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Implementing Math Discourse and Rich Math Tasks to Develop Mathematical Thinking

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IMPLEMENTING MATH DISCOURSE AND RICH MATH TASKS TO DEVELOP MATHEMATICAL THINKING

By

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TABLE OF CONTENTS

CHAPTER ONE

Introduction

Math education in the United States has historically centered around procedural instruction, driven by teacher modeled concepts and directed completion of specific algorithms and strategies. This traditional model of instruction was my experience in the math classroom and, largely, this practice continues to be the experience of many students today. Yet, according to researchers, “In contrast to a procedural teaching and learning approach, teachers who invite students to consider the meaning of mathematical methods, to choose and discuss approaches and to think creatively, enable students to develop a sense of agency and mathematical authority” (Lamar et al., 2020, p.2). My desire is for my students to not only experience success in the classroom, but develop into life-long learners who can apply what they have learned in larger contexts as they move through school and life.

In this chapter, I will discuss my personal experiences with math, as a student, educator, and parent, and how those experiences have shaped my understanding of the subject. I will go on to evaluate my professional experience as an educator, struggling to effectively and impactfully teach math in both an intervention and classroom setting. The combination of these experiences has sparked my interest and initiated questions in regard to current practices in math instruction. From there, I will go on to explain the rationale behind my research question: *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?*

Personal Math Experiences

As a student, math always came easy to me. I remember being told, and believing, that I was simply lucky to be “math minded.” In elementary school I quickly earned the stars on the math facts chart that hung in our classroom, moved through the lessons with ease and received an invitation to our Math Olympiad program. My teachers would present a method to solve a math problem, and I would successfully replicate the method with both speed and accuracy. In middle school the pattern continued and I began participating in the advanced math track, which continued to be my path all the way through high school. My classroom math experiences from elementary through high school were all very similar: quiet, teacher centered classrooms where students replicated modeled procedures, strategies and algorithms, with the goal of quick, correct answers. Speed was valued. Occasionally I needed some extra explaining at home, but most often it made sense to me and I scored very well.

In college, my math experience changed. As an elementary education major, I was required to enroll in a math class titled “Math for Elementary Teachers.” The experience in the class was eye opening for me on two levels. First, I was surprised by the number of my classmates who generally struggled with math. In my previous experience in the advanced math track, I was surrounded by students who approached and experienced math in the same way that I did and with relative ease. In this class, students spoke of their struggles learning math in their journey through school, and many of them believed they were simply bad at math. They lived under the notion, like me, that some people were “math minded” and some were not. The other surprise I faced was a personal confrontation with the fact that although math computation came easily, I struggled to explain the number relationships or problem solving strategies I used to solve math problems and I was not particularly skilled at explaining the concepts to others. I knew the algorithms and could complete the math task, but I had limited language around the how, why or other ways to solve the problem.

As a parent, I have had the experience of watching my own children interact with and learn math. I have four very different children who have opened my eyes to child development, different ways of sense-making and how children process through language. My children often come to solutions in math in different ways and are able to refine their thinking and broaden each other's understanding through their explanations. As math homework and formal education began for my children, I was surprised to see the demand for higher level, abstract and procedural math thinking without much number sense or concept development. I have also noticed that, even though my children memorize math facts, they don't necessarily make problem solving connections between math concepts and skills. In their math classrooms, they generally learn math the same way that I did, largely waiting for the teacher to model what procedure to follow.

Professional Math Experiences

Professionally speaking, I began my teaching career as a third grade teacher. I brought with me an awareness of the importance of hands-on math experiences and concrete examples for my learners. Even with the addition of these new strategies, my math instruction largely mimicked what I had experienced as a student. I used timed tests religiously, taught students key words and endlessly practiced procedures. I also sent home pages and pages of computation practice. I was surprised that even with the addition of math manipulatives, only about one third of my students were able to perform the math tasks with consistent success. The other two thirds of students experienced little to moderate success, even with me modeling and reteaching the procedures again and again. For my next five years of teaching, I was not completely satisfied with all of my students' outcomes in math, but ultimately believed that some of my students were math minded and some were not. They would simply have to struggle through math in school. I worked hard to help them understand the procedures, but for many students it was not enough.

After a thirteen year hiatus from teaching to spend time at home with my children, I returned to the classroom as an intervention teacher working with K-5 students who were identified as more than one grade level of proficiency behind in math. My first few months in the position, I fell back into the pattern of traditional, procedure-focused math teaching that I experienced as a student and had used in my first years as a classroom teacher. During the first months on the job, it became clear that reteaching in a small group setting was still not accomplishing the intended outcome of concept development, understanding, and progression through math skills. Nonetheless, I carried on reviewing the key words and strategies in a traditional, teacher-centered instructional model.

Discouraged by the lack of progress, I began to implement intentional questioning strategies and increased concrete math experiences with the hope of helping students develop number sense and apply their understanding to other number concepts but still, the learning was teacher-focused. I was working to draw them to a pre-identified strategy and connect that to the procedural knowledge being taught in the mainstream classroom. I noticed that students would experience short term success, yet when the learning would spiral back to previously discussed concepts, there was little retention or growth. I was also increasingly aware of students' inability to transfer strategies to other connected mathematical situations. They could accomplish the task, as long as the problem matched the specified procedure they had practiced.

During that same timeframe, I attended a district professional development session that introduced the practice of number talks. Before any background or training was given, we, as educators, participated in a number talk as learners. We were presented with a problem and tasked with finding as many possible routes to a solution. Initially, I was frustrated and uncomfortable being forced to look beyond my initial strategy. After some time, though, I did comply and forced myself to dig deeper and look again. I was amazed to find other ways to develop a solution. We then shared solutions and strategies as a group, and again my eyes were opened to strategies I had not considered. It was also interesting to see how difficult it was for us, as educators, to describe our mathematical thinking with words rather than in writing. It was clear that we were not accustomed to that type of math dialogue. This experience not only opened my eyes to a new strategy to implement in my classroom, it opened my mind to a different way of approaching math instruction as a whole; an approach that values meaning making and dialogue over procedure and quick, correct answers.

Rationale

The challenges I was experiencing trying to teach math in an intervention setting drove me to engage in research and enroll in professional development in the area of math. Through that process, I began to deepen my understanding of number sense, conceptual math progression and the importance of dialogue and rich math experiences. I was noticing authors and teachers discussing a transformation of math instruction. As I acquired more knowledge around math, new questions emerged about the relationship between traditional, procedural math instruction and overall development and performance in math. I also began to notice that there were more students identified as "below grade level" than the intervention department was able to service; around one third of the students in many grade levels were not achieving the desired outcomes in math. Classroom teachers were expressing frustration that students who had previously demonstrated mastery were not retaining math skills in subsequent grade levels. I was also concerned with the impact of the repeated frustration in math on student attitudes toward math after hearing many students express that they simply couldn't do it. Yet, when given increased opportunities to discuss and create meaning through open ended tasks in a guided setting, I observed them engaging in meaning making and problem solving with success, but this was only occurring in the intervention setting. A larger portion of their math instruction was still procedure based. My intrigue about traditional math approaches in the classroom and their relationship to underdeveloped number sense, number relationships, flexible thinking and problem solving skills grew.

Now, as a classroom teacher working in a system that continues to approach math instruction through traditional instructional methods largely focused on algorithms and procedures, I feel an increased sense of urgency to learn more and change my practice. I am concerned that students do not develop adequate number sense to achieve the desired proficiency in math. In addition, I have a growing concern that traditional math instruction fosters students to wait to be told how to do things, rather than engage in problem solving and meaning making. Often, when given a problem solving task or open ended question, students express high levels of frustration or an inability to even begin.

Their classroom math experiences have trained them to search for the correct answer using the correct procedure or strategy. This research is important to me because I see the potential impact of transforming math instruction to practices that center around student dialogue and rich math tasks in order to develop deeper understanding, improve performance and impact problem solving skills, which could ultimately benefit students beyond the math classroom.

Summary

Math is a subject in which I experienced much success as a student, yet as a teacher, I am aware that what and how I learned math is not sufficient for the majority of my students. My experiences as a math student, parent, intervention teacher and classroom teacher have contributed to my desire to look for a new approach to math instruction; one that removes the teacher from the center and instead engages students in math dialogue and rich math tasks in order to promote deeper understanding and improve overall performance in math.

Chapter Two will provide a literature review related to the question: *How can elementary teachers use math discourse and rich math tasks to develop mathematical thinking?* This review will analyze the current research in math instruction and explore the subtopics of traditional math instruction, mathematical thinking, math discourse, and rich math tasks. Chapter Three will describe, in detail, the project to be created, and will include rationale, project description, an overview of participants and setting, as well as a project timeline. The project will be a curriculum resource for upper elementary teachers, designed using Minnesota's Academic Standards in Mathematics with the purpose of developing mathematical thinking, transforming math mindsets and classroom culture.

There will be two units of instruction intended for the beginning of the school year to establish discourse practices and promote the development of mathematical thinking through rich math tasks. Chapter Four will provide a personal and professional reflection of the capstone chapters and project.

