Summer 2021

Combining Cognitively Guided Instruction With Required Mathematics Curriculum To Improve Overall Number Sense And Problem Solving Skill

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COMBINING COGNITIVELY GUIDED INSTRUCTION WITH REQUIRED MATHEMATICS CURRICULUM TO IMPROVE OVERALL NUMBER SENSE AND PROBLEM SOLVING SKILL

by

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A capstone project submitted in partial fulfillment of the requirements for the degree of Master of Arts in Teaching.

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Saint Paul, Minnesota

April 2021

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Chapter One

Introduction

From a very young age, I would have been quick to tell you that my least favorite subject in school was math. I was always placed in the lowest math group, I never raised my hand during class, and was constantly covering up my work and answers for fear of a classmate pointing out a mistake. The standard algorithms and equations did not make sense to me and I quickly began to think of myself as stupid. I, like many other students, had never built up a strong number sense and never learned the necessary problem solving skills for high levels of mathematics. It was not until I was in my senior year of college, in a course called Teaching Mathematics in Elementary School, that I realized I was not alone in this.

It was during this course that I first became aware of Cognitively Guided Instruction (CGI). This type of instruction, developed by a number of teachers and researchers including Thomas Carpenter, Elizabeth Fennama, Megan Loef Franke, Linda Levi, and Susan B. Empson is one that focuses on student thinking and strategy. Rather than teaching algorithms and strategies, as most traditional math classes do, a CGI classroom lets students come to solve problems naturally and using their own way of thinking. Students work to explain their thinking to peers and teachers. From there they begin building up understanding of mathematical concepts and principles. Now, as a kindergarten teacher, I am finding it quite challenging to incorporate this type of teaching with my school's curriculum. Throughout the year I have found myself asking the same question: *How can teachers use Cognitive Guided Instruction in conjunction with current* *required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?*

Personal Experiences with Mathematics

Through out elementary school I was consistently placed in the lowest math group. My friends were always in the highest group, solving multiplication problems while I, and others in my group struggled with two digit numbers.The lowest group was filled with students who were slow to catch on, who couldn't explain their thinking, and who took the entire year to memorize their math facts. Students in this group never raised a hand and would cover up their work for fear of a teacher or peer pointing out a mistake. As I grew older, things did not change much and I remained a part of this 'bottom' level however, I did begin going in for extra help. I would spend multiple mornings a week in my math classroom, asking questions, doing practice problems, and forever trying to memorize that basic algorithm. I ended high school with the understanding that I was just bad at math, that even if I worked harder than other students, I would still be bad, still get below average scores, and never be able to explain my thinking. I told myself it was not that important and moved on.

Once in college, I entered the School of Education at Hamline University. I had a large work load but was getting it done and doing well. Then students began talking about the mathematics course, Teaching Mathematics in Elementary School. Everyone said that it was the most difficult course in the program and was to be taken in your last semester. Immediately, I told myself I would fail. I was anxious and constantly thinking about my experience in school. I was doing so well in college and this course was going to ruin it. I worried that I would be seen as the 'stupid' girl again. However, on the very first day we were told to solve problems without using the basic algorithm. Rather than solving 67+ 37 by regrouping, I added 60 and 30, then 7 and 7, and finally 90 and 14. It was shocking to find how easy and intuitive it was to solve in this way. For the first time, the way I wanted to do math was okay. Students continued working on small problems, finding new ways to solve each time. The professor prompted students to play with numbers and shapes. I began to see so much more clearly that I was not lacking the ability to do math, I was just doing it and learning it in a different way. Slowly but surely, my number sense grew. I felt as if I had a whole new understanding of what math was, of what I was supposed to learn as a child, and truly felt like I could help others feel the same.

Cognitively Guided Instruction

Over the course of the semester we learned more about Cognitively Guided Instruction (CGI). This type of instruction is student centered and focuses on strategies created or used by the students. Cognitively Guided Instruction steers away from the standard algorithms because they are not intuitive or natural to students. One of the founders of Cognitively Guided Instruction, Thomas Carpenter, writes in his book, “Initially, young children’s conceptions of addition, subtraction, multiplication and division are quite different from adults’. This does not mean that their conceptions are wrong or misguided. In fact, their conceptions make a great deal of sense, and they provide a basis for learning basic mathematical concepts and skills with understanding” (Carpenter, et al., 2015, p.1). Cognitively Guided Instruction uses what students already know, or the strategies they are already using, as a basis for teachers to build off of. CGI is an effective way of building up students number sense and mathematical understanding because it allows the teacher to use student thinking within their own instruction.

Using CGI in the classroom may involve the use of word problems written so that they are relevant to student's lives. Rather than walking students through the problem step by step, a teacher may present the word problem, talk through the real world situation given, and then have students use materials in front of them to solve in which ever way they see fit. Students may use cubes, draw pictures, skip count, or already see that it is a basic addition problem and write a number sentence to solve. Teachers watch carefully and discuss with students which allows teachers to become aware of the level of mathematical understanding each student possesses. Having students explain their thinking is a crucial part of a CGI classroom. By doing so, students are building up their understanding while also explaining strategies in a new light that may help improve other student's understanding.

Importance of the Question

Today, I work as an elementary teacher in a private school. I began as a second grade teacher but will be moving to kindergarten this fall. I came into my first year of teaching confident with my ability to teach math. However, as I began to look at the schools choice of curriculum, Sadlier Mathematics, I realized that it would be difficult to incorporate CGI into the school day. After talking with the two other second grade teachers, I found that they followed the curriculum almost exactly as written and also tend to teach things in a traditional, direct instruction setting. This comes from a belief that it will frustrate parents if teachers are teaching math differently and that students may struggle when they move onto third grade if they do not have a firm understanding of algorithms and equations. The focus is not on number sense nor supplying students with a firm base for explaining their thinking. Rather, the focus is on preparing students to do well on the assessment. I have found myself turning to this as well. If we do not teach these specific problem types or algorithms, how can we ensure that students do well on essential assessments. Second grade also happens to be the year of the common core standards CCSS.MATH.CONTENT.2.NBT.B.5 and CCSS.MATH.CONTENT.2.NBT.B.6. Both standards focus on addition and subtraction with two and three digit numbers. This means teachers and curriculums turn to the standard algorithm with regrouping rather than focusing on number sense or problem solving. After talking with my colleagues, there seemed to be no getting around teaching it, after all it was in the curriculum and the overwhelming belief was that it was the only way students should be doing math.

