and overall thoughts on math tools. After discussing, I plan to counteract the barriers by sharing the benefits that have been found. For example, many have found that the use of manipulatives has increased students' academic performance (Battle, 2007; Boggan et al., 2010; Carbonneau et al., 2013; Liggett, 2017) and students who correctly used manipulatives were excited and motivated to learn (Battle, 2007). The teacher participants will also see the benefits firsthand through the mini lessons we go over together.

I will also add in my own personal experiences with manipulatives. To avoid the manipulatives being just seen as “fun” (Moyer, 2001), I’ll explain how when I use manipulatives with my students, we go over expectations and procedures for using them. It seems to help tremendously. Other teachers just do not feel comfortable or know the correct way to use manipulatives (Graham, 2013). To relieve some anxiety with using the manipulatives themselves, I will be presenting mini lessons where I will demonstrate how to effectively use cuisenaire rods and fraction tiles. There will also be a slide that includes effective strategies as well as time for teachers to explore with manipulatives during our initial discussion.

According to Gilakjani (2012), teachers tend to prefer the lecture style of teaching, because they think their students will be more successful. The professional development will include plenty of opportunities for manipulative use. We will use them to further understand the pythagorean theorem, and to add fractions. By demonstrating with more hands-on lessons, hopefully teachers will note the importance and effectiveness of using manipulatives. Some teachers are also concerned that students can have a difficult time transitioning from concrete to abstract understanding while using manipulatives (Graham, 2013; McNeil & Jarvin, 2007). I plan to share observations from my classroom. I have noted that it can take some students more time than others to move on from using manipulatives as a strategy. Regardless, it is not that they do not understand a concept, it is just what makes sense to them at the time.

For teachers to provide the necessary tools for student success, they need to be aware of the various levels of understanding. Teachers can get a better idea of students’

understanding when they are using manipulatives if they listen to their students talk about their thinking, observe students working independently and in cooperative groups, ask them “Why?” questions instead of yes or no questions, and have students write the whole answer sentence to their math problem (Hedden, 1997, as cited in Larbi & Mavis, 2016). This information will be shared during the ‘theories of understanding’ portion of the presentation with a discussion around what teachers use to check for student understanding now. I will also be actively highlighting these skills during my mini lesson on cuisenaire rods and fraction tiles.

I also plan to use Bloom’s et al. (1956) taxonomy of understanding. I included an image of all the stages to help teachers see how manipulatives can support students at different levels. This will allow for an overview of each category. For example, educators should start with questions concerning basic recall, and gradually work their way up to composing or relating new ideas (Bloom et al., 1956). This will also lead to discussions on how to progress from one stage to the next. Lastly, I will share my personal experiences. My students often have a hard time moving past the “knowledge stage” and manipulatives have helped them progress to the next step (Bloom et al., 1956). Through these discussions and examples teachers will see how students need to first understand a topic, then apply it. This will also allow them to better gauge where their students are at now versus where they ideally should be and how manipulatives could play a role in their progression.

Every teacher’s goal is to get students to apply their knowledge and use it in their everyday life, which is another reason why the developmental stages of students played

an important part. Piaget (1977) stated that children went through four stages of cognitive development. Moving through each stage can vary, depending on the student. During my presentation, especially during the mini lessons, I plan on relating these stages to mathematical development. For example, the Preoperational stage should involve some problem solving with blocks while also having the student explain their thinking (Ojose, 2008). Teachers will plan on referring to this information for their current and future lesson plans. They will be able to meet students at their level and are more likely to provide math manipulatives as needed. While it is not the perfect formula for each student, it is a building block that teachers now have as a resource when creating and implementing math lessons.

Having a positive or growth mindset when it comes to mathematics is difficult for many people, which means it is in need of reform (Boaler, 2016). During the presentation I plan on talking about what mindset means, the differences between a growth and fixed mindset, as well as ways to assist students with this topic. Before we unpack how to help students, teachers will have the opportunity to share. In a previous slide, teachers are asked to discuss their own experiences around mathematics. In order for teachers to help their students, they themselves need to know their history and process of coming to understand math. People can carry a lot of baggage in this subject area, and it can be difficult to let go of. I plan on talking about alternative thinking and language teachers and students can use in order to work towards a growth mindset. Manipulative use will also come up in this discussion, and how it could be used as a tool to counteract a fixed

mindset. Some of our students need a physical representation to understand and apply a concept and maybe what has been holding them back.