CHAPTER TWO

Literature Review

As educators, we desire learning for all of our students that results in deep understanding and application not only to classroom situations, but to life beyond the classroom. We consistently work to improve learning by incorporating new strategies and techniques as research informs our instruction. Yet in the area of mathematics, the overall instructional approach has been slow to respond to a call for reform that is based on years of research and consistent poor performance on domestic and international exams. Almost two-thirds of adults in the United States express fear of math and won't even consider a career that would involve higher level math (Parrish, 2010). Something is wrong in math education. The culture of our classrooms, our deep beliefs about what math is, and our commitment to traditional practices are holding us back. To transform math classroom culture, instructional practices, and outcomes, an examination of mathematical thinking is necessary. Change is possible through an increase of strategic, high-quality mathematical discourse and the implementation of rich math tasks (Sztjan et al., 2021; Liljedahl, 2021). Teachers need to examine and redefine their understanding and practice as it relates to math education, which leads to the question, *how can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?*

This chapter will provide an overview of current research on the components of mathematical thinking and discourse. Before analyzing mathematical thinking, it will begin with a discussion of the history and endurance of traditional math practices in order to provide context for the discussion of redefining mathematical thinking, as well as classroom practices. In addition, there will be analysis of the impact of the cultural nature of the mathematics classroom and the core beliefs associated with the persistence of traditional math practices. This foundation will set the stage for further analysis of mathematical thinking, including the relationships of number sense, mindset and metacognition. Understanding the definitions provided by and position of research on those concepts allows for discussion of practices that can transform math education. The second half of the literature review will discuss and analyze two different classroom practices that can be implemented to develop mathematical thinking; math discourse and rich math tasks, including specific teacher moves, as well as routines and strategies that will support their implementation.

Traditional Math Practices

In order to analyze the research related to current mathematical practices, it is important to discuss the historical context of math education and its relationship to the United States's performance in math. Over the past fifty years, American students' performance on domestic and international tests has remained stagnant, ranking consistently below average, with an increasing disparity between students of contrasting socioeconomic statuses (NCTM, 2020; Nations Report Card, 2019). In response to the most recent data from the National Association of Educational Progress (NAEP) assessment, Trena Wilerson, president of the National Council of Teachers of Mathematics (NCTM), released the following statement:

> For students who are leaving high school when our democracy, economy, and personal safety all require more ability to understand, use, and apply math, holding steady is not success. Each and every student must be equipped to use

> math to make sense of our world and to increase their opportunities moving forward. (paragraph 3, NCTM release 2020)

American math classrooms are not producing students who are able to demonstrate mathematical proficiency at the same level as students internationally nor engage in the mathematical thinking needed in the twenty-first century. It is important to note that the lowest scoring nations on the Program for International Student Assessment (PISA) exam are consistently those whose instructional practices focus to a great extent on memorization strategies, while in contrast the highest scoring nations emphasize the relationships and connections between "big ideas" in math (Boaler, 2015).

Previously, computation was a highly valued mathematical skill and focus in the United States. In 1970, Fortune 500 named computation as the second most sought after skill in the upcoming workforce, yet by 1999 it had fallen out of the top ten and was replaced with problem solving (Boaler, 2015). At that time, there was a realization that computational skill alone was not enough for the modern workforce, but rather, problem solving and deeper mathematical thinking was needed. By the mid 1990's there was an awareness of, and national consensus for, the need of math reform in both policy and practice (Smith, 1996). More than twenty years later, not only has American performance in mathematics shown no significant change, but computation based classroom practices remain common practice. Despite the legitimization of a more open, process-based approach to teaching mathematics supported by the published NCTM process standards as well as the Common Core State Standards, traditional practices and curriculum promoting them have largely remained. For the sake of clarity in this literature review, the definition of traditional practices include the following characteristics:

- Teacher-centered math classrooms where instruction is predominantly “teaching by telling”; defined as the action of the teacher demonstrating specific, isolated strategies and procedures presented as a sequence of steps.
- Emphasis on the development of routine expertise (isolated procedural knowledge not necessarily rooted in conceptual understanding).
- Importance of accurate and fast computation, including practices such as timed tests and repeated computational practice (worksheets).
- Daily homework, typically in the form of a worksheet.
- Memorization of facts, isolated algorithms, tips and tricks.
- Understanding of math as a performance subject, where students are to perform by answering questions correctly and taking tests (Ben-Hur, 2006; Boaler, 2015; Karp et al., 2014; McCloskey, 2014; Smith, 1996; Verschaffel et al. 2009).

This is not to say that there is no place for instruction that develops procedural competence and memorization in effective math classrooms; it is just not enough on its own. Researchers agree that procedural knowledge has a significant role in math development and performance, yet there is debate around the emphasis and timing of instructional practices predominantly focused on its development (Verschaffel et al., 2009). Carpenter et al. (2003) make the distinction that computation alone is regarded as arithmetic, not mathematics, and continues as an important aspect of math education. The work of Verschaffel, et. al (2009) does present the existence of an opposite perspective that argues it is optimal to first develop a strong foundation of routine expertise through explicit, direct instruction of isolated strategies, particularly for students who are not “mathematically minded”. However, other research rejects the notion of mathematical mindedness altogether (Boaler, 2015).

Core Beliefs

Driving the continuation of traditional practices in spite of a call for math reform are pertinacious core beliefs about math education and learning that are held not only by teachers, but students and parents as well. These beliefs are centered around the idea that math content is a fixed set of facts and procedures that exist in order to find predetermined, correct answers; the facts and procedures exist in the authority of math textbooks (Smith, 1996). In addition, there is strong conviction about the role of the student and teacher in the math classroom. Traditionally, the expectation of the teacher is to provide clear directions, opportunities to practice, and repetition of instructions for the prescribed procedures; while students are to watch, listen and mimic the procedure, often without understanding of the underlying math concepts, until computational mastery is demonstrated (Ben-Hur, 2006; Boaler, 2015; Smith, 1996). These beliefs support teacher-centered instruction; students passively receive prescribed content or algorithms from direct modeling of the process or procedure, and then follow up with practice.

Routines and Rituals

In addition to core beliefs, sustaining traditional instructional practices are the routinized experiences of math education. Teachers and parents alike have a nostalgic commitment to the math tasks and experiences (for example, the traditional long division algorithm) they experienced during their years of learning math. This expectation contributes to the cultural nature of mathematics classrooms, based on formalized and routinized math rituals and performance expectations that are continuously passed down and reinforced (McCloskey, 2014). An example of this is the continued use of timed tests and the daily practice of sending homework consisting of skill and drill worksheets. McCloskey goes on to state that regardless of recommendations and new technologies that exist to provide students with more meaningful and varied forms of learning and assessment, math rituals endure and perpetuate traditional practices.

Teacher Role

It cannot be overstated how significant the role of the teacher is in the endurance of traditional math practices. Several factors influence teachers' instructional practices including personal beliefs, experience learning math and teacher training programs. Many teachers are trained in classrooms that model teacher-centered, traditional math instruction, which ultimately reinforces their own beliefs from their experiences in the math classrooms of their childhood (McCloskey, 2014; Smith, 1996). In addition to personal beliefs and experiences, there is also the contributing factor of insufficient mathematical knowledge. According to Faulkner (2009), there has been a qualitative demonstration that "elementary teachers in the United States tend to lack a "profound understanding" of the fundamentals of the mathematics they teach," (p. 24). In addition, when teachers are engaged in the practice of "teaching by telling" they are encouraged to continue in that method due to the strong feelings of efficacy and control provided by the predictable, detailed sequence and model of what they should do and what students should know (Smith, 1996). In other words, the practices largely continue because it is understood, comfortable, and is simply the way it's always been done.

Call for Reform

There has been significant research in the field of math education that has advocated for a new focus and approach, promoting a "new" view of math: one that goes beyond traditional core beliefs, fixed methods and isolated procedural knowledge. This reformed view ascribes to the definition of math as the study of patterns and properties of number systems, involving creativity, number relationships, and exploration and conjecture, with an understanding of the need for deep thinking and sense making (Boaler, 2015; Smith, 1996). In terms of teacher and student roles, it calls for a shift from teacher-centered classrooms to a socio constructivist approach to teaching and learning, emphasizing that math is both an individual and collective activity (Ben-Hur, 2006; Cobb et al., 1992). In the reformed approach, the goal is for students to work to construct math knowledge and make sense of the world, building on and modifying their current understanding through social interaction, experiences and supportive instructional representations, shifting the emphasis from content to the mental processes involved in sense making (Ben-Hur, 2006; Cobb et al., 1992). The Common Core State Standards (CCSS) for Mathematical Practice address this shift in thinking with their goal for "students to become problem solvers who can reason, apply, justify and effectively use appropriate mathematics vocabulary to demonstrate their understanding of mathematics concepts," (Karper et al., 2014, p. 20). In order for classrooms to engage and equip students for the kind of math and thinking needed in the twenty-first century, beliefs and class practices must change. A new understanding of what it means to do math and engage in mathematical thinking is essential.

Mathematical Thinking

There is a significant disconnect between the understanding and description of math based on many classroom experiences and the essence of math itself, as defined by researchers and mathematicians. In contradiction to the traditional core math beliefs held by many teachers, students and parents, mathematicians have a different view, describing the discipline of math with words including: intuition, creativity, art, poetry, curiosity, passion and mystery (Katz, 2014; Zager, 2017). Traditional classroom beliefs and practices do not align with those descriptors. The mathematician James Tanton states (as quoted by Zager, 2017, p.6):

> The true joy in mathematics, the true hook that compels mathematicians to devote their careers to the subject, comes from a sense of boundless wonder induced by the subject. There is transcendental beauty, there are deep and intriguing connections, there are surprises and rewards, and there is play and creativity. Mathematics has very little to do with crunching numbers. Mathematics is a landscape of ideas and wonders.