Unfortunately, as a first year teacher I had to adjust by teaching the standard algorithm and following the curriculum just as my colleagues do. However, using small group rotations along with whole group problem solving can balance out the curriculum and add in some CGI practices along the way. Unfortunately, there is only so much time in a math lesson and as the school year progresses it becomes increasingly difficult to work on number sense and problem solving while also ensuring that students are getting what they need to do well on the assessments. This is also the chief complaint from colleagues. With limited whole group instruction, they feel they will move too fast for students or skip parts of the curriculum needed for students to pass the test.

Teachers need to feel confident and comfortable with what they are teaching. That is the benefit of using a curriculum from year to year. You become accustomed to each lesson and unit, learn which problems to skip, or which ones you need more examples of, and eventually become so comfortable that you are teaching exactly what is in the curriculum and skipping over valuable concepts and understandings. For example, a teacher may teach each lesson on base ten concepts while their students fly through worksheet after worksheet. Then the teacher shows a word problem on the board and multiple students do not know how it relates to base ten. The teacher proceeds to show students how to solve the problem using the method shown in the curriculum. From there on students do well on curriculum assessments because they use the same method over and over. The trouble here is that students are missing out on the essential problem solving skills. The teacher told the class how to solve the problem, they did not need to work through it, connect concepts, or explain their thinking because that was already done for them. A cognitively guided classroom would allow students to work through the problem and solve it in their own way.

Without training, CGI can be incredibly daunting and difficult. Many questions arise and many teachers are outwardly against diverging from the curriculum. This is where my research question comes in. *How can teachers use Cognitively Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?*

Summary

In this chapter, we discussed my own experiences in mathematics throughout my life and schooling. I am not alone in these experiences and students all over the country are struggling to build their mathematical understandings because they are lacking in basic number sense and problem solving skill. Students are following algorithms and equations and doing well through memorization of facts, but struggle greatly when it comes to true problem solving. A cognitively guided classroom would allow students to build up their own understandings of mathematical principles and concepts. Teachers would come to better understand student thinking and therefore be better prepared to help them grow. However, teaching mathematics using Cognitively Guided Instruction is difficult and proper understanding of its' goals and objectives are important. Teachers need to feel confident in what they are teaching, which is a benefit of current curriculum.

The remainder of this paper will seek to answer the question: *How can teachers use Cognitively Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?* In Chapter 2, research on Cognitively Guided Instruction, current curriculums, and best practices in the math classroom will be reviewed. Many sources will be explored and analyzed. In the following chapter we will delve into the development of the project itself. The paper will conclude with my findings, a reflection and a summary of research I found most important to the question.

Chapter Two

Literature Review

Introduction

Understanding of the best way to teach, the best way to assess students, and the best way to prepare students for their futures is constantly changing. However, the overarching and most common teaching practices are not. Many students are having similar mathematics experiences to that of their parents even though our understanding of how students learn and grow has changed. Students are still taking timed tests, memorizing algorithms, and completing worksheet after worksheet. The country's state and national standards have changed to allow for a more individualized approach, yet core curriculums still work off of uniform and rote methodologies. Chapter Two will discuss the reasoning behind this. First, it will delve deeper into the Common Core State Standards, the Nebraska State Standards, and the Archdiocese of Omaha Standards. Next, the chapter will look closely at traditional mathematics instruction and the current curriculum used at the project site. Then it will begin to look at Cognitively Guided Instruction and the impacts its implementation can have on student learning.

Ultimately the chapter seeks to better answer the question, *How can teachers use Cognitive Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?*

Mathematic Standards

It is important to note the final capstone project will be completed in a Catholic grade school in Nebraska. The lesson plans will need to meet standards set out by the

Archdiocese of Omaha and the State of Nebraska. The lessons will also need to align with the curriculum required by the administration. This section of the review will explore the differences between the three sets of standards as well as the reasoning behind them. This comparison will allow for a better understanding of what is being asked of students and what strategies are essential to teach.

According to the Common Core website, state education standards have been around since the early 1990s. By 2000, every state had adopted its own standards for students from grades three to eight. These standards outlined what every student should know and be able to do by the end of each grade level. However, each state had its own definition of proficiency and understanding. Ultimately, this led to the development of the Common Core State Standards for students in Kindergarten to Twelfth grade. This initiative was led by state leaders such as governors, state commissioners of education, and members of the Council of Chief State School Officers. These standards set out to improve mathematical achievement in this country through substantially more focused and comprehensive goals.

The Common Core State Standards stress the conceptual understanding of key ideas and continually return to those ideas. For example, students begin developing an understanding of place value in only first grade, but should continue to come back to it and build upon that knowledge until much further into their schooling. The Common Core Standards Initiative goes as far as defining the word understanding. This simple definition allows teachers and administrators across the country to be aligned in what a student should be able to do at the end of each grade level. According to their official website, true understanding is when a student can explain and justify why a statement is true as well as where a mathematical principle or rule comes from. It is important to note that these standards do not stress a particular way of thinking or way of teaching these concepts. Students may begin to understand concepts in new and different ways while still meeting these standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010).

Although the goal is complete alignment throughout the country, not all states have officially adopted the Common Core State Standards. One of those states is Nebraska. However, Nebraska's standards do align closely with the Common Core. Educators in Nebraska are also pushing to build conceptual knowledge and number sense (State Board of Education, 2015). The school building in which this project will be completed is a Catholic elementary school in Nebraska that follows the Archdiocese of Omaha's standards. Each set of standards begin with a brief outline of the key concepts students should master as they work towards either college or a career. These are essentially outlining the characteristics of a student who is proficient in mathematics. Nebraska calls them the "Mathematical Processes" while the Common Core labels them as "Standards for Mathematical Practice", and the Archdiocese of Omaha labels them as "Math Program and Essential Standards". Nebraska's "Mathematical Processes" are simple and to the point. The Archdiocesan "Essential Standards" are even more so.

The Common Core's "Standards for Mathematical Practice" are both more drawn out and more comprehensive. For example, the Common Core states that students should be able to make sense of problems and persevere in solving them while also analyzing constraints, relationships, and goals. Students should be able to monitor their own progress and check their answers with different methods (National Governors

Association Center for Best Practices and Council of Chief State School Officers, 2010). The Nebraska Standards ask for students to be able to solve mathematical problems by drawing upon prior knowledge, employing critical thinking, reasoning, creativity, innovative ability, and computing accurately (State Board of Education, 2015). The Archdiocese is even more simplistic stating only that problem solving is an essential skill and students should explore ways to solve problems (Archdiocese of Omaha, 2013).