Another part of the literature review that I plan on implementing during the series are motivation and engagement strategies. Motivation in math strongly correlates with success in the subject. This is formed early in elementary school (Orosco, 2016). I plan on motivating the teacher participants by helping them create a lesson plan they can take right to the classroom. Student choice is another factor that contributes to higher student engagement (Shevin & Klein, 2004). Their lesson plan will incorporate this idea by offering teachers a choice in their math topic and tools used. To create a motivating environment students need to feel respected, feel prideful of their accomplishments, and work collaboratively with their peers (Bowman, 2007). Teachers will have this opportunity through their own hands-on practice during a mini lesson on cuisenaire rods where they can work in groups with their peers. Building a strong community of learners is key in order to achieve positive outcomes for all students.

Student interest was something that I did not work into the professional development series. While it absolutely can make learning more meaningful for students (Johns, 2015), this professional development was focused more on manipulatives in relation to understanding and mindset. With limited time and important information to share, it seemed wise to leave the component of student interest to individual teachers; they know their own students better and recognize the importance of connecting learning to those interests across all subject areas.

**Implications**

The professional development I created is meant to benefit groups of K-5 educators and their students. It is a 4-5 month presentation series that provides a starting point to incorporate manipulatives in elementary math lessons. There may be some teachers who are dedicated to making a change in their math classrooms, others in between and some indifferent. Whatever the case, my hope is that manipulatives can become a part of every teacher's daily routine during their math instruction, not only at my school, but others as well.

***Impact of Professional Development***

The teachers who do actively engage or already have experience effectively using manipulatives while teaching math would work to include the tools in their lessons as much as possible. I would recommend incorporating manipulatives into lessons twice a week at first, while gradually working towards five days or what is best for their students. In the following years I would hope that the teachers who do believe in this change would become a part of future professional development. It would be great to have them assist in leading some of the sessions and giving their experiences first hand. They could also assist their team members through shared lesson plans or having them come and observe a lesson. Students who have the benefit of being in these math classes will flourish in their understanding of the content and their growth mindset. Their class will not just learn the algorithm and move on to the next topic, but they will be able to apply what they learn to new situations and concepts.

The educators who want to use manipulatives effectively for learning but do not yet do so would also make major gains. There may be a number of factors holding them

back, from planning time constraints to being new in the profession. These teachers will have a positive experience using manipulatives during the professional development sessions, then benefit from being supported by their instructional coach and peers as they work to incorporate manipulatives into their lessons once or twice a week. Earlier in the year, teachers can use these supports to guide their lesson plans and to provide feedback. Closer to the end of the year, they will hopefully only need them for questions. In the years to come, these teachers will hopefully be able to coach others. Students of these teachers would benefit from using manipulatives in the classroom through opportunities of exploration, higher engagement in lessons, and overall deepening of their understanding.

For the more reluctant teacher during the professional development, there will still be positive takeaways. I hope that, at a minimum, their outlook on mathematics will be altered, moving toward more of a growth mindset. It would be wonderful to see them realize how powerful manipulatives are for their own understanding during the activities in the first session, then imagine how that could be beneficial for students. I also hope that--either from discussions or examples--teachers would learn an alternative way to teach their students a few math topics. For example, using an area model for multiplication, base ten blocks to add and subtract, or cubes to simply compare numbers. Overall, I hope they have at least one positive takeaway and are willing to work proactively alongside their peers in discussions and planning sessions aimed at including manipulatives into math lessons. Hopefully, these teachers' students will benefit from the

growth mindset practices, and multiple strategies used to practice a few skills, which will ultimately lead to a deeper understanding.

