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# How Discourse And Collaboration Can Be Used In Mathematics Classrooms To Promote Engagement And Learning

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HOW DISCOURSE AND COLLABORATION CAN BE USED IN MATHEMATICS  
CLASSROOMS TO PROMOTE ENGAGEMENT AND LEARNING

By

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A capstone submitted in partial fulfillment of the requirements for the degree of Masters of Arts  
in Education

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## Project Summary

For my research, I explored the question: *How can math teachers promote engagement and learning through collaboration and discourse in the secondary mathematics classroom?* The literature review process guided me to learn more about various discourse routines and qualities of group-worthy tasks to support engagement and learning in the mathematics classroom through mathematical conversations and collaboration. For my project, I took the literature and applied it to create unit guides. This project consists of 10 unit guides that correlate with the seventh-grade Minnesota mathematics standards. All of the unit guides are adapted from the Understanding by Design (Ubd) framework created by Wiggins and McTighe (2011).

When designing the template for this project, I had to think about what qualities of the Ubd framework I wanted to incorporate. This type of framework suggests to start the design of a unit or lesson with the end in mind. This is beneficial because it gives a unit a clear focus through the lesson planning process (Wiggins & McTighe, 2011). However, the unit guides I created are not a complete curriculum unit. Rather, the guides provide additional resources to mathematics teachers that encourage collaboration and discourse. Since it is not a complete unit, I chose to adapt the Ubd template to best serve the needs of the project. In the next paragraph I will explain the different stages and elements that are incorporated into each of the guides.

For each of my unit guides, I decided to include all three of the stages of Ubd: desired results, assessment evidence, and learning plan. In the desired results stage, you will find all of the Minnesota standards that correlate with that particular unit of study. The first stage of the guides also include the essential questions and skills. Both essential questions and skills are shared and communicated with the students. In the second stage, assessment evidence, there are

discourse routines and performance tasks listed and linked. Discourse routines are shorter activities in length that promote mathematical conversations for students. Performance tasks are group-worthy learning activities. Students are given group roles and work collaboratively with each other on the given tasks. Lastly, in the third stage (learning plan) the performance tasks and discourse routines are described and explained.

The intended audience for this project are seventh grade math teachers specifically, but all secondary math teachers also could benefit from this project. Seventh grade math teachers benefit the most because they can use some/all of the learning tasks listed in their own classroom. A seventh grade mathematics teacher could review the document, go to the unit of study that their students are exploring, and add a discourse routine or group-worthy tasks into their lessons. The activities are already created, and the teachers would only need to plan when to use the activity and execute the instruction. Teachers using the unit guides know that the task they find will support their students in collaboration and/or mathematical discourse. Other secondary math teachers can review the different group-worthy tasks and discourse routines and use them as a guide to create tasks that will support the same goals, but with different mathematical skills.

In conclusion, this project supports student engagement and learning through students collaborating together on various group-worthy tasks and student discourse through mathematical conversations. Seventh-grade math teachers can use this project as an additional resource in their classrooms. They can implement different discourse routines and group-worthy tasks with their students. Other math teachers, can review the guides and get ideas on how to incorporate similar learning activities in their own classroom.

# Unit 1 - Ratios & Proportions

## Stage 1 - Desired Results

### Established Goal(s):

- 7.1.2.5 Use proportional reasoning to solve problems involving ratios in various contexts.
- 7.2.2.2 Solve multi-step problems involving proportional relationships in numerous contexts.
- 7.2.2.3 Use knowledge of proportions to assess the reasonableness of solutions.
- 7.2.4.2 Solve equations resulting from proportional relationships in various contexts.
- 7.3.2.1 Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors.
- 7.3.2.2 Apply scale factor
- 7.3.2.3 Use proportions and ratios to solve problems involving scale drawings and conversions of measurement units.

### Essential Question(s):

- How can proportional reasoning help in everyday life?

### Skill:

*Students will be able to...*

- I can find ratios, rates, and unit rates
- I can determine if two ratios are proportional
- I can write and solve proportions
- I can determine if two figures are similar
- I can find the scale factor of similar figures
- I can convert measures from scale drawings
- I can use the scale factor to find missing lengths and areas of similar figures
- I can use proportional reasoning to solve problems involving scale factor

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Catching Goldfish](#)

### Discourse Routine(s):

[Would You Rather - Unit Rate Honey](#) (Would You Rather, n.d.)