The table in Appendix A shows each Common Core Practice aligned with that of its counterparts. The blank areas in the table represent the areas not covered in the other's processes (or principles). Nebraska's processes are not only fewer in number, but also objectively simpler and easier to understand. They are clear and concise. However, this leaves room for misunderstandings or a more generalized approach to meeting these standards. The Common Core Standards are much more specific and cover more topics. Teachers and administrators looking at the Common Core to format instruction need to delve much deeper into the principles before being able to truly understand and teach each of them.

The following is a table comparing a few of the Common Core kindergarten standards with the Nebraska kindergarten mathematics standards and the Archdiocesan standards. It is interesting to note that once again, the Common Core standards are more specific than Nebraska's standards; however, neither of them ask for specific strategies or methods. The Archdiocesan standards however, do state specific problem solving techniques that students must master (Archdiocese of Omaha, 2013).

Figure 1

Comparison of Standards

Common Core State Standards Kindergarten Mathematics	Nebraska State Standards Kindergarten Mathematics	Archdiocese of Omaha Standards Kindergarten Mathematics
CCSS.MATH.CONTENT.K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings1, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations	MA 0.2.3.a Solve real world problems that involve addition and subtraction within 10 (e.g. by using objects, drawings, or equations to represent the problem)	2.27 Recognizes and applies math ideas in everyday experiences
CCSS.MATH.CONTENT.K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	MA 0.1.1g compose and decompose numbers from 11 to 19 into tens ones and some more ones by a drawing, model, or equation (e.g., 14=10+4) to record each composition and decomposition	2.1.2 Demonstrates the concept of subtraction from 20. Uses subtraction strategies - Number lines
CCSS.MATH.CONTENT.K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	MA 0.2.1.b For any number from 1 to 9, find the number that makes 10 when added to the given number, showing the answer with a model, drawing, or equation.	

The Common Core Standards are comprehensive and based on what is known today about student development (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Nebraska's standards are less specific about student thinking but very similar to the Common Core. The Archdiocese standards are simple and straightforward but also include specific strategies that students must master. All three, however, are operating with the main goal of building students' conceptual understanding and number sense. Neither forces teachers into a box of teacher centered instruction based on procedural knowledge. Yet, current elementary school mathematics classrooms overwhelmingly base themselves in this approach (Star, 2016).

Traditional Mathematics Instruction and Curriculum

As test scores fall, researchers continue to search for ways to correct the system. However, in order to correct something, one must first discover what went wrong in the first place. Traditional mathematics instruction is typically teacher centered. The teacher gives the problem, the students solve the problem on their own, and the teacher tells them if the answer is correct. Many contemporary US elementary classrooms continue to use practices such as timed tests, homework, and standard algorithms regularly. Students are being taught procedural knowledge that is not easily applied outside the classroom. This section of the paper will analyze traditional approaches to teaching mathematics as well as the apparent disconnect between procedural knowledge learned in the classroom and real world mathematics.

In 2019, 12th-graders from across the country took part in the National Assessment of Educational Progress for mathematics. The scores from this national assessment showed no significant change in average mathematics scores when compared to scores from 2015. Although no change may be better than negative change, the President of the National Council of Teachers of Mathematics (NCTM), Trena Wilkerson, states, “No one should be pleased that the NAEP math scores have merely held steady,” (National Council of Teachers of Mathematics, 2020). Wilkerson goes on to explain that students today are headed into a world that requires them to understand, use, and apply mathematics more than ever. Staying even with scores from a time where that was not necessarily the case is not success. The goal should always be that the NAEP scores continue to rise. The NCTM is now taking a closer look at teaching practices in classrooms of all ages, not only working towards a change in scores, but a change in understanding and application of mathematics for future generations (National Council of Teachers in Mathematics, 2020).

One does not have to look too closely at the American school system to get an idea of what a typical math classroom looks like. Most American students and their parents would describe their experiences in mathematics classrooms in similar ways. Whether in elementary school, middle, or high school the American education system is set up to teach procedural knowledge (Star, 2016). Students listen as the teacher explains a problem step by step, the students solve a problem individually, the teacher tells them if they are correct. This easily results in rote memorization of algorithms rather than strong number sense or problem solving skill.

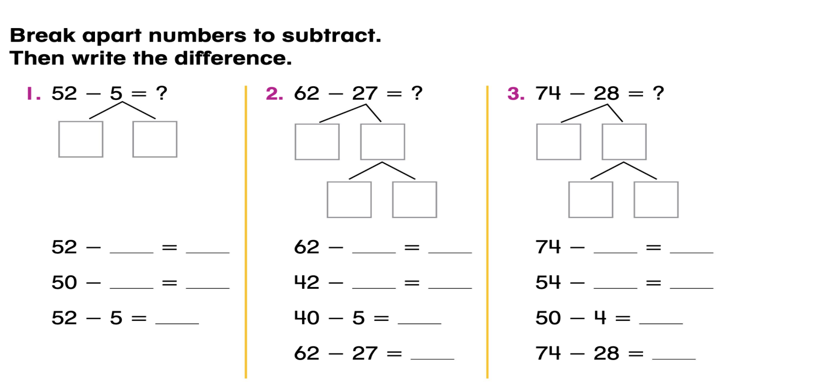
This way of teaching is often due to teachers using published curriculum materials to guide instruction, curriculum that is based in procedure and rote practices. Students become proficient in this singular way of solving problems and do well on curriculum based assessments. However, once out of the classroom, students struggle to solve problems due to a lack of number sense (Boeler, 1998).

The final project based on this research will be completed within a school using the Sadlier Curriculum for Mathematics as their core mathematics curriculum. According to the Sadlier website, it is a comprehensive core math program that fosters students' conceptual understanding. The curriculum seeks to develop skill through a combination of explicit instruction, guided practice, and independent practice. At first glance, the curriculum provides teachers and students with a detailed outline of knowledge and skills to teach main concepts (Sadlier, 2021). However, the curriculum is so structured that it allows for very little flexibility in teaching methods and student strategy.

After teaching with this curriculum for an entire year, it is easy to see how teachers begin to feel boxed into one way of teaching. In order for students to pass a chapter test, students must be able to solve the problem using a very specific method. A teacher may want students to solve in whatever way is intuitive to them, but if required to use the Sadlier assessments, the students must also know and understand how to complete the problem using one specific method. Below is an example of a specific way that students are required to solve two digit subtraction problems when working with the Sadlier Curriculum. It comes from Chapter Five of the Second Grade student workbook.