To participate in the math promoted by researchers and described by mathematicians, a new definition and understanding of mathematical thinking must be developed, one that creates space for ideas and wonders. Although precision, accuracy and computation are important in the work of learning and doing math, there is clearly more. In order to identify the practices that support the development of mathematical thinking, its many facets must be identified, defined, and discussed.

Components of Mathematical Thinking

To do the transformational work of changing persistent math practices, it is important to have a shared understanding of what it means to learn and do math, as well as the specific components that must be developed. In the traditional mindset, mathematical thinking is associated with correct computation, thus the traditional practices used provide a sense of efficacy for teachers and support that instructional approach. According to years of research, however, "doing math" is the act of sense making (Schoenfeld, 2016), yet beneath the surface of that simple definition lies highly complex concepts and processes of learning to generate, express and justify math ideas and relationships through words and symbols (Carpenter et al., 2003). For the sake of this literature review, the definition of mathematical thinking will include the following interdependent components:

- *Procedural fluency* - This component is often misinterpreted to simply mean the ability to be quick and accurate at computation. In fact, it is grounded in conceptual understanding and includes the skill to carry out procedures accurately, efficiently, precisely, and appropriately, distinguishing when to use one procedure over another (Katz, 2014).
- *Strategic competence* - This is similar to what many consider or label 'problem solving'. It involves the ability to formulate, represent and solve different math problems by flexibly and selectively choosing different methods, strategies, or procedures from a broad repertoire to suit the demands of the problem (Katz, 2014; Schoenfeld, 2016).

- *Adaptive reasoning* - This is foundational to sense making and is the glue that holds together many components of mathematics and provides a basis for creative transfer to other procedures, concepts, and solution methods. It includes the capacity for logical thought, reflection on thought and process, and also involves explanation, analysis and justification of thinking (Baroody et al., 2007; Katz, 2014; Schoenfeld, 2016).
- *Productive disposition* - This is the students' view of themselves as effective learners of math, understanding that they have the ability and capacity to make sense and "do" math. Math is not limited to those who are "math minded). (Boaler, 2018; Katz, 2014).

A large contributing factor to the development of the individual components of mathematical thinking is mental math. Research supports the notion that mental computation training leads to increased connections and transfer among big ideas, as well as increased use of varied strategies and the ability to choose those that are situationally most efficient (Liu, et al., 2015). In addition, mathematical thinking involves developing a mathematical point of view that not only values the processes and mental representations of mathematics, but applies them, by competently connecting and using strategies and procedures in order to understand math structure (Schoenfeld, 2016).

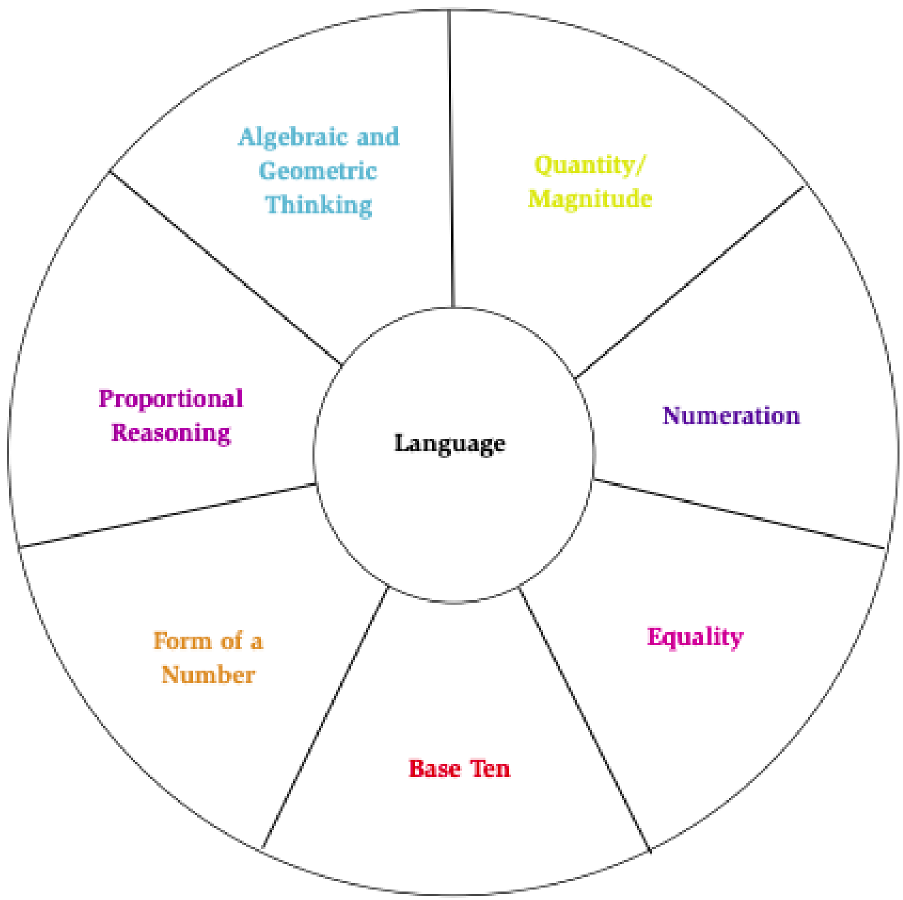
Connection Between Procedural and Conceptual Knowledge. Procedural and conceptual knowledge are essential to the discipline of math, so it is important to look at the different layers involved in understanding these concepts. The work of Baroody, et al. (2007) discusses the distinction between superficial procedural knowledge (understood as superficial, rote knowledge of procedures) and deep procedural knowledge (the connection of procedures to "comprehension, flexibility and critical judgement") (p.116). They go on to describe conceptual knowledge (knowledge of concepts) in the same manner, pointing out that although conceptual knowledge involves relationships and connects concepts, it doesn't inherently mean that the connections are rich ones (Baroody et al., 2007). Superficial procedural knowledge and conceptual knowledge can exist relatively independent of each other, yet deep procedural and conceptual knowledge are mutually reinforcing (Baroody et al., 2007; Katz, 2014; Verschaffel et al., 2009). The goal, then, is to simultaneously promote deep procedural and conceptual knowledge through the intentional development of the components of mathematical thinking.

Mathematical Mindset. Another contributing factor to mathematical thinking is mindset. Contrary to the common belief that some people are just "math minded", there is research that breaks down that misconception and supports the notion of "brain plasticity" (Boaler, 2015). In Boaler's (2015) study, she analyzed brain activity and the connections that are formed into structural pathways (learning) through discourse, play, and experiences. In other words, the brain is not fixed; everyone can learn and do math. When the ability to learn math is understood as predetermined and interpreted through the lens of performance culture, where students believe their role is to get correct answers, many students are left to think that math learning is out of their reach. Boaler states that a fixed mindset can damage students' learning potential for fear of mistakes and an unwillingness to try, while a growth mindset results in higher participation and increased success. The needed mathematical mindset also requires a shift in the understanding and messaging around mistakes. Boaler goes on to say that when mistakes are reframed as opportunities for growth, fear and hesitation to engage in the process of taking mathematical risks subsides and is replaced with a desire for sense making.

Equally important to the role of growth mindset in mathematics is metacognition. Metacognition is defined as the knowledge of one's own thought process and includes regulating and monitoring one's thinking and activity during a task (Lester, 1994). Lester (1994) states that students' metacognitive actions have an influence not only on their mathematical processes but on their math beliefs and attitudes which impacts their fixed or growth mindset. Lester (1994) goes on to express that, much like the unlearning that must take place in regard to traditional beliefs about math learning, often learning new metacognitive strategies that benefit the development of math concepts and techniques requires unlearning and reteaching. As a result, it is imperative that teachers attend to the development of metacognitive skills in order to promote a positive mathematical mindset and develop mathematical thinking.