### Stage 3 - Learning Plan (Learning Activities)

#### Learning Activities:

**Catching Goldfish (45 minutes):** Students work in pairs. The group roles are resource manager and recorder/reporter. Students will be looking at a population of goldfish crackers in a paper bag. They will “tag” 10 goldfish by collecting 10 goldfish and replacing them with 10 pretzel goldfish. They shake the bag, and begin sampling. They reach in and collect a sample of 5-10 crackers. They record how many fish were pretzels, the total, the ratio of pretzel to total fish as a fraction, decimal equivalent, and the percent equivalent. They do this 10 times. After they have recorded that information for 10 samplings, they add up the total pretzel goldfish and total fish columns. They write that information as a ratio of pretzel goldfish to total fish as a fraction, decimal, and percent. They then use that ratio to create a proportion and predict how many total fish are in the bag knowing that 10 are “tagged” pretzel fish. Students then open their bag and count how many fish are actually in their bag and they compare their prediction with the actual amount.

**Would You Rather - Unit Rate Honey (10 minutes):** Students are posed with the question of would you rather buy a 5 pound jug of honey for \$15.35 or three 1 pound 8 ounce bottles of honey for \$14.39. Students will use their mathematical knowledge to calculate how much product is in each option, and then calculate the unit rate for each option. They can calculate unit rate in cost per ounce or in cost per pound. Once students have had time to think about which option they would buy, students share their choices with the class and explain why.

**Would You Rather - Unit Rate Speed (10 minutes)** Students are posed with the question of would you rather drive a car at a rate of 40 miles per hour or drive a car at 50 feet per second. Students will use their mathematical knowledge to compare the two speeds. They could either figure out how fast 40 miles per hour is in feet per second, or find out how fast 50 feet per second is in miles per hour. Once students have had time to think about which option they would choose, students share their choices with the class and explain why.

#### Resources:

Would You Rather - Unit Rate Honey retrieved from

<http://www.wouldyourathermath.com/would-you-rather-48/>

Would You Rather Speed retrieved from <http://www.wouldyourathermath.com/racecar/>

# Unit 2 - Probability

## Stage 1 - Desired Results

### Established Goal(s):

**7.4.3.1** Use random numbers generated by a calculator or a spreadsheet or taken from a table to simulate situations involving randomness, make a histogram to display the results, and compare the results to known probabilities.

**7.4.3.2** Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

**7.4.3.3** Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities.

### Essential Question(s):

- What is the probability of particular events happening?
- How can probability be used to make predictions?

### Skill:

*Students will be able to...*

- I can express probabilities as fractions, decimals, and percents
- I can make predictions when given the theoretical and experimental probability

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Heads Up](#)

### Discourse Routine(s):

[Would You Rather - Probability](#) (Would You Rather, n.d.)

## Stage 3 - Learning Plan (Learning Activities)

### Learning Activities:

**Heads Up (30 minutes):** Students will be working in pairs. Group roles will be the resource manager and the recorder/reporter. Students will be given three coins. They begin by flipping one coin 25 times. The record in a table how many times it landed on heads and how many times it landed on tails. They then answer questions about what was more likely and make predictions based on if they flipped the coin 100 times. The pair does the same thing by flipping 2 coins and 3 coins at a time. They make the different outcomes in the tables provided. Students can extend their learning by flipping 4 coins 25 times in a row and

marking the outcomes in a table that they create themselves.

**Would You Rather - Probability (10 minutes):** Students are presented with the question, would you rather flip 3 coins and win if all match or roll 3 dice and win if none match. Students have time to think, and will decide which option they would pick. They need to find the probability of each compound event. The teacher will then ask students to share what they choose and to justify their answers.

**Resources:**

Would You Rather - Probability retrieved from

<http://www.wouldyourathermath.com/would-you-rather-38-2/>



# Unit 3 - Integers

## Stage 1 - Desired Results

### Established Goal(s):

**7.1.1.3** Locate positive and negative rational numbers on a number line, understand the concept of opposites, and plot pairs of positive and negative rational numbers on a coordinate grid.

**7.1.1.4** Compare positive and negative rational numbers expressed in various forms using the symbols  $<$ ,  $>$ ,  $=$ ,  $\leq$ ,  $\geq$ .