Figure 2

Sadlier Chapter Five- Student Workbook



Although breaking apart to subtract is a common, intuitive strategy for students at this age, they may not break it down using this method and therefore can easily get lost in the question "*What number goes where?*". This leaves teachers with a major dilemma; to teach to the test, or to truly build up their number sense and build upon those intuitive strategies.

Even as national and state standards begin to focus more on broad ideas and concepts to be mastered, procedural mathematics still dominates in the United States because of the curriculums that are required by districts and schools. Teaching standard algorithms and administering timed tests are still commonplace in contemporary elementary classrooms throughout the country (McCloskey, 2014).

These teaching practices persist even as research recommends varied and differentiated forms of assessment and instruction (McCloskey, 2014). In his article, Improve Math Teaching with Incremental Improvements, Jon R. Star suggests that teachers are so concerned with their students passing the test that they are hesitant to change the way they are teaching. So they continue with what they know, teacher centric, procedural knowledge. Star states:

> Choosing this strategy reflects the reality that many teachers see themselves in: Faced with conventional curriculum materials and pacing guides that require them to move quickly through materials, they may see themselves as unable to overtake a complete overhaul of mathematics instruction. (p. 59)

Although the Common Core and State standards are working towards the goal of better conceptual understanding, better problem solving, and a higher level of number sense, there still is a disconnect between them and the way required curriculums are built and taught. Teachers are left to choose between teaching to the test, teaching to the standards, or building on intuitive strategies and understanding. Convincing teachers that they can both pass the test and succeed in bringing about true understanding and strong number sense is a huge undertaking. It requires a new way of thinking and a better understanding of what is being asked of teachers (Star, 2016).

Cognitively Guided Instruction

In the introduction to the 2010 Common Core State Standards in Mathematics, the committee states, “Mathematics education in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country” (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). They go on to explain that the focus of these standards is to improve conceptual understanding of key ideas and concepts. They are written with an understanding of how students' mathematical thinking develops overtime (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). This lends itself well to the pedagogical approach, Cognitively Guided Instruction (CGI). Using this approach, a teacher bases instruction on student thinking and strategy rather than on standard algorithms and memorization (Carpenter, et al., 2015). This section will discuss the key components of Cognitively Guided Instruction as well as how it further develops student understanding.

Thomas P. Carpenter, one of the founders of CGI, explains that there are two main principles of this approach. The first principle is that instruction should be built on existing student knowledge (Carpenter et al., 1989). Research suggests that teachers do not always base their teaching off of the cognitive understanding of their students. Rather, they focus on the curriculum and figure that it must be in line with what most students are doing developmentally (Carpenter et. al., 1989). For example, a second grade teacher may teach the standard algorithm for double digit subtraction at the end of the second quarter because they have followed the curriculum completely and now their students should be ready. However, that is not a natural method for most children and teaching it can easily stifle the intuitive understandings students already have.

In their article, Learning to See Students' Mathematical Strengths, Abbe Skinner, Nicole Louie and Evra M. Baldinger write about the importance of seeing students as mathematically smart. Their article suggests that teachers work to provide students with opportunities that allow for diverse ways of problem solving. This will allow teachers and other students to see the multitude of ways to solve any one problem. They also suggest grouping kids without assumptions based on what they are good at or what they know (Baldinger E. M., et. al, 2019). This furthers the first principle of CGI by allowing students to find and demonstrate their own strategies. CGI allows students to explore their own intuitive understandings while solving problems and gaining understanding of mathematical principles.

The second main principle of Cognitively Guided Instruction is the idea that instruction should develop understanding of mathematical concepts, skills, and principles through the use of problem solving. This will further their understanding and number sense because they are in charge of finding the solution in their own way (Carpenter et. al., 1989). From a young age, students can begin solving word problems. Jessica F. Shumway and Lauren Pace research preschool students' mathematical thinking and integrate Cognitively Guided Instruction into their preschool classroom. The teachers use a play based curriculum and create mathematical tasks based on the learning themes already present in the classroom. It is important to note that rather than teaching students how to solve each problem type, the teachers provide students with a large variety of materials so that they can solve in a way that makes sense to them. For example, they may provide students with apples and buckets, cubes, crayons, or colorful marbles (Shumay and Pace, 2017). Teachers also must explain the problem in a way that is relatable to the young student's life.

Rather than asking young students to read word problems, the teacher can read the problem aloud to them. The problem itself should be relevant to their lives. Instead of asking kindergarten students to find the total number of miles driven to school, ask them to find the total number of cupcakes they baked with their mother or father. Then, provide them with manipulatives and tools to model the problem. They will use these tools to directly model, essentially acting out, the steps of the problem. In a 1993 study of three kindergarten classrooms, Carpenter and his colleagues found that, when provided with manipulatives, kindergarten students can use direct modeling to solve word problems and problem types that are often not addressed until second grade. Even without formal instruction on how to solve word problems, kindergarten students can solve these problems using methods and strategies intuitive to them (Carpenter et. al., 1993).

Older children often struggle to solve real world problems because they focus too much on the memorized algorithm they were taught in the classroom. Rather than read and understand the problem, a fifth grade student may pull out the numbers and one or two key words. In his 1993 article, Carpenter writes about the fundamental problem solving skills children begin to abandon. He gives the example of a common assessment question for third grade students, “Students were asked to find the number of buses needed to transport 1128 soldiers if 36 soldiers could ride on each bus” (1993, p.49). He goes on to explain that the majority of third graders recognized that they needed to use division to solve the problem; however, only one third of them realized they needed to round to the next whole number. After all, you cannot have only part of a bus (Carpenter et. al., 1993). This is where CGI comes in. Cognitively Guided Instruction gives students the tools and the support to think through, model, and make the problem relevant before rushing to solve.

Another key concept of CGI is a teacher’s ability to understand students' thinking in a helpful and relevant way. Studies have shown a positive correlation between students' problem solving ability and teachers' understanding of it. Most teachers already have a baseline understanding of where their students are at and how they are thinking. The key component that is missing, the component that teachers also find overwhelming, is the organization of it all. It would be very difficult to base teaching off of student thinking without first organizing it (Carpenter et. al., 1996).