Number Sense. In addition, a discussion of mathematical thinking would be incomplete without an understanding of the role of number sense. The importance of number sense is increasingly mentioned in research, yet a consistent definition and overall understanding of what it, and other math concepts discussed in research actually entails, is often lacking (Faulkner & Cain, 2009; Ryve, 2011). Teachers speak of it frequently and recognize when it is evident or lacking, but often don't know how to help with its development. The definition of number sense is like that of common sense: difficult to specifically define, yet you know it when you see it (McIntosh et al., 1992). Number sense is commonly understood as foundational for success in math, particularly for deep conceptual understanding. McIntosh et al. (1992) describe it as a general understanding of numbers and operations that gradually develops and evolves from an early age. It includes the proclivity to use that understanding in flexible ways to develop strategies, procedures, and conjectures in math situations (McIntosh et al., 1992). Faulkner and Cain's (2009) work builds on that definition and provides more detailed examples of the critical structures of number sense and a model to better understand their interconnected nature. Each equally important component, and their connection through language, is shown below in Figure 1:

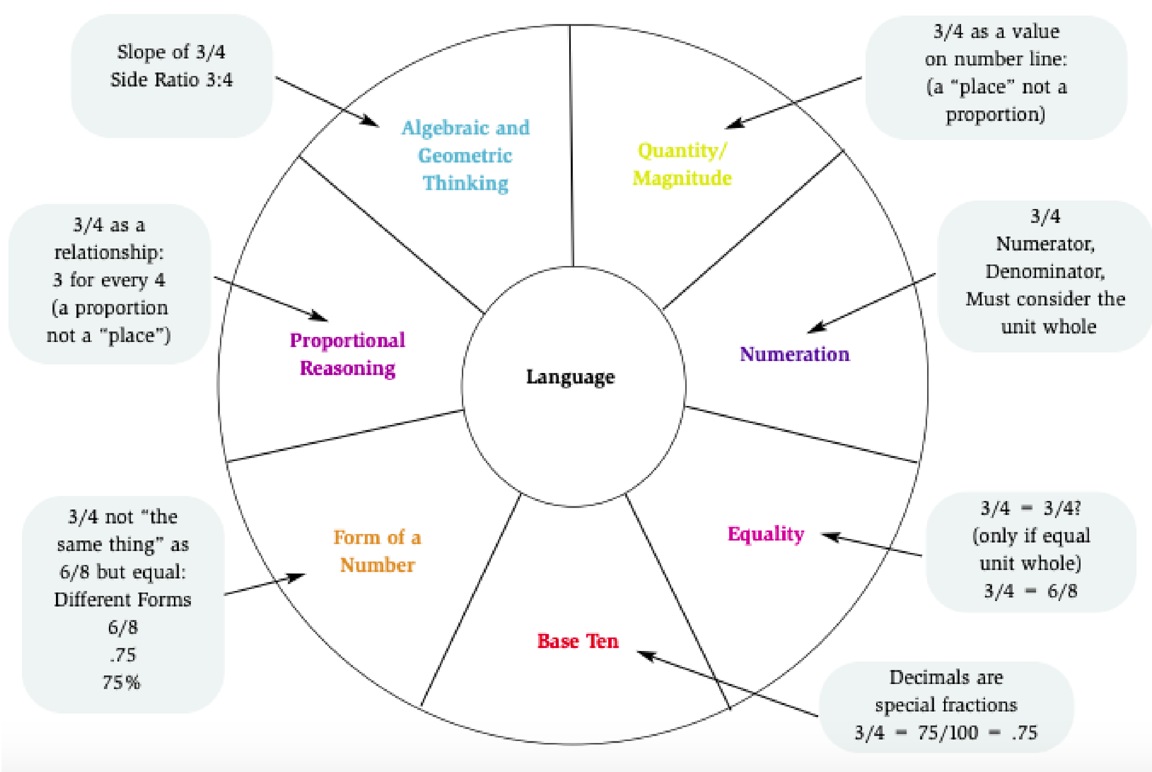
Figure 1 (Faulkner & Cain, 2009, p.25)



The teacher can use the model to identify the number sense components related to specific concepts and skills and then connect them to the big ideas, strategies and procedures in the math classroom. To illustrate how the model aids in this, Faulkner and

Cain (2009) use the example of estimation. They explain that estimation is dependent on the understanding and relationship between the number sense components of quantity, magnitude and proportional reasoning. Understanding the different components allows teachers to identify and scaffold learning opportunities for students. Different mathematical tasks and procedures are more dependent on certain components of numbers sense than others. Faulkner & Cain provide another helpful example of their model in relation to fractions, which is represented in Figure 2.

Figure 2 (Faulkner & Cain, 2009, p.29)



Faulkner and Cain's (2009) work illustrates that when a teacher considers and builds the connections and relationships within the number sense model for themselves and their students, mathematical thinking is developed.

Fueled by this definition of mathematical thinking, one must rethink traditional instructional practices. The following section will discuss types of mathematical discourse and the strategic implementation of discourse practices to achieve that end.

Math Discourse

As research has analyzed mathematical thinking, there has been increased discussion around discourse in math education. According to the Oxford Dictionary (2021), discourse, simply stated, is the written or verbal communication of ideas. Much of the research in math reform has roots in Vygotsky's theories of social constructivism, emphasizing that knowledge is constructed through social interactions and occurs in the "zone of proximal development" (Hufferd-Ackles et al., 2004). It is not a new idea that learning is primarily a social endeavor (Schoenfeld, 2016), yet increased attention must attend to its social nature in order to maximize learning and engage all learners. Increased interaction in and of itself, is not sufficient. It is important to investigate specific types and components of math discourse to determine what makes it effective in supporting and increasing mathematical thinking.

The National Council of Teachers of Mathematics (NCTM) identifies the facilitation of discourse on their list of crucial practices in effective classrooms (Sztajn et al., 2021). In NCTM's (2014) publication, *Principles to Actions,* it is stated that discourse includes the "purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication," (p.29). Purposeful, productive discourse creates space for students to build mathematical thinking by sharing conjectures, analyzing strategies, comparing and defending ideas, and constructing accurate conceptions (Nathan & Knuth, 2003), which ultimately builds procedural flexibility and shared conceptual understanding (Sztajn et al., 2021). Math discourse practices are foundational to the goal of math education, which is to create mathematical thinkers who can not only accurately compute, but can flexibly solve and communicate in varied mathematical situations.

Types of Discourse

In order to clarify the discussion around math discourse, it is necessary, then, to discuss and define the types and characteristics of discourse-rich classrooms. According to Sztajn et al. (2021), meaningful discourse is defined as "patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners," (p.7). Sztajn et al. (2021) also identify the following four types of math discourse to be used for different purposes:

- *Correcting* - This type of discourse is widely used in traditional math classroom settings. It involves teacher-initiated questions requiring student response, followed by teacher evaluation of the response. Although it does not promote the development of strategic competency and higher-order thinking, it does allow assessment of students' accuracy and speed with procedures.
- *Eliciting* - Teachers draw out strategies and solutions from a broad group of students. This type of discourse expands participation in the discourse and allows for both correct and incorrect approaches to be presented, valued, and discussed.
- *Probing* - In probing discourse, teachers ask further questions to examine the what, how, and why of student thinking and strategic approach. In this type of discourse, students engage in justifying their answers as well as evaluating the

reasoning of their peers. This type of discourse supports procedural fluency and strategic competence.

- *Responsive* - In this type of discourse, the students take over responsibility for asking the questions and probing the thinking of others. The teacher poses challenging tasks and requires all students to justify their thinking and establish connections among different solutions. It supports the development of adaptive reasoning and productive disposition.

Hufferd-Ackles et al. (2004) characterize discourse-rich classrooms as those composed of individuals who attend to building one another's mathematical understanding through engagement in meaningful discourse. In their research, Hufferd-Ackles et al. (2004) describe a framework for math-talk communities which includes the following essential components: questioning, explaining mathematical thinking, authority (source of mathematical ideas), and responsibility for learning. Hufferd-Ackles et al.'s framework identifies specific teacher actions and student roles for each of the four components, which ultimately transform classroom culture and maximize the math-talk community's goal of developing deep mathematical thinking. Hufferd-Ackles et al. go on to describe a math-talk learning community as one that is focused on meaning making rather than finding correct answers, and is marked by students acting in central roles in the process.

Discourse Norms and Classroom Culture

To transform traditional math classrooms from environments characterized by passive student learning and teacher-directed instruction to vibrant math-talk communities that develop deep mathematical thinking, classroom culture cannot be overlooked. The strategies and routines used to develop discourse patterns that foster math growth work simultaneously to transform the class culture. A foundational element of transformed math culture is the establishment of norms (Kazemi & Hintz, 2014). The Oxford Dictionary (2021) defines a norm as “a standard or pattern, especially of social behavior, that is typical or expected of a group”. In order to reshape norms, new learning needs to occur in order to unlearn the traditional beliefs about math. New norms should be centered around the research about mindset and brain development and their relationship to mathematical thinking. Other concepts to consider in norm development include shared beliefs around math as sense making, the role of mistakes in learning, the value of all voices, listening practices, and responsibility for individual and collective learning (Kazemi & Hintz, 2014). Math norms should be discussed and agreed upon by the teacher and the students after new understanding around the previously mentioned topics occurs. The norms, informed and supported by repeated actions and patterns (Sztajn et al., 2021), can then become part of the class belief system and ultimately provide stability for the class culture.

Questioning. Building on the foundation of norms, the questioning practices in the classroom also have a significant impact on both the class culture and the development of mathematical thinking. Discourse culture is built through the types of questions asked, who is empowered to ask the questions, the expected and accepted explanations to questions, and how members of the community listen, participate, and respond to the discussion (Sztajn et al., 2021). Bofferding and Kemmerle (2015) state that the style and substance of teachers’ questioning skills have a significant impact on the classroom learning environment. In order to engage in deeper mathematical thinking and build a discourse culture, teachers must examine and develop their questioning practices.

The questions need to change from attempts to elicit correct responses and keep students' attention on the teacher, to open-ended questions that emphasize student thinking. Hufferd-Ackles et al.'s (2014) framework illustrates how questioning strategies employed by the teacher not only increase mathematical thinking, but impact the students' conceptions of mathematical authority and responsibility for learning, which transform math beliefs and the culture of the classroom. This occurs as students hear more student perspectives and strategies which leads to recognition and value of student voice and understanding of their ability to defend and evaluate other responses. As a result, students see increased value in their mathematical perspective and grow to share the mathematical authority, resulting in increased responsibility for their own learning and for the learning of the community as a whole (Hufferd-Ackles et al., 2004). It is a significant cultural shift when students are no longer reliant solely on teachers' expertise and passively wait for information, but rather see themselves and classmates as strategic thinkers, offering valuable insights that deepen and transform their learning.