**7.1.2.1** Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

**7.1.2.2** Use real-world contexts and the inverse relationship between addition and subtraction to explain why the procedures of arithmetic with negative rational numbers make sense.

**7.1.2.4** Solve problems in various contexts involving calculations with positive and negative rational numbers and positive integer exponents, including computing simple and compound interest.

**7.1.2.6** Demonstrate an understanding of the relationship between the absolute value of a rational number and distance on a number line. Use the symbol for absolute value.

**7.2.3.2** Evaluate algebraic expressions containing rational numbers and whole number exponents at specified values of their variables.

**7.2.3.3** Apply understanding of order of operations and grouping symbols when using calculators and other technologies.

### Essential Question(s):

- How can we represent integer operations?
- What are the properties of negative numbers?

### Skill:

*Students will be able to...*

- I can define and find absolute values of numbers
- I can add, subtract, multiply, and divide integers
- I can evaluate expressions
- I can plot positive and negative points on a coordinate grid

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Extreme Weather](#) (Weather Extremes, 2015)

**Discourse Routine(s):**

Counting Circles - Addition (Horn, 2017)

Counting Circles - Subtraction (Horn, 2017)

Number Talk - Multiplication (Krall, 2018)

**Stage 3 - Learning Plan (Learning Activities)****Learning Activities:**

**Extreme Weather (45 minutes):** Students are placed into pairs. The group roles are resource manager and a task manager. In their pairs, students look at a list of the 50 states in the United States and their corresponding high and low temperatures. Students find the range of temperature for each state and record their findings. Students then answer questions about which states had the highest high temperature, lowest low temperature, biggest range, smallest range, and about the range of temperature for the state they live in.

**Counting Circles - Addition (10 minutes):** Students gather in a circle so everyone can see each other. We will start at the number -50 and go up by 4. The teacher begins by saying -50, and the student next them will say the next number in the sequence. In this example, the student would say -46. The pattern continues until everyone has said a number. At that point the teacher stops the group and asks students to predict what number would be said in 4 turns. When students know the answer, they will silently put a finger to their chest to indicate that they are ready. After the teacher has given students time to think, they will ask students to share the number they got and the strategy.

**Counting Circles - Subtraction (10 minutes):** Students gather in a circle so everyone can see each other. We will start at the number 44 and go down by 3. The teacher begins by saying -44, and the student next them will say the next number in the sequence. In this example, the student would say 41. The pattern continues until everyone has said a number. At that point the teacher stops the group and asks students to predict what number would be said in 6 turns. When students know the answer, they will silently put a finger to their chest to indicate that they are ready. After the teacher has given students time to think, they will ask students to share the number they got and the strategy.

**Number Talk - Multiplication (10 minutes):** The teacher will display the problem  $-15 \times 23$  on the board. Students silently figure out a strategy for solving this mentally in their heads. When students have a strategy they will silently put a finger to their chest. If they can figure out another strategy they will put two fingers to their chest. After everyone has had time to solve the problem, the teacher will ask students to just share what number they got. The teacher writes down all of the different numbers on the board. After students have shared their solution, students are asked to share their strategy. The teacher will scribe what the students are describing visually on the board. Multiple students will share their strategies

and the class will have a discussion about the connections and differences amongst the different strategies presented.

**Resources:**

Extreme Weather adapted from <https://www.yummymath.com/2015/weather-extremes-2/>

# Unit 4 - Rational Numbers

## Stage 1 - Desired Results

### Established Goal(s):

**7.1.1.1** Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal. Recognize that  $\pi$  is not rational, but that it can be approximated by rational numbers such as  $\frac{22}{7}$  and 3.14. **7.1.1.2** Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.-

**7.1.1.2** Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.

**7.1.1.4** Compare positive and negative rational numbers expressed in various forms using the symbols  $<$ ,  $>$ ,  $=$ ,  $\leq$ ,  $\geq$ .

**7.1.1.5** Recognize and generate equivalent representations of positive and negative rational numbers, including equivalent fractions.

**7.1.2.3** Understand that calculators and other computing technologies often truncate or round numbers.

**7.1.2.4** Solve problems in various contexts involving calculations with positive and negative rational numbers and positive integer exponents, including computing simple and compound interest.

### Essential Question(s):

- How can we represent rational number operations?
- How can we convert between different rational number representations?