Most teachers around the United States today are being asked to use standardized assessments and their data to inform their teaching practices. Megan L. Franke and her colleagues explain that this can easily lead to questions for the teacher and large gaps in understanding for the student (Franke et. al., 2016). In a study published in the *International Journal for Mathematics Education*, authors Jacobs, Lamb, and Phillip discuss the professional observations and noticings of teachers within the mathematics classroom. They explain that teachers cannot preplan in the moment responses to student understanding and therefore they must be prepared to constantly analyze and connect the strategies used to what they understand about student's mathematical development (Jacobs et.al., 2010). A true Cognitively Guided Instruction classroom is centered around a true and deep understanding of how students learn and think. In order to truly understand students' thinking, teachers must have knowledge regarding the many problem types and the differing strategies that students may use to solve them (Franke & Kazemi, 2001).

Students typically begin their journey as mathematicians in the direct modeling phase of development. Take this problem for example: *Sara has 4 cupcakes. Her friend Liz has 11. How many more cupcakes does Liz have then Sara?* Most adults would solve this problem by subtracting four from eleven. A student using direct modeling would create a set of four manipulatives, and a set of eleven. Then the student would count the number of 'extra' cubes in the set of eleven. The student is directly modeling the situation as if they had real cupcakes in front of them. Next, a student moves into the counting strategies phase. This is a natural jump from direct modeling. Counting strategies are more efficient and more abstract than direct modeling. Take the example problem from before, a student using counting strategies would most likely start at four and count up to eleven. As the student counts, they would extend a finger and their final answer would be the number of fingers raised (Carpenter et.al., 2015).

From here, students move forward into more abstract strategies, typically around second or third grade. Students begin solving without the use of manipulatives. They typically have a solid understanding of the base ten concept and much of their thinking builds off of this. However, this is a time where rote practices and memorization of strategies begins to be introduced. It is a time in school where students' intuitive number sense can either begin to thrive, or decline. Cognitively Guided Instruction allows for it to thrive. A teacher that knows and understands this sequence of strategies, will be better able to understand why a problem is difficult for a student to solve, or what errors they made that led them to an incorrect solution. With this knowledge, teachers are able to bring out knew understandings and push students towards higher level thinking (Franke & Kazemi, 2001).

Understanding these progressions allows teachers to follow student thinking directly through development. If a student is working in the direct modeling phase, using cubes and counting each one to solve the problem, a teacher with good understanding of student thinking will be able to prompt the student to move forward with new, more efficient counting strategies. It allows teachers to guide students through these intuitive strategies and support their own understandings, rather than pushing new thinking or strategy. Cognitively Guided Instruction is a method of teaching that supports students and their own intuitive thinking. It paves the way for an intuitive understanding of mathematics. Once teachers begin to understand their students' problem solving strategies and methods, students' problem solving skills will begin to grow (Carpenter, Thomas P, et al. 1989).

Cognitively Guided Instruction in the Classroom

A classroom based in CGI is one filled with collaboration, with hands-on activities, and real world problem solving applications. Cognitively Guided Instruction is a method of teaching that supports students and their own intuitive thinking. This section will explore in great detail what a CGI classroom looks like and the ways in which it can be implemented. It will also take another look at the Sadlier curriculum and compare it to CGI strategies.

Inside a truly CGI based classroom, the teacher is deeply in touch with their student's thinking and strategies. Many teachers argue that the idea of knowing how every student is thinking and solving problems is completely impractical. It is overwhelming to think about. How can one person watch every student solve problems, decipher each learner's work, and listen to each student talk through their own strategy all at one time? In actuality, most teachers start with a baseline understanding of where their students are at anyway (Carpenter et. al., 1996).

Most teachers would be able to explain which students were still counting on their fingers and which ones were able to rattle off math facts without pause. The challenging and overwhelming part is taking what a teacher knows about their students, and turning it into instruction and then basing a lesson almost solely off of what the students know, or are ready to learn. If teachers were able to organize this student data, they would be better able to extend student thinking and engage them in learning (Ball, 1993) Teachers however, are unsure of how to organize it all and instead follow the curriculum figuring that it must be in line with what students should be able to do developmentally. Unfortunately, this is not always the case. As exemplified earlier in Figure 2, assessments written for specific curriculums are not often written with students' intuitive understandings in mind. Although students may understand a certain strategy, it may not be intuitive and would most likely move them away from their original problem solving method.

Here is another example of a Sadlier Curriculum assessment question from a third grade workbook.

Figure 3

Sadlier Student Assessment Question

Retrieved From Sadlier Online workbook

This is shown in a professional development video and labeled as a real world application for third grade students. Although some third graders may have heard of a decapod, it is safe to assume that most do not encounter them on a daily basis. It would be difficult to conceptualize this problem at all but especially if just introducing multiplication. Rather than asking this question, a CGI classroom would ask a question including students' names and have the students completing or envisioning something they could model. Students can use manipulatives, drawings, or any strategy they feel comfortable with to solve. Students need to be able to clearly visualize the scenario to successfully solve it, making it relevant to their own lives and allowing them to do so (Sherman and Gabriel, 2016). In a 2016 article on using word problems in elementary mathematics, Khristine Sherman and Rachael Gabriel state, "Beginning mathematicians need to be able to imagine the relationships and actions being described in the problem to determine which count as ‘key’ to the operation required (2016, p. 473)”.

For example, the teacher may ask them to solve this problem “*Evelyn brought mini erasers to school for every student in the class. There are 12 students in the class and she brought exactly enough for every student to get 3. How many erasers did she bring to class*?” Students hear, or read, this problem and can almost immediately imagine what the story would look like. They may get out cubes and physically place twelve piles of three, they may draw a picture, or they may count by three twelve times. In any case, they will have solved the problem without having to fit their method into a box in order to score points on a test. Once students have solved the problem, the teacher will open the room up to discussion of both strategies and answers.

A CGI classroom is filled with communication and conversation between students. Erin Wagganer writes in her 2015 article, “‘Creating Math Talk Communities”’, that teachers are asked to provide meaningful explanations of problem solving strategies after a child in their class uses one. However, the student that uses the strategy should be able to explain it as well. With this method, students become leaders in discussion and take on the responsibility of justifying their own thinking (Wagganer, 2015). By explaining and talking through their own thinking, students will also begin to re-examine their own thought process. They may find a more effective way to solve the problem, or they may recognize an error as they solve. In a 2015 study of the role of the teacher in student conversation and engagement, Megan Franke, Angela C. Turruo, and their fellow researchers found that teachers can use a number of different support strategies to engage their students. These strategies included asking students to explain a classmate’s solution, compare their solution to another student's, and to use a solution shared by another student. Their study found that a combination of these teaching strategies, as well as continuous encouragement and probing helped students to increase their interactions with one another and become contributing members of discussion (Franke et al., 2015).