Talk Moves. In combination with questioning strategies, there are several talk moves that teachers can employ that simultaneously build mathematical thinking and develop high quality discourse culture (Kazemi & Hintz, 2014). The term "talk moves" describes the intentional questions or statements that teachers use to elicit student thinking, respond to their discussions and shift mathematical authority and learning responsibility from the teacher to the students. Some examples of talk moves are:

- *Revoicing or repeating* back what a student has shared to verify what was stated and potentially clarify or highlight an idea or strategy.

- *Rephrasing* what another student has said to emphasize important ideas or to slow a conversation to make space for contemplation.
- *Reasoning* by asking questions such as: Do you agree or disagree with your classmates' claims? Why or why not? Why does that strategy work?
- *Adding on* to what someone else has shared.
- Providing *wait time,* which allows students space to evaluate their thinking or the thinking of others.
- *Revising* thinking with any new insights that have been shared (Kazemi & Hintz, 2014).

Along with the strategic talk moves, teachers must also increase their attention to student strategies and thinking in order to support connections, draw meaningful contributions, build flexibility, and plan future lessons (Kazemi & Hintz, 2014; Sztajn et al., 2021).

Student Actions. In like manner, the student actions in discussions have a significant impact on the quality of discourse and the classroom culture that ensues. In order for students to participate in meaningful discourse, teachers must explicitly teach discussion strategies, supported by practices of accountable talk and nonverbal cues. As a result, participation is not limited to those who typically find quick, correct answers; all students can participate, share their thinking and advance the discussion (Kazemi & Hintz, 2014). The way both teachers and students speak in the mathematics classroom is critical not only to developing mathematical thinking, but to the way students view themselves and value their voice and the voices of others in math. Hufferd-Ackles et al. (2004) discuss how discourse-rich communities support the development of mathematical thinking for all students, but specifically mention the positive impact and significance for those who are considered not “linguistically prepared” (p.82). When intentional space is given for discourse, classroom culture is centered around the belief that all students have valuable mathematical ideas to contribute, and students are equipped with discourse skills to discuss conjectures, analyze thinking, justify responses, make connections, and revise strategies, then the onus for learning is shared by everyone.

Discourse Routines. There are several specific discourse routines that can be used to build and transform discourse culture in the mathematics classroom. These routines can be used in whole group and small group settings, and modified to align with many big ideas and math tasks.

- *Number Talks* - During this brief activity, teachers present a purposeful computation problem for students to solve mentally. Students indicate, with a physical cue (often a thumbs up), when a solution has been found and then work to find other strategies to solve the problem (Parrish, 2010; Sun et. al, 2018). After all students have indicated the discovery of at least one strategy, the teacher records the various answers for the group to see. Next, students share the strategies they used, while the teacher records their process visually, so that it can be examined by the group (Parrish, 2010). The discussion allows students to question each other’s methods, justify their thinking, clarify their strategies, examine mistakes or misconceptions, and revise their thinking (Parrish, 2010; Sun et. al, 2018). Number talks build mathematical thinking by supporting sense making, promoting flexible thinking, and encouraging the connection of number relationships (Sun et al., 2018).

- *Number Strings* - Students are presented with a sequence of math problems that call attention to a particular relationship, strategy or big idea (Bofferding & Kemmerle, 2015). The problems are written horizontally and introduced one at a time. The students are given ample time to solve the problems mentally and then share their thinking while the teacher represents each new strategy using a particular mathematical model (i.e. rekenrek, array, open number line, etc.) (Lambert et al., 2017). Teachers elicit student contributions and accept all answers, whether correct or incorrect, encouraging comparison, clarification and justification of mathematical thinking (Bofferding & Kemmerle, 2015). After engaging in number string discussion, students are able to use the visual models (intentionally chosen by the teacher) as tools to solve mathematical future problems (Lambert et al., 2017).
- *Think Pair Share* - Teachers present a problem or math task to the class and students are given time to generate a solution strategy. Students share their thinking with a partner through conversation, taking turns sharing strategies and solutions, building on or questioning each other's solution, and revising their answer if desired. In this routine, all students in the classroom are engaged in the discussion and have the opportunity to improve their verbal math explanations, build confidence and take an active role in their learning (Bofferding & Kemmerle, 2015).

As teachers develop deeper understanding of mathematical thinking and broaden their repertoire of discourse practices, they can transform their math classrooms.

Opportunity for high-quality math discourse is dependent on the problems and tasks students are asked to ponder. The discourse previously described is not possible without tasks that are designed to provide freedom and possibility. The next section of this literature review will define rich math tasks, provide concrete examples, and discuss their implementation in the classroom.

Rich Math Tasks

Math instruction that develops mathematical thinking requires instructional balance between conceptual understanding, procedural fluency, and application (SanGiovanni, 2017). In order to achieve that balance, teachers must consider the types of problems and exercises used in the classroom. Math tasks are a fundamental component of learning and doing math, and there is a correlation between the nature of the tasks and the quantity and quality of the learning that occurs in the classroom (SanGiovanni, 2017; Stylianou & Blanton, 2018). It is paramount, then, for teachers to thoughtfully select and design the math tasks used for instruction.

Definition of Rich Math Tasks

Rich math tasks are high-quality math problems or exercises that shift the focus of math learning away from prescribed algorithms and techniques and instead are designed to provide opportunity for students try, get stuck, experiment, apply knowledge in new ways, and develop their own strategies to get unstuck (Liljedahl, 2021; Verschaffel et al., 2009). The tasks are more open in nature and promote risk-taking, different ways of thinking, and can be extended for high levels of deep challenge, yet are still accessible to all students (Boaler, 2015; Zager, 2017). Although there is not a specific formula to create or identify a rich math task, there are common characteristics that guide their development and help teachers identify them. Rich math tasks work to accomplish the purpose of developing either conceptual understanding, procedural fluency, or math application. Rich math tasks have the following characteristics:

- Are aligned to content standards and/or big ideas in math
- Utilize representations
- Provide opportunities for students to engage in discourse about reasoning
- Are accessible through multiple entry points
- Provide possibilities for varied strategic approaches within fixed constraints
- Allow students to make connections between concepts (allow for pattern seeking to see consistencies rather than rules)
- Make students think and draw on the diversity of math knowledge
- Are interesting, authentic, and problem based (Boaler, 2018; Liljedahl, 2021; SanGiovanni, 2017).

Although rich math tasks don't produce proficient math students on their own (there are other instructional factors involved as was discussed in previous sections), they do foster the development of mathematical proficiency (San Giovanni, 2017).

Low-Level Tasks. To deepen the understanding of what constitutes a rich math task, it is important to examine what does not. Many math classrooms are abounding with assignments and activities that are categorized as low-level tasks, seemingly influenced by a "quantity over quality" mentality (SanGiovanni, 2017). Examples of low-level tasks include worksheets or workbook pages filled with skill and drill practice, word problems that are designed for a particular procedural solution, memorization tasks, and any procedure based tasks that do not provide opportunities to connect the thinking or procedure to deeper mathematical concepts (Liljedahl, 2021; Stylianou, 2018). Abundant practice of narrow questions not only negatively impacts students' math attitudes, it does not promote reasoning or transfer to new mathematical situations or connect to real life mathematical situations (Boaler, et al., 2018; SanGiovanni, 2017) which are foundational elements of mathematical thinking. SanGiovanni (2017) also warns that low-level tasks can yield correct answers in spite of incomplete or flawed understanding and the repetition can ultimately reinforce misconceptions. Teachers are then faced with the job of helping students unlearn what they have practiced.

Examples of Rich Math Tasks. There is an increasing number of resources available to find rich math tasks to incorporate into the classroom. There are books and websites that provide ideas and tasks for teachers to use and contribute to in order to expand the use of this type of learning in the classroom.

- *Open Middle* - In this type of task students are given a problem with a closed beginning and a closed ending, meaning that the starting point and ending point are the same for everyone. However, the path to the solution can be accomplished in many different ways. These problems address the same standards and topics, yet engage students in examining strategies and debate around the journey toward the solution. They must notice patterns and use conceptual understanding in order to find an efficient solution. An example of an open middle problem compared to a typical problem is as follows:

  Standard problem: $21 + x = 70$.

Open Middle Problem: Using the digits 1 to 9 at most one time each, create an equation where x has the greatest possible value. _ _ + x = _ _ (Kaplinsky, 2020).

- *Three Act Tasks* -As the name suggests, there are three parts to these tasks, designed to engage students in discourse, strategic thinking and modeling. In Act 1, students are introduced to a challenge with a visual (image or video) and proceed to discuss, wonder, and generate questions about what was seen. For Act 2, students are given a little more information and then they work to find a solution, making adjustments to their questions and strategies as they proceed. The task is concluded with Act 3, in which the solution is revealed and students then examine, compare and discuss the different strategies and connections they notice among them (Wolf, 2015).
- *Youcubed* - This is not a specific type of open math task but rather a hub of rich math tasks, curated by Boaler and a small group of educators. The tasks are designed or modified to be open, visual, and creative and integrate brain science and mindset. The purpose of youcubed is to provide a resource for teachers in order to transform math experiences and learning for students (Stanford University, n.d.).