### Skill:

*Students will be able to...*

- I can add, subtract, multiply, and divide rational numbers
- I can identify and define rational numbers
- I can convert between decimals, fractions, and mixed numbers
- I can compare positive and negative rational numbers
- I can evaluate expressions involving rational numbers

## Stage 2 - Assessment Evidence

### Performance Task(s):

Adding and Subtracting Rational Numbers Escape Room (Adding and Subtracting Rational Numbers, n.d.)

**Discourse Routine(s):**

[Desmos Polygraph - Rational Numbers](#) (Rational Numbers, n.d.)

[Which One Doesn't Belong - Rational Numbers](#) (Which One Doesn't Belong, n.d.)

Counting Circles (Horn, 2017)

**Stage 3 - Learning Plan (Learning Activities)**

**Learning Activities:**

**Adding and Subtracting Rational Numbers Escape Room (60 minutes):** Students are placed into groups of 4. The roles include a resource manager, recorder/reporter, facilitator, and task manager. Students are given a link to a google survey and 4 puzzles to complete. When they complete a puzzle about adding and subtracting rational numbers they will unlock a code. They enter that code into the google survey, and if it is right they can move onto the next puzzle. If the code is wrong, they will need to keep working on the first puzzle. When they have unlocked all of the puzzles they will have completed the escape room.

**Desmos Polygraph - Rational Numbers (20 minutes):** Students log into the activity and are paired up with another student within their class. Within a pair, is a picker and a guesser. The picker selects one of 16 images on the screen. The images for this polygraph are different positive and negative fractions. The guesser will ask a series of yes or no questions that the picker will need to answer. The guesser can eliminate images after gathering more information from the questions until they are able to accurately identify the mathematical object. This activity would be perfect for having students develop their vocabulary knowledge with fractions, such as denominator and numerator. They also could work on converting fractions to decimal representations through this polygraph.

**Which One Doesn't Belong - Rational Numbers (5 minutes):** In this routine students are presented with 4 different rational numbers. There are fractions, positives, negatives, decimals, and percents. Students are posed with the question, which one doesn't belong. For this activity students need to share which one they picked and why. Each of the rational numbers presented has a reason for not belonging. As a class, the goal is to find a reason for each number to not belong.

**Counting Circle (10 minutes):** All students gather around in a circle so everyone can see each other. We will begin with the number 11 and then count up by  $\frac{3}{4}$ . Students will go around the circle sharing the next number going up by  $\frac{3}{4}$ . Students can use mixed numbers, improper fractions, decimals, or any representation that they choose. After everyone has said a number, I will stop the class and ask the students what would be the number in 5 turns. Once students know the answer they will put a finger to their chest to show that they are

ready. After I have given all students time to think, I will ask for volunteers to share the number they got and their strategy.

**Resources:**

Adding and Subtracting Rational Numbers Escape Room retrieved from <https://www.teacherspayteachers.com/Product/Adding-and-Subtracting-Rational-Numbers-Game-Escape-Room-Math-Activity-3648166>

Desmos Polygraph - Rational Numbers retrieved from <https://teacher.desmos.com/polygraph/custom/560ad68b7701c303063305b1>

Which One Doesn't Belong - Rational Numbers adapted from <https://wodb.ca/>

# Unit 5 - Algebra

## Stage 1 - Desired Results

### Established Goal(s):

**7.1.2.4** Solve problems in various contexts involving calculations with positive and negative rational numbers and positive integer exponents, including computing simple and compound interest.

**7.2.2.4** Represent real-world or mathematical situations using equations and inequalities involving variables and positive and negative rational numbers.

**7.2.3.1** Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.

**7.2.3.2** Evaluate algebraic expressions containing rational numbers and whole number exponents at specified values of their variables.

**7.2.3.3** Apply understanding of order of operations and grouping symbols when using calculators and other technologies.

**7.2.4.1** Represent relationships in various contexts with equations involving variables and positive and negative rational numbers. Use the properties of equality to solve for the value of a variable. Interpret the solution in the original context.

### Essential Question(s):

- How do you solve for a missing variable?
- What are like terms?
- When are algebraic equations and expressions useful for solving problems?

### Skill:

*Students will be able to...*

- I can simplify algebraic expressions
- I can use the distributive property
- I can solve one-step equations
- I can write a simple equation
- I can solve two-step equations

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Pool Border Problem](#) (Pool Border Problem, n.d.)

### Discourse Routine(s):

[Which One Doesn't Belong - Algebra](#) (Which One Doesn't Belong, n.d.)