Cognitively Guided Instruction can be used in many different ways throughout the classroom. A teacher may use problem solving and word problems everyday as a math warm-up. Students may discuss strategies with each other and explain their thinking to the class. Hands-on projects and activities are used often so that students can make mathematics relevant to their own lives. However it is used in the classroom, the key principles stay the same. CGI is based in student's own intuitive strategies, and number sense is developed through problem solving.

Conclusion

Chapter Two went into great detail regarding a number of topics, each of which will help me to better understand my final project and question: *How can teachers use Cognitive Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?* The project will be a set of lesson plans that combine the use of the Sadlier Math curriculum required by the project site, and Cognitively Guided Instruction.

The first topic discussed, current state and local standards, allows me to better understand exactly what my students will need to know at the end of the year. By analyzing the standards at each level, I am able to develop a clear understanding of the overarching principles that my students should be working towards. The Archdiocese of Omaha sets a clear standard for students understanding how to use a number line by the end of kindergarten. Although this may not align directly with CGI, I know that I must include it in my lessons and help students to work towards mastery of this particular strategy.

The second topic of discussion is traditional mathematics instruction used throughout the United States as well as the Sadlier curriculum which lends itself to that method of teaching. By further analyzing current practices, I am better able to understand why teachers may be apprehensive about moving to a nontraditional approach. Curriculums, like Sadlier Math, teach specific strategies and allow for little differentiation from those strategies. By looking closely at the curriculum, I am better able to develop lessons that will help students succeed on assessments while also allowing them to discover and build upon their own intuitive thinking.

Finally, the chapter looked closely at Cognitively Guided Instruction. CGI helps teachers to bring out the intuitive strategies their students want to use naturally. By using Cognitively Guided Instruction, a teacher develops students' problem solving skills through relevant real world word problems. CGI better prepares students by developing a strong number sense and understanding. This research helps me to prepare lessons with the true CGI principles in mind.

Chapter Three will discuss the project component related to this research. The chapter will answer many questions. It will discuss the setting of the project, the timeline on which the project will take place, as well as the intended audience. It will also explore the ways in which data may be collected and documented. The capstone project will be supported and written using research stated throughout Chapter Two.

Mathematics is complicated. It is both complicated to learn about, and complicated to teach. There has been constant, drawn out debate regarding the best, most effective way to teach mathematics. All the while, America's test scores began to lower. Today, America is working towards a better understanding of mathematics for its students and future generations. The implementation of the Common Core State Standards paved the way for Nebraska and other states to redesign their own. Now the large, overarching goal is to improve students' conceptual knowledge and number sense. This goes well with Cognitively Guided Instruction, where the goal is to bring out student understanding through their own intuitions. The trouble comes when curriculums require teachers to teach to the test in order for students to pass. Or when they require concrete, specific strategies to be taught and mastered by students. Ultimately, this project has the same goal as that of the Common Core State Standards, to improve conceptual knowledge and number sense. This can be accomplished through a combination of the current curriculums required by schools and the pedological approach Cognitively Guided Instruction.

Chapter Three

Project Description

Introduction

This chapter will outline the framework used, as well as the strategies implemented in order to produce a set of primary level mathematics lesson plans that help to answer the question: *How can teachers use Cognitive Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?*

Chapter Three first discusses the method by which the question will be answered. Each lesson is intentionally designed to teach strategies and techniques specific to the Sadlier Curriculum as well as the State Standards through the use of Cognitively Guided Instruction. Next, Chapter Three delves into the theories and practices behind the final product. This research includes methods and theories from the founders of Cognitively Guided Instruction, studies from Martha Ing connecting student conversation with higher levels of understanding, and Sherman and Gabriel's writing on the importance of hands-on learning. Finally, Chapter Three walks through the parameters of the project site. This includes a description of both the participants and the audience.

Project Design

The capstone project is a culmination of research relating to Cognitively Guided Instruction, State Standards, and the Sadlier Curriculum required by the project site. This research is used intentionally in order to create a set of seven lesson plans for Chapter Ten and a chapter outline for Chapter Three. These lessons build off the required Sadlier Curriculum; however, they are based in Cognitively Guided Instruction and it's pedagogy. The primary goal of the lessons is to improve students' overall number sense and problem solving skills while also allowing them to solve problems using their own intuitive strategies.

The lessons themselves contain information regarding the materials needed, the assessments used, and possible differentiation strategies that may be implemented by the teacher. They are designed intentionally to teach both the State Standards as well as the techniques and strategies specific to the Sadlier Mathematics Curriculum. For example, in Chapter Ten of the Sadlier Mathematics curriculum for kindergarten, students are working on adding one to any number below ten (Sadlier 2021). They are also learning new mathematical symbols and vocabulary. Obviously, addition and the vocabulary that goes along with it is an essential part of math fluency. However, the Sadlier curriculum teaches students in a very specific format. The lesson I have created includes the Sadlier worksheet as part of the lesson so that students meet the necessary understandings to move forward in the curriculum. However, ultimately, the lesson need to meet Nebraska State Standard, MA 0.1.2 “Operations: Students will demonstrate the meaning of addition and subtraction with whole numbers and compute accurately” (State Board of Education 2016). The standard is met by using hands-on, problem solving techniques as described in key theories and methodologies. For example, students are asked to be a part of the story problem told to exemplify adding one. Students bring items to the front of the room, count them together, and write addition sentences based off their actions. This helps students to begin making connections between mathematical concepts and their everyday lives.

The final capstone project is a set of lesson plans created and designed to include CGI pedagogy while also teaching the strategies required by the curriculum and the skills required by the standards. Students need to complete the daily worksheet from the Sadlier curriculum, however, by basing the daily lessons in problem solving and word problems, and providing students with hands-on materials and activities, students are better able to find their own intuitive strategies.

Theories and Practices

These methods are primarily based off of the pedological approach Cognitively Guided Instruction but also include practices from many other compatible sources. One of the two main principles of CGI is the use of problem solving to build understanding of mathematical concepts. Students are asked to solve word problems that are relevant to their own lives. They are given hands-on activities and materials to solve the problems. Conversation between students plays an important role in each lesson. Another key component of CGI is building instruction off of student knowledge. In order to build off a student's knowledge, the teacher must have a firm understanding of it. This understanding is best built through listening to conversation between students and about their own intuitive thinking (Carpenter et. al., 1989). Marsha Ing and her colleagues wrote an article for the journal, Educational Studies in Mathematics, in which they connected students' participation, teaching practices, and student achievement. Once students begin comparing their own thinking to that of their classmates, both the teacher and student are able to see errors in thinking, gaps in understanding, or a new perspective (Ing 2015). Throughout the lessons, small group conversations are utilized. Students are asked to explain how they solved the problem, what strategy they used, if they could have solved differently, and if they can justify the answer they came to. These conversations are essential to understanding student thinking.