Strategic Implementation

Unfortunately, most textbooks and curriculum do not provide rich math tasks due to the fact that they are not designed for isolated procedures or tied to a specific strategy or algorithm. Most rich math tasks are non-curricular in nature and don't necessarily map nicely to a prescribed curriculum (Liljedahl, 2021). However, it is possible to find rich math tasks that connect to big ideas and themes, although it is important to keep in mind that due to their open nature, students won't necessarily choose a path that aligns with the curricular goal. Ultimately, using rich math tasks requires a loosening of the grip on prescribed curriculum and a shift in focus to the interconnected concepts of mathematical learning and thinking. Even with the constraints of a prescribed curriculum, teachers can incorporate rich math tasks. They can be used to build the type of thinking or mental processes that are desired before transitioning to the curricular tasks. Liljedahl (2021) states that teachers can use curricular tasks in more of a rich math task fashion if they are presented to students before explicit teaching of strategies and procedures. When students encounter a task or problem without being told how to solve it, they are given the opportunity to find, share, and connect their own solution strategies.

Summary

For too long, math has been misunderstood as a performance subject, reduced to the experience of pursuing correct answers through isolated, repeated procedures and fast computation. Decades of research contradict this belief and the associated practices, yet they persist. There was a time when the goal of math instruction was largely focused on computation; traditional methods and practices were sufficient and ultimately accomplished the intended outcomes, but that is not the case anymore. At this time, our society and world require deeper mathematical understanding and increased ability to reason, discern, discuss, and communicate solutions to problems rather than simply compute with accuracy. Students must be given ample opportunities to think mathematically and proficiently practice computation (Parrish, 2010).

Mathematical thinking goes beyond arithmetic, it is about teaching ways of thinking rather than curriculum concepts (Carpenter et al., 2003). It is foundational to deep conceptual and procedural understanding in mathematics, but has applications for improved problem solving and impact beyond the classroom. The implementation of high quality math discourse and rich math tasks has the potential to transform mathematical mindsets and develop mathematical thinking, ultimately transforming overall math culture and resulting in improved performance and deeper thinking. My research question, *how can elementary teachers use math discourse and rich math tasks to develop mathematical thinking?* analyzes how rich math tasks and discourse implementation work in tandem to promote the development of mathematical thinking.

In the next chapter I will be presenting an outline and overview of my capstone project which will provide a curriculum resource to implement the practices and information presented and synthesized in the literature review. Chapter Three will provide the rationale behind the project, the frameworks used to create it, and a detailed description of the curriculum created in response to the question: *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?* The project will be a curriculum resource for upper elementary teachers, designed using Minnesota's Academic Standards in Mathematics with the purpose of developing mathematical thinking, transforming math mindsets and classroom culture. After a detailed description of the project, Chapter Three will go on to discuss the assessments that will be included in the curriculum, a comprehensive description of the setting and participants, and conclude with a timeline for project completion and implementation. The project will be created to impact teachers, as they work to transform traditional practices, and students who engage in this transformed perspective and approach to mathematical thinking development.

CHAPTER THREE

Project Overview

For too long, students have experienced math as primarily a subject of rules, procedures, and correct answers. On the contrary, the heart of math is meaning making. Mathematical thinking is multi-faceted and not only involves accurate computation, but the simultaneous development of deep, conceptual understanding, the ability to flexibly solve problems, and the capacity to explain, analyze and justify one's process (Baroody et al., 2007; Katz, 2014, Schoenfeld, 2016). In order to achieve this type of mathematical thinking, instructional practices must change. One area where this can be addressed is through an increase in high quality discourse. According to the National Council of Teachers of Mathematics, math discourse is considered a crucial practice for effective math instruction (Sztajn et al., 2021). In addition, students need to engage in rich math tasks that allow them the opportunity to make meaning by exploring possibilities, making conjectures, analyzing connections and investigating strategies.

In Chapter Two, a review of literature provided a foundation of research and understanding for the themes of traditional math practices, mathematical thinking, math discourse, and rich math tasks. Chapter Three provides an outline of the capstone project which will include standards-aligned number talks and rich math tasks to be used in a fourth grade classroom to address the question: *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?*

The chapter will provide an outline of the two unit supplemental curriculum project, beginning with a description of the driving rationale. From there, it will discuss the modified Understanding by Design (Ubd) framework (Wiggins & McTighe, 2011) which was used to design each of the lessons. The Minnesota State Standards (2007) in math and reading and the NCTM (2014) standards provided a guide for the essential questions and student outcomes. Lastly, it will discuss the school setting, student participants, assessments, and the timeline for development and implementation.

The next section will lay the foundation for the project development by discussing the rationale for the project.

Rationale

The development of mathematical thinking is essential in high-quality math education. It fortifies and promotes problem solving and thinking skills that benefit students beyond the classroom. An extensive body of research has identified how traditional math practices have emphasized computational proficiency and presented procedures and algorithms in isolation, resulting in underdeveloped conceptual understanding and student inability to apply mathematical understanding across the discipline (Boaler, 2015). Learning to think mathematically involves more than proficient computation and mimicked procedures; it goes deeper, to the understanding of big ideas, the ability to engage in evaluation and communication around those ideas, and the flexible application of appropriate representations and strategies in different mathematical situations (Baroody et al., 2007; Carpenter et al., 2003; Katz, 2014; Schoenfeld, 2016). Number talks and rich math tasks provide supported, open-ended opportunities for students to develop mathematical thinking through tasks and discussions that give space for exploration, conjecture, critical analysis, and rich discourse (Hufferd-Ackles et al., 2004; Knuth, 2003). In order for the needed change to occur in math education, classroom practices must change through the strategic implementation of curriculum that provides opportunities to engage in rich math tasks and quality math discourse.

In addition to the research, the rationale for this project includes personal observations within the elementary setting. In both an intervention and classroom setting, I have observed students consistently struggle to solve math problems that address a “learned” concept, yet were presented in a different format, requiring different steps or procedures than they had been taught. In addition, I’ve observed students' hesitancy to attempt new problems and mathematical situations out of fear of an incorrect answer or simply because they are conditioned to be told how to solve the problem first. In response to these observations, I began to encourage students to attempt to problem solve using what they know, and provide open-ended opportunities for students to collectively explore potential solutions, releasing them from using a particular strategy or procedure. With the freedom to use manipulatives, draw models, and discuss with classmates, I noticed increased enthusiasm and engagement in the tasks, varied solution strategies, and critical thinking about the process. The addition of discussion and space to develop their own strategies appeared to support higher engagement and mathematical thinking with these groups of students. In response to research and personal observations, this project will create a curriculum that is centered around building mathematical thinking through intentional discourse and rich math tasks.

The next section of this chapter will discuss the curriculum framework and standards that will be used to drive and develop the curriculum in this project.

Framework

There are several frameworks that will intersect to guide the development of this curriculum including Understanding by Design (UbD), the Minnesota K-12 Academic Standards in Mathematics, and the National Council of Teachers of Mathematics' (NCTM) five content standards from their published *Principles and Standards for School Mathematics* (2000) as listed in their executive summary (n.d.).

The curriculum for this project will follow the three stages of backward design outlined by Wiggins and McTighe (2011). Understanding by Design not only provides a framework for the development of this curriculum, but also has a strong connection to the heart of mathematical thinking through its design process which places similar emphasis on big ideas, the goal of understanding defined as student meaning-making, and the importance of student ability to transfer ideas (Wiggins & McTighe, 2011).

Wiggins and McTighe's (2011) UbD framework lays out three stages in the curriculum design process: identifying the desired results, determining appropriate evidence to demonstrate that the desired results have been met, and planning the learning experiences to achieve that end. The Minnesota K-12 Academic Standards in Mathematics and NCTM's content standards will provide guidance for the essential questions and what students must know and be able to do.

Project Description

This capstone project will be a supplemental curriculum for fourth grade math classes. It will contain two units that will develop classroom discourse practices and rich math tasks to build mathematical thinking. The units are meant to be a catalyst to transform the overall math culture and instructional practices in the classroom. Further math instruction beyond the two units can continue to use the discourse practices established in this curriculum.

This curriculum project will incorporate the MN K-12 academic standards in math (2007) 3.1.1.4, 3.1.1.5, and 3.1.2.1, which state that students will "compare and represent whole numbers up to 100,000 with an emphasis on place value and equality," and "add and subtract multi-digit whole numbers" (p.12). In addition to the MN K-12 math standards, the discourse development in this project will include the MN K-5 Speaking, Viewing, Listening and Media Literacy Benchmark (2010) 4.8.1.1 which states that students will, "engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on *grade 4 topics and texts,* building on others' ideas and expressing their own clearly," (p.34). In tandem with the state standards, the project curriculum will develop components of NCTM's (n.d.) five process standards, including:

- *Problem Solving* - apply and adapt a variety of appropriate strategies to solve problems.
- *Reasoning and Proof* - develop and evaluate mathematical arguments and proofs
- *Communication* - communicate mathematical thinking coherently
- *Connections* - recognize and use connections among mathematical ideas
- *Representations* -create and use representations to organize, record and communicate mathematical ideas (Executive Summary, p.4)

Unit one will be called "Introduction to Math Tasks & Mathematical Thinking". There will be five lessons in unit one, each designed for roughly one hour of math instruction. Students will participate in tasks designed to develop a mathematical mindset and new understanding of what it means to learn and do math. It will include research about the brain and what it means for students and their mathematical learning. These lessons will support the development of class math norms. The unit will also provide open-ended, rich math tasks that are not tied to a specific procedural outcome or strategy, but focus on the development of the process standards and allow students space to fully engage in meaningful discourse and meaning making. Number talks and number strings will also be included in this introductory unit so that students can learn and practice the discourse routines.