Lastly, it would be wonderful if other schools could implement my professional development series to benefit their teachers, staff, and students. Before sharing it out, I would revise my presentation based on the feedback from teachers at my school. I could then see this occurring in a few possible scenarios: presenting it in person over the course of several months, presenting it over a video conference in a similar format, or providing another educator with the tools I have built to share with their school community themselves. Regardless of the circumstances, it would be an honor to share my findings on what I am most passionate about. It would help other teachers learn to enjoy teaching math and their students to love learning more about it.

**Limitations**

While there are many opportunities for success in this professional development series, there are also some barriers. One of the first challenges is time management and how much time could actually be set aside for this important work by teachers. Another challenge may be that teachers and staff who are involved in the professional development are not as eager or willing to participate as I hope. One last limitation came from the video recording I created with some of my students.

There are a few factors that could impede a perfect professional development series, time being at the forefront. As mentioned previously, the most effective professional development takes place over a longer time frame (Bayar, 2014; Darling-Hammond et al., 2017). Due to numerous factors, that may not be able to take

place. There may not be a sufficient number of professional development days throughout the school year, the slots could also already be filled, or something administration deems important may take priority. There also may only be time for part of the initial presentation. While this would be better than nothing, the sessions would need to be revised to have close to the same effect.

Some teachers may be very open to the idea of learning more about math manipulatives while others are not. Their openness could depend on what their school year looks like and what they have already committed to. Including instructional coaches in the work may help motivate those who do not have as much buy-in, as well as being able to work alongside other teachers in their grade level. The administration's outlook on the professional development series may also hold some weight when it comes to teachers taking it seriously. I would hope all teachers see this as important work that would greatly affect student outcomes, but everyone has a different mindset when it comes to the subject of math.

One of the last limitations comes from using video recording with students. I am very grateful to the students and families that offered to help make the presentation successful. Students were able to give their perspective on math manipulatives in the classroom, but there were missed opportunities. Having the students in front of the camera lost some of the authenticity. I tried to talk to them beforehand about ignoring the camera and thinking of it as just having a normal conversation, but it still was a distraction. It created some anxiety which produced answers that were not as clearly thought through. I had thought about re-recording the sessions with students, but I did not

want them to think we had to redo it so they could alter their responses. All of these aspects are important to consider when thinking about future research and professional development projects.

**Future Research**

My hope is that this work will influence the lives of many educators and students in the area of mathematics. As an Elementary Math Specialist, I am always trying to better my teaching practices and share my findings with others, which is why I want to continue to develop professional development series for teachers and staff. My goal is to create a follow-up presentation series that would be a continuation and addition to *the benefits of using mathematical manipulatives to help foster elementary students' understanding and engagement.* This would give returning teachers a chance to further their practice using manipulatives in their classroom and give new staff a chance to get started. It would be optional for teachers who had already gone through the previous training, but highly recommended. New staff would be required to attend. It would be extremely beneficial if teachers who had attended the last professional development helped contribute to the new one in some way. It could be as simple as sharing a testimonial or helping lead a certain part of it. If new teachers can see how effective it has been for their colleagues, the buy in will be much higher.

**Personal Learning**

While reflecting on my capstone process, I had two major takeaways. The first is how interesting the research process is, and the second is how my passion as a math teacher has grown. My capstone experience has shown me how important and intriguing

research can be. You can take a topic you want to know more about or that interests you and contribute to it. It also feels good to know that your contribution can impact the lives of others and, in my case, the world of education. Future students will benefit from what I developed and presented, and that is powerful. This process has also reaffirmed how enjoyable learning and teaching math can be. There are so many different strategies to be used, but time and time again manipulatives seem to play a large role in understanding and mindset. I am very grateful for what I do, and I hope to continue to inspire learners and educators in the area of math for many more years to come.

**Conclusion**

Understanding the optimum math pedagogies is challenging, and implementing them adds a whole new level of understanding. Every teacher wants what is best for their students and they should provide high quality math instruction every day. I believe math manipulatives play a large role in the success of students, especially their understanding and mindset. I was a student who struggled with math and I hope that by sharing my professional development series with educators, students like me will not have as difficult a time as I did.

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