The lessons created for this final capstone are based on hands-on learning through the use of manipulatives. In their 2016 article, Sherman and Gabriel emphasize the importance of hands-on, real world mathematical experiences. Young students need the opportunity to visualize and walk through the problems they are solving because of the complexity of mathematics (Gabriel and Sherman 2016). Each lesson is built off of a base of problem solving. After being read a problem, students are first given the opportunity to talk through the events in the problem. For example, if students are given this problem: *Jeremy went and had a snack. He had 13 pretzels on his plate. He ate 6 of them. How many pretzels does he have left?* The students and teacher would talk through the problem. The teacher may ask students if they liked pretzels and if they could imagine them in front of them. Then ask students to explain what happened in the problem to you. A student may say, *Jeremy had 13 pretzels but he ate 6 of them.* Next, the students will solve using hands-on manipulatives, by drawing, and some more advanced may work through it in their head. Walking through the problem, and then visualizing the problem allows the students to grasp the concept in a deeper way.

Although these methods primarily come from research relating to the Cognitively Guided Instruction approach, many of the practices built in throughout the lessons come from other researchers and their explanations of how students learn. These lessons were designed while looking closely at research regarding student conversation in the mathematics classroom, the use of manipulatives and visualization, and the use of word problems with young children.

Project Site and Demographics

This project was designed prior to the beginning of the school year and implemented at the start of the year. Ultimately, the lessons will be implemented in a Pre-School through Eighth, Catholic Grade School in Nebraska. This is important to consider throughout the curriculum development process for multiple reasons. First, the curriculum must meet the Nebraska State Standards and the standards laid out by the Archdiocese of Omaha. Secondly, the Sadlier Curriculum is a required curriculum throughout the school. It is important that students learn and work through the strategies set out by Sadlier so that they continue to be successful in coming years.These lessons are closely aligned with Sadlier Mathematics; however, if teachers and administration used them at school sites different from my own, they could be adjusted easily to fit the requirements of their district.

Finally, the size and population of the school was one of the most important factors to consider when designing these plans. This Catholic school is one of the largest in the city with over 800 students enrolled. I will be moving from second grade to kindergarten at the same school. Therefore, the intended setting of this project is my own Kindergarten classroom. Each grade level, kindergarten and up, has three sections that are capped off at thirty students per section. Each of the three kindergarten classrooms have full time teacher assistants, allowing for much more small group instruction and differentiation. This is an important detail to consider. The teacher assistant within the project classroom will be utilized often throughout the lessons. The class will have 25 to 30 kindergarten students for the 2021-2022 school year. The administration and teachers within the school are considered the audience for this project. Before beginning the project I received permission from my administration as well as approval from the kindergarten team.

Ultimately, the success of this project will be measured by student achievement in a couple of main areas. First, students' understanding of the Sadlier curriculum. Test scores will be recorded and compared to test scores in both control groups. Secondly, through their ability to solve word problems and explain their thinking.

Conclusion

Throughout this chapter, the implementation of the lesson plans, as well as the methods and theories behind them has been introduced and discussed. The main principles of Cognitively Guided Instruction are intertwined with the Sadlier Curriculum and the State Standards to create a coherent set of lesson plans. Chapter Four will discuss the project's final findings and summarize it as a whole. Ultimately, the following chapter is the culmination work and research related to the project question: *How can teachers use Cognitive Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?*

Chapter Four

Conclusion

Chapter Overview

Chapter Four will delve deeper into the project as a whole.The project itself is a set of seven lessons that intertwine Cognitively Guided Instruction and the Sadlier Math curriculum. Ultimately, it seeks to answer the question: *How can teachers use Cognitive Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?*

This chapter will first discuss the major findings and learnings that resulted from this project. Next, it will discuss the literature review and the role it played in the development of the lessons and chapter outline. Two main sources guided many of the activities and much of the process. Then, the chapter will discuss three main limitations of the project. Finally, the benefit to educators and implications of the project as a whole will be laid out.

Major Learnings

Just one year out of undergraduate school, I was already ready to be back in a classroom. Not as a teacher but as a student instead. The capstone process has been invaluable to me as both a lifelong learner and an educator. Throughout my undergraduate courses I completed many research projects, created many lesson plans, and wrote too many essays to count. However, focusing on one subject that I am passionate about was much more fulfilling than I could have imagined. I delved deeper into APA guidelines and research techniques while learning quite a bit about myself as a writer.

As an educator I found myself falling down rabbit holes filled with teaching strategies and educational theories. The research has inspired me to be a more creative teacher. It has reminded me of the importance of hands-on learning that is relative to the lives of students. With each lesson, I hope to give students problems and activities that relate to their own lives as kindergarteners. The literature review explored ways in which teachers can incorporate CGI principles into the classroom and the strategies they may use to do so.

Literature Review

The Literature Review portion of this project was essential to the development of each lesson plan. Through research, not only did I learn about the main principles of CGI as laid out by Thomas Carpenter, but also many other theories and strategies that built off these principles. I frequently looked at Khristine Sherman and Rachael Gabriel's 2016 article, *Math Word Problems: Reading Math Situations From the Start,* as I developed problem solving portions of the lesson. As I wrote the unit plan, I often came back to their findings regarding the students' need to be able to clearly visualize the scenario so that they can make it relevant to their own lives, and then successfully solve it (Sherman and Gabriel, 2016).

CGI is based on a deep understanding of student thinking. Teachers then use this knowledge to push students towards higher levels of thinking (Franke & Kazemi, 2001). While writing the unit plan I found myself working to give students ample time to explore their own strategies while guiding them toward deeper thinking with prompting and questioning. Franke and Kazemi's article, *Learning to Teach Mathematics: Focus on*

Student Thinking, was one I frequently went back to for guidance regarding these prompts.

Throughout the process, I often looked back at the standards. Although for the project site, it was most important that I follow the Archdiocese of Omaha standards, I also felt it important to align with the Nebraska State Standards and the Common Core. By doing so, others outside of the district can use these lessons and see that they align with certain key standards.

Ultimately, the Literature Review played an important role in the creation of the final project. The set of lesson plans combines the use of the Sadlier Math curriculum required by the project site, and Cognitively Guided Instruction with the overarching goal of improving number sense and problem solving skill.