Unit Two, "Review of Place Value and Addition/Subtraction Strategies," is a beginning of the year review. It will develop conceptual understanding of place value, extended to millions, through tasks that provide opportunities for students to examine and connect numerical relationships and strategies. This unit will review the third grade math standards while extending to the fourth grade multi-digit addition and subtraction benchmarks as well. A key element of this curriculum project is the combination of rich math tasks and discourse. As a result, this unit will incorporate discourse strategies and practices such as number talks and number strings in tandem with rich math tasks. The unit is intended to be used as a supplement to classroom curriculum that focuses more on procedural competence.

The goal of this curriculum is to develop a rich discourse culture and engage students in mathematical thinking, while deepening their conceptual understanding of place value and addition/subtraction strategies. This curriculum will provide both the teacher and students with new discourse practices and tasks that can be modified and applied to other units throughout the year.

Assessment

Both units developed for this project will include a variety of assessments, including formative and summative assessments that will be used to provide information about mathematical mindset, identify areas of needed growth, illuminate student misconceptions, and evaluate student understandings of mathematical concepts. The assessments for the units will take different forms, including questionnaires, observations, performance tasks, performance task rubrics, self reflection rubrics and a summative performance task.

The following section of this chapter will discuss the participants and setting for which this project will be created.

Participants and Setting

This project will be implemented in a suburban school district in the St. Paul area. According to the Minnesota Report card (2021), the school district has consistently performed slightly higher than the state average on proficiency assessments, yet has demonstrated consistent decline in math performance since 2015. The student population is 71.6 percent White, 7.8 percent Hispanic or Lation, 7.2 percent Asian, 6.5 percent Black or African American and 6.4 percent two or more races. 4 percent of the students are English learners and 27 percent of the students receive free and reduced lunch (2021). The school where I teach is a Kindergarten through fifth grade Title I elementary school. There are 389 students enrolled. Each grade level has three homeroom teachers, supported half time by an intervention teacher. However, due to the pandemic, math intervention services have been suspended. The school also has one half time English Language learner teacher. The school district has been providing professional development opportunities over the past several years to grow mathematical understanding and support a transformation from traditional practices. This curriculum is being designed to help support that effort.

More specifically, the setting for this curriculum implementation is a fourth grade self-contained classroom, although the open-ended nature of the discourse practices and tasks allows for modification across grade levels and use beyond that one setting. Currently, 25 percent of the students in this classroom are classified in the ‘Some Risk’ category on the FastBridge aMath assessment and 20 percent are identified as ‘High Risk’. There are two other teachers on this fourth grade team that could potentially engage with this curriculum, however it will be piloted first in my classroom. The third and fifth grade teams will have the opportunity to use this curriculum, particularly the first unit used to launch the school year.

In the following section, I will discuss the timeline of the project, including its creation and implementation in the classroom setting.

Timeline

This project will be created during the summer of 2021 and be implemented during the 2021-2022 school year. It is intended to be used to launch the school year, with the goal of starting the year establishing math norms, transforming mathematical mindsets and developing a foundation of discourse practices in the math setting. From there, unit two will provide opportunities to develop mathematical thinking, building on the foundation of unit one. However, unit one of this project could be reused at any point throughout the school year to reinforce discourse practices or reestablish norms.

As stated in the project description, this project will include two units. It will be developed specifically for a fourth grade classroom, although it can be modified to be used in other elementary grade levels as well. Each unit will provide curriculum for approximately 15 days, keeping in mind the realities of needed flexibility and potential interruptions in the school calendar. The first unit, "Introduction to Discourse, Rich Math Tasks & Mathematical Mindsets," is intended to be used during the first weeks of school. The second unit, "Place Value," will immediately follow the first unit, and introduce specific big ideas in mathematics while incorporating and building on the norms and practices established in unit one.

Summary

This chapter provided an outline for the curriculum developed in response to and support of the question: *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?* The rationale for this project is based on research that supports the use of discourse and open-ended math tasks as key components of high-quality math instruction. The rationale also includes observations of math behaviors and struggles from classroom experiences. The curriculum will be designed using Minnesota K-12 Mathematical Standards, Minnesota K-5 Speaking, Viewing, Listening and Media Literacy Benchmarks, NCTM's process standards, and the framework of Understanding by Design (Wiggins & McTighe, 2011). The project description provided an overview of the components and purpose of the units, specifically mentioning key outcomes, assessments, strategies and experiences used in its development. After that, the specific school setting and project participants were provided to put the capstone project in context and address the challenges faced due to these factors. Finally, a timeline for project completion and implementation was provided. Chapter Four will provide an evaluative reflection of personal and professional insights gained as a result of the capstone project development.

CHAPTER FOUR

Reflection

Introduction

As an educator, I have a strong desire for my students to become lifelong learners who are equipped with the tools and experiences to be meaning makers and problem solvers, both in the classroom and beyond. In my classroom, I have observed a significant number of students demonstrate mastery of a particular procedure or concept, only to forget it months later or "freeze up" when faced with a math problem that varies from the specific strategies or formulas that were modeled. In addition, I have perceived an underlying understanding that math is a subject of quick computation and correct answers and for too many, it feels out of reach. These patterns have caused me to pause as an educator and look around at the practices that perpetuate these beliefs and behaviors, and have ultimately led me to this capstone project.

This capstone was created to answer the question: *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?* In Chapter One, I shared my journey through math education as a student, parent, and teacher, discussing the experiences that have shaped not only my own understanding of math, but the instructional practices I use in my classroom. In Chapter Two, I explored literature that discussed the practices, culture, and mindsets that exist in math education and the outcomes of their continued use. I also examined the components of mathematical thinking and the relationship of mathematical discourse practices and rich math experiences on its development. In Chapter Three the central framework used to develop the curriculum, an adaptation of Wiggins & McTighe's (2011) Understanding by

Design, and the guiding standards used were explained in depth. This chapter also included an explanation of the rationale for the curriculum design, details about the intended upper elementary audience, and a general timeline for its implementation in the classroom. In Chapter Four, I examine what I have learned throughout the process of completing this capstone. Included in this chapter is analysis and discussion of key research that was influential in the creation of the curriculum. In addition, implications and limitations will be considered, as well as discussion around potential future research that will extend the work of this capstone. To conclude, I provide an overview of Chapter Four and revisit the foundational research question.

Learnings

Overall, the capstone process resulted in significant learning for me. Although engaging in this level of research, writing, and curriculum development was a challenge, I found the entire process to be highly rewarding. The capstone experience pushed me to dig deeper as a researcher, honing my skills at unearthing and connecting different research sources that were specifically related to my question. I was enriched through the process of following concepts and questions that were raised in my study and that led me to other areas of research, unearthing new layers of connection and depth in my understanding of math education and refining my conception of mathematical thinking. The research process also pushed and developed my ability to read, understand, analyze and synthesize the academic writing of others. Much of what I have studied in my professional work before this capstone experience was typically written in a simpler style than the studies and research articles I used for my literature review. I had to employ different reading strategies to analyze and interpret some of the text I encountered. As a writer, I was pushed to expand my skills to meet the requirements of formal academic and curriculum writing. I believe that both of these challenges will help me immensely as an educator as I relate this experience back to what my students experience as learners in my classroom.

My project sought to create a supplemental math curriculum that is built on shifting the understanding of the subject of math to sense making, rather than the mastery of unrelated facts and procedures. In order to "do" math, students need opportunities to think and explore through engagement in high quality discourse and open-ended rich math tasks. The curriculum consists of two units. The first unit is designed to be used to launch the school year and taught over five consecutive days. The second unit is a review of third grade standards and can either be taught consecutively, or used in tandem with a district curriculum.

As I constructed the curriculum, I was continually confronted with the reality that the traditional assessment strategies I was accustomed to were inefficient at assessing the growth and development of mathematical thinking. Approaching assessment as a process of refinement, digging deeper, and observed growth, rather than a summative result surfaced as the necessary method. I continuously wrestled with my tendencies to want to assess math learning in the traditional manner I was accustomed to as a student and what has typically been provided in previous district curriculum. However, the focus of the curriculum on mathematical thinking development through discourse and rich math tasks, rather than computational fluency, ultimately led me to create assessments that rely heavily on teacher observation, student reflection and continuous analysis and revision of student work on open-ended tasks. It is a shift from the simple computational quiz, marking equation after equation either correct or incorrect, and demands the teacher and the student learn to analyze the thinking that is represented or discussed and reflect on the meaning making process. The unit assessments consist of a collection of observations, discussions and evidence of thinking that is revisited and used to build mathematical thinking, in the specific unit and beyond. The summative assessment is a final performance task which requires students to use the mathematical thinking developed throughout the unit.

The learning around the transformation of assessment practices also impacted the curriculum writing process in that I had to pay close attention to the potential connections students could make when constructing meaning during number talks and open-ended math tasks. I focused on the alignment of the discourse strategies and rich math tasks to the intended outcomes that were selected for each lesson. The discussion and tasks created for the curriculum needed to provide opportunities for students to create meaning, yet still highlight specific standards. I included probing questions to help guide students in their meaning making and to construct math connections, as well as help the educator highlight student discoveries that align with the intended outcomes. The different components of the lesson design and unit construction form a web of knowledge, rather than a traditional linear progression. Reframing and approaching math instruction with this lens was significant learning for me.