### Stage 3 - Learning Plan (Learning Activities)

#### Learning Activities:

**Pool Border Problem (45 minutes):** Students are paired into groups of two. The pair of students will share one chromebook and be given a link to a desmos activity. In the activity, students work to identify how many tiles will be needed around the border of a pool. They do this for multiple sized pools, and then pairs need to come up with an expression to generalize how many tiles are needed for any pool border.

**Which One Doesn't Belong - Algebra (5 minutes):** In this routine students are presented with 4 different algebraic terms. There are terms that are constants, have a variable, a variable with an exponent, as well as positive and negative terms. Students are posed with the question, which one doesn't belong. For this activity students need to share which one they picked and why. Each of the terms presented has a reason for not belonging. As a class, the goal is to find a reason for each term to not belong. This is a good introduction to the algebra unit when we discuss like terms.

#### Resources:

Pool Border Problem retrieved from

<https://teacher.desmos.com/activitybuilder/custom/58798fc7e338613f05a42feb>

Which One Doesn't Belong - Algebra adapted from <https://wodb.ca/>



# Unit 6 - Data & Statistics

## Stage 1 - Desired Results

### Established Goal(s):

**7.4.1.1** Design simple experiments and collect data. Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets, and make predictions.

**7.4.1.2** Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet to examine this impact.

**7.4.2.1** Use reasoning with proportions to display and interpret data in histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology.

**7.4.3.1** Use random numbers generated by a calculator or a spreadsheet or taken from a table to simulate situations involving randomness, make a histogram to display the results, and compare the results to known probabilities.

### Essential Question(s):

- What is the best way to represent data?
- What happens to measures of central tendency when points are added and deleted?

### Skill:

*Students will be able to...*

- I can determine the mean, median, mode, and range for data represented
- I can describe the impact that inserting or deleting a data point has on the mean and median of the data set
- I can use the mean, median, mode, and range of a data set to draw conclusions about the data
- I can create and interpret histograms
- I can create and interpret circle graphs
- I can choose the best display for a given data set

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Typical Bag of Skittles](#)

### Discourse Routine(s):

[Always/Sometimes/Never Data & Statistics](#) (Carter, 2017)

### Stage 3 - Learning Plan (Learning Activities)

#### Learning Activities:

**Typical Bag of Skittles (60 minutes):** Students are placed in groups of three. There is a resource manager, facilitator, and recorder/reporter. Students are given a bag of skittles and they need to separate the skittles by color. They count each color (purple, green, yellow, red, and orange) and create a line plot to represent their bag. They fill in the table where they mark the count of each color, the ratio of count to the total number of skittles in the bag, and the percent of that color within the bag. After filling out the table, students will create a circle graph to represent the percent of each color within the bag. Students will use coloring utensils to color in their line plot and circle graph. The reporter will share with the class how many of each color skittle was in their bag. Every group shares this information, and records the class data in another table. Students look at the whole class' data to calculate the percent of each color, mean, median, and mode for each color skittle.

**Always/Sometimes/Never Data & Statistics (20 minutes):** Students are given a set of statements where they need to decide if they are always, sometimes, or never true. These statements are about the mean, data, mode, and range of data sets. Students will work with one other person at first to review the statements. Students will be discussing how could a statement be true always, sometimes, or what makes it so the statement could never be true. We will come back together as a class and go through the statements. Pairs will share out what they selected and why. At the end of the discussion the teacher reveals what are the correct responses.

#### Resources:

Always/Sometimes/Never Data & Statistics adapted from  
<https://mathequalslove.blogspot.com/2017/04/always-sometimes-never-with-measures-of.html>

# Unit 7 - Geometry

## Stage 1 - Desired Results

### Established Goal(s):

**7.3.1.1** Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is  $\pi$ .

Calculate the circumference and area of circles and sectors of circles to solve problems in various contexts.

**7.3.1.2** Calculate the volume and surface area of cylinders and justify the formulas used.

### Essential Question(s):

- What are the parts of circles and cylinders?
- How are circumference and diameter related to pi?