Limitations

Although overall I believe I have created seven engaging and thought provoking lesson plans, it is important to note a few significant limitations one may encounter. The first of these limitations is the reliance on a teaching assistant throughout the lesson.The project site has teacher assistance present in every classroom kindergarten through third grade and therefore allows for more one on one or small group instructional time. Although with smaller class sizes these types of lessons are certainly possible, it would become difficult to work directly with students while managing the rest of the classroom. For example, MATH rotations would need to be adjusted and the three groups not meeting with the teacher would likely need a more self explanatory activity. Teachers without a teaching assistant may want to give students a more structured activity during small group time and adjust MATH rotations accordingly.

It is important to note that this project was created specifically for a kindergarten classroom using the Sadlier Mathematics Curriculum. Therefore, only teachers in this same situation will be able to use the lessons directly. Others would need to adjust the activities and the curriculum worksheets used. MATH rotations could still be used frequently, but the activities may need to be adjusted for higher grade levels or different standards. The layout of lessons themselves, the problem solving portions could likely stay with the understanding that problem types may need to be adjusted.

Another limitation that may be noticed throughout the sequence is the length of each lesson. I am fortunate to work in a school that allows a lot of autonomy for teachers. Teachers and grade level teams have the ability to create and adjust their schedules as they see fit. Typically, math lessons take between one hour and an hour and a half. However, another teacher may only have an hour to teach the lesson for the day. Rather than doing MATH rotations, the teacher may teach a short whole group mini lesson, and then meet with small groups while others work on the worksheet assessment. Although each lesson contains quite a few components, they are designed so that they may be taken apart, or simplified for different time constraints. Meanings, the lessons can be adapted to the needs of many different environments.

Implications for Educators

At first glance, Cognitively Guided Instruction can be overwhelming. Publishers and authors ask educators to read pages and pages of research, with many examples but no true guide on how it can be implemented into a traditional classroom environment. This project provides a practical lesson planning framework for teachers who are looking to change the way they teach mathematics.

The lesson plans provided, as well as the short unit outline, give teachers access to many different strategies, activities, and prompts that will engage students while improving their overall number sense. Each full lesson plan includes what the teacher will say, possible student responses, problem types, and the goals behind each instructional period. This gives teachers and administrators a full view of how and why that can incorporate Cognitively Guided Instruction. The short unit outline also included in the project is a simple scope and sequence that provides possible activities, prompts, and problem types for another unit.

This project will lead me into my next quite seamlessly. After implementing these lesson plans within my own classroom, I plan to create a short professional development and present it to the faculty at my current school. This will require a short, 2 hour presentation on not only my project, but Cognitively Guided Instruction. The goal of the presentation itself is to introduce Cognitively Guided Instruction in a practical, straightforward way so that teachers can begin using it right away. With this small professional development session I hope to begin creating a simple guide to bringing CGI, small groups, and problem solving into the classroom alongside required curriculums.

Conclusion

This chapter has summarized the process through which I answered the question, *How can teachers use Cognitive Guided Instruction in conjunction with current required curriculum to better understand student thinking and therefore improve overall number sense and problem solving skill?* The chapter first addressed the major learnings including, the importance of incorporating hands-on, engaging activities, even while following a curriculum. Next, the chapter discussed the key components of the literature review. These played a key role in the development of each lesson. Next, three limitations of the project were addressed. Although time, the projects focus on the use of the Sadlier Math curriculum, and the use of a teaching assistant all make it more difficult to transfer into other schools, with careful consideration and thoughtful implementation this transfer is possible. Finally, the chapter walks through the implications for educators. The main focus being that it provides teachers with a simple, and practical lesson planning framework with the goal of improving number sense and problem solving skills.

As an early elementary school teacher, it is important to instill in students a firm foundation on which to build their knowledge. Elementary school mathematics instruction often moves quickly from one to one correspondence to algorithms and rote exercises, leaving students with procedural knowledge but lacking in number sense and problem solving skills. Teachers follow the required curriculum to ensure success on assessments but skip over important instructional time that leads to number sense and a deeper understanding. Bringing Cognitively Guided Instruction into the classroom will improve students' problem solving skills while also strengthening their number sense.

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Appendix A

A Comparison of Mathematics Standards

Standards for Mathematical Practice Common Core (Common Core, 2010)	Mathematical Processes Nebraska (Nebraska, 2015)	Math Program and Essential Standards Archdiocese of Omaha (Archdiocese, 2013)
Make sense of problems and persevere in solving them: - Students analyze constraints, relationships, and goals. - Students monitor their own progress and check their answers with different methods.	Solves mathematical problems: - Students should be able to draw upon prior knowledge, employ critical thinking, reasoning, creativity, and innovative ability. - Students should be able to compute accurately.	Problem Solving: - Explore ways to solve problems
Reason abstractly and quantitatively: - Students should be able to both conceptualize the data or a math problem and deconceptualize it. Meaning they should be able to put the problem into a real world context to better understand, and also take it out in order to solve abstractly.		
Construct viable arguments and critique the reasoning of others: - Justify conclusions and explain them to others - Listen or read the argument of others and decide whether they make sense and can be justified.	Communicates mathematical ideas effectively: - Students will be able to utilize appropriate communication approaches both individually and collectively.	Communication: - Recognize and use mathematical symbols and words.

	- Students will be able to communicate their thinking through both writing and speaking.	
Model with mathematics: - Students should be able to apply the mathematical knowledge they have to situations in everyday life.	Make mathematical connections: - Students will be able to connect mathematical knowledge, ideas, and skills beyond the classroom.	Connections: - Recognize mathematical ideas in everyday experiences.
Use appropriate tools strategically: - Students should be able to consider the available tools when solving a given problem. - Students should be able to recognize the given restraints or possible errors made with the given tool. - High school students are able to use various technological tools to deepen their understanding.	Models and represents mathematical problems: - Students should be able to analyze relationships in order to build a mathematical model given a real world scenario. - Students should be able to look at a mathematical model and analyze relationships based on it.	Representation: - Model mathematical concepts using materials.
Attend to precision: - Students should be able to communicate precisely in discussion. - Students should use clear definitions of words and symbols to explain their thinking.		Reasoning: - Explain and show work to justify answers
Look for and make use of structure: - Students should be able to recognize a pattern in numbers or in problems. - Students may recognize		

that 3+7 is the same as 7+3, or sort shapes by the number of sides they have.		
Look for and express regularity in repeated reasoning: - Students should be able to recognize if a calculation is being repeated and begin looking for shortcuts or a general method.		

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