Along the way, I found it surprising to discover how far back the body of research regarding the recognition of a necessary shift in teaching practices and focus in math education dates. Researchers have been examining math instructional practices for more than fifty years. It is clear that the awareness of and desire for change is practice does not translate to actual change quickly. It was also interesting to recognize the complexities involved in transforming math education practices, including tradition, culture, teacher training, overall understanding of the nature of mathematics, and mindset. It became apparent that there are numerous, interconnected factors that influence the perpetuation of traditional math practices, all with their own bodies of research. After writing the curriculum, I have a new appreciation for the difficulty of this change, experiencing the pull of my traditional training at almost every step of the construction.

Revisiting the Literature Review

There is an incredibly large body of research in the area of math education. I found that initially the research I was doing seemed to continuously lead me to more research. Through that process, I greatly expanded my own understanding of the complexities of math education, the persistence of traditional practices, and the components of mathematical thinking. As time went on, I had to refocus my learning and research around my question, *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?,* and determine which facets were the most important to my specific question.

There were several sources that emerged as particularly impactful for the creation of my project. Alan Schoenfeld's (2016) work, *Learning to think mathematically; problem solving, metacognition, and sense making in mathematics,* and Jo Boaler's (2015) work regarding brain research and mindset had significant impact on my reframing of mathematics as sense making rather than the act of mastering a collection of facts and procedures. Schoenfeld's (2016) research highlighted and defined the importance of strategic competence and adaptive reasoning; both of which are key for designing math experiences that focus on flexibility of thinking, multiple representations, connection of concepts and reflection. I consistently relied on this new understanding as I crafted the structure and lessons of the curriculum units. Boaler's (2015) focus on intentional mindset development and open-ended tasks presented in *Mindset Mathematics* and the website *youcubed.org*, provided specific examples and tasks that I used in the creation of both units. Unit One was written with significant focus on the development of a math mind set that can serve as the foundation of all math learning. Both authors were instrumental in the focus on open-ended tasks that leave room for students to make meaning.

Finally, in the area of discourse development, Elham Kazemi and Allison Hintz's (2014) book, *Intentional talk: How to structure and lead productive mathematical discussions,* provided tools to use as I crafted lessons to build a math discourse culture. The different Number Talks, Number Strings and other discourse strategies in the curriculum include directives for specific teacher talk moves and student actions that foster both the development of discourse and mathematical thinking (Kazemi & Hintz, 2014). The literature review research provided a strong foundation for the development of this curriculum project.

Implications

My intention was to create a curriculum to help transform not only my math instruction, but to hopefully help other teachers do the same as well. My building is currently in the process of redesigning our upper elementary classrooms from individual grade levels to a multi-grade level approach. This coming school year, I will be one of three teachers who will transition into a classroom that is a combination of fourth and fifth grade students. In the past, the math instruction in both grade levels has followed the district provided curriculum, which is grounded in more traditional math practices and has a significant focus on computation and procedures. This curriculum will be helpful as we launch our school year and build a new math culture that is intentional about fostering a math mindset, high quality discourse and meaning making across two grade levels in one classroom. We will be able to use this curriculum as the foundation of our first two units and build consecutive units, continuing the use of the discourse strategies and rich math tasks. As users of this curriculum, students will be able to engage in discourse and rich math tasks that provide equitable access, opportunities for a wide range of solutions, connection and extension possibilities (low floor, high ceiling), and essential experience with the state math standards. This project could also have implications for future curriculum decisions or development in my building and district, based on its potential impact and success.

Beyond the implications for my classroom and building, this curriculum could be used across my district and beyond. The literature review provides a foundation of understanding for the need to transform math classrooms into student centered, discourse rich communities. It also provides detailed information to build teachers' understanding of the components of mathematical thinking and specific practices to implement discourse and rich math tasks. At any grade level, Unit One could be modified and used to establish a math mindset culture and train students in discourse practices. The Number Talks, Number Strings and other discourse routines can be easily adapted for the academic needs of primary, middle school or high school classrooms. Unit Two could serve as a springboard for other teachers to initiate the implementation of rich math tasks, whether the specific tasks are used or other tasks are selected from the resources attached to the project. Although the capstone chapters and curriculum project can be beneficial to teachers and students, there are some limitations to the project as well.

Limitations

One of the limitations of this curriculum project is that it does not address all of the math standards for an entire school year in an upper elementary classroom. It is a starting point, establishing discourse practices and a math culture centered around a growth mindset and meaning making. The curriculum provides an introduction unit and a unit that addresses place value, addition and subtraction standards. It was designed to align with the current scope and sequence used in fourth grade classrooms in my district. This is specific to my setting and may not be applicable for the timeframe of another setting. In order to continue in this transformed math culture, I will need to continue to design lessons using this template and based on this research. It will continue to be a work in progress as it is developed and modified over the course of a school year.

Another limitation is that it does not provide specific structures for repeated computational practice and review. This curriculum relies heavily on a teacher's understanding of the components of mathematical thinking as well as the interconnectedness of mathematical concepts. Using this curriculum requires the teacher to be responsive to student learning, providing structure and guidance through questions and student examples. A teacher must skillfully continue to provide the procedures and algorithms connected to the strategies students use and develop throughout the units.

Future Research

As I researched for the literature review and created this curriculum project, I have identified other areas of research that could further enhance this discussion. I would like to learn more about how to enhance computational proficiency within a meaning making model. I would be interested to discover how to best incorporate computational practice and efficient procedures into meaning making, without reverting back to traditional practices. In addition to researching more about this balance, I would be curious to investigate the relationship between math classrooms marked by discourse and rich math tasks and standardized test scores, particularly if the skills developed would translate to more traditional testing methods. As math practices shift, more research will need to be done to analyze its success.

Throughout the research process for the literature review, I encountered many references to the connection of teacher's mathematical understanding and their ability to guide students in math development. This research leads me to believe there is future work to be done in the area of professional development for both future and current teachers in order to support their own mathematical understanding. In particular, I would be curious to research how to develop this for teachers who learned as students and were trained as teachers with traditional math practices. Math learning is a complicated web of interconnected understandings, there are many opportunities for further research.

Communication

Throughout the curriculum units, I will collect observational data and artifacts that exhibit the components of mathematical thinking and discourse strategies students are using to make meaning. By the end of the two units, students and myself will have concrete examples and representations of the progression of their mathematical thinking, which will be used for future lessons and discussions. The data collected will be analyzed by students and myself, in both small and large group settings, and continuously revisited to construct new meaning, make mathematical connections, and guide future discourse experiences and interactions with open-ended tasks. I will be sharing these results and the student progression with teammates, the instructional coach, and the principal through our discussions in our professional learning communities and in our examination of practices for our multi-grade approach. There will be opportunities for observation of discourse and analysis of the artifacts that are collected from the rich math tasks.

Benefits to the Profession

As I have previously mentioned, there is no shortage of research on the need for change in mathematics education. The data from the last fifty years supports the notion that traditional math practices are not cultivating the type of mathematical learning that is necessary in our current world. I believe that this curriculum, and the research that supports it, is a step toward the kind of experiences necessary to build mathematical thinking, rather than solely computational proficiency.

I believe that this capstone is a benefit to all teachers, across all grade levels. This project will benefit teachers who have felt apprehensive to depart from their prescriptive curriculum, or for those who are uncertain of how to build discourse practices. The curriculum helps establish a math culture that is rooted in the belief that everyone can do math, make sense, and contribute to the mathematical conversation. It provides concrete experience with math that focuses on connecting big ideas, rather than memorizing a series of separate skills and procedures. My hope is that it will give teachers confidence to try another way and empower students to engage in their thinking and learning.

Conclusion

This project was created to answer the question: *How can upper elementary teachers use math discourse and rich math tasks to develop mathematical thinking?* Through my research and creation of the project, my foundational understanding of mathematical thinking expanded and I was able to recognize its connection to intentional discourse development and rich, open-ended math experiences. The research provided clear definitions of the components of mathematical thinking, specific discourse practices and group-worthy tasks that could work together to foster a change in math culture and practices. Using the research as a guide, I developed two units of study that correlate with Minnesota State Standards in Mathematics and Speaking, Listening and Viewing. Each unit provides in depth lessons with essential questions, student objectives, discourse routines, assessments and rich math tasks. The first unit works to establish mathematical mindsets, class norms and a discourse community within the math classroom. The second unit focuses on specific place value, addition and subtraction standards and provides a variety of discourse routines and rich math tasks. While the capstone project is now complete, I plan to continue developing units and lessons with the same framework, continuing to use discourse practices and rich math tasks with the remaining math standards for the school year. This project will continue to be adapted as time goes on, using student and colleague feedback to refine it.

Mathematical thinking is critical for students to become lifelong learners and problem solvers, both in and out of the classroom. It is no longer enough for students to just be fast and accurate. The world needs thinkers, analyzers, and problem solvers who can discover multiple solutions, justify their thinking and analyze the thinking of others. This project will hopefully be a starting point for change, equipping students to see and understand themselves, each other, and math in new and exciting ways.

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