### Skill:

*Students will be able to...*

- I can find the diameter of a circle given the radius
- I can find the radius of a circle given the diameter
- I can find the radius and diameter given the circumference of a circle
- I can use the formula to find the circumference and area of a circle
- I can understand that pi is the ratio of a circle's circumference to its diameter
- I can calculator measurements of a sector of a circle
- I can find the volume of a cylinder
- I can find the surface area of a cylinder
- I can justify the formula for the volume and surface area of a cylinder

## Stage 2 - Assessment Evidence

### Performance Task(s):

Cylinder Dissection

Measures Around a Circle

### Discourse Routine(s):

[Would You Rather - Circle vs. Rectangle](#) (Would You Rather, n.d.)

[Pie Estimation 180](#) (Estimation 180, n.d.)

### Stage 3 - Learning Plan (Learning Activities)

#### Learning Activities:

**Cylinder Dissection (20 minutes):** On the first day, table partners are given a net of a cylinder. They are given the task of cutting the net out and constructing a cylinder from the net. On the next day in class, the students get their cylinder returned and asked to find the surface area of the cylinder. They will “dissect” the cylinder and return it to its net form if that helps them. They will use a ruler to measure the radius, circumference, and height. They will have to discover that the length of the rectangle is actually the same distance as the circumference of the base of the cylinder.

**Measures Around A Circle (30 minutes):** Students will be working in groups of 3. There is a resource manager, facilitator, and recorder/reporter. All groups are given a ruler, string, and a recording sheet. In the front of the classroom there is a variety of cylindrical objects that can be measured. The resource manager will select one item at a time for the group to measure. The group measures the circumference and diameter of each object and lists those measurements in the recording sheet. The resource manager will return the object they measured and grab a new one. They do this until time runs out. After measuring all their objects, they will take the circumference of each and divide it by each of their diameters and list the number they compute. All of the values they compute should be close to 3.14 if they measured precisely. This helps students see how the formula for circumference of a circle is formed where circumference equals pi times the diameter.

**Would You Rather - Circle vs. Rectangle (10 minutes):** Students are presented with the question would you rather use table A or table B for serving a large feast. Table A is a rectangle with a base of 96’’ and a height of 42’’. Table B is a circular table with a radius of 45’’. Students will work independently to select a table that they would rather use. Students might find the area of both of the tables and/or find the perimeter of each of the tables. After each student has had time to think I will ask for volunteers to share their choice and why.

**Pie Estimation 180 (5 minutes):** Students are presented with an image of a pie that has had a slice taken out. Students are asked to estimate what percent and what degrees of the pie had already been eaten. The teacher first asks students to make an estimate that is too low and have students share out their responses and why. The teacher then asks students to give an estimate they know is too big. Students share out their responses and explain why their estimate is too big. Finally, students make their best estimate. Students share their estimates and teachers record the estimates on the board. Once students have shared, the teacher reveals the true percent and degrees. This estimation activity segways into thinking about sector area of circles.

#### Resources:

Pie Estimation 180 retrieved from <http://www.estimated180.com/day-112.html>

Would You Rather - Circle vs. Rectangle retrieved from

<http://www.wouldyourathermath.com/thanksgiving/>

# Unit 8 - Proportional Relationships

## Stage 1 - Desired Results

### Established Goal(s):

**7.2.1.1** Understand that a relationship between two variables,  $x$  and  $y$ , is proportional if it can be expressed in the form  $y/x = k$  or  $y = kx$ . Distinguish proportional relationships from other relationships, including inversely proportional relationships

**7.2.1.2** Understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate (constant of proportionality). Know how to use graphing technology to

**7.2.2.1** Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another. Determine the unit rate (constant of proportionality or slope) given any of these representations.

**7.2.2.2** Solve multi-step problems involving proportional relationships in numerous

**7.2.2.3** Use knowledge of proportions to assess the reasonableness of solutions.

**7.2.4.2** Solve equations resulting from proportional relationships in various contexts.

### Essential Question(s):

- How are steepness and slope related?
- What are the different types of slope?

### Skill:

*Students will be able to...*

- I can determine a proportional relationship from a table
- I can determine a proportional relationship from a graph
- I can determine a proportional relationship from a verbal description
- I can find the slope of a line from a graph
- I can translate from verbal descriptions, symbols, equations, and graphs

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Grind Rails](#) (Kaplinsky, n.d.)

### Discourse Routine(s):

[Desmos Polygraph - Lines](#) (Polygraph-Lines, n.d.)

[Which One Doesn't Belong - Lines](#) (Which One Doesn't Belong, n.d.)

### Stage 3 - Learning Plan (Learning Activities)

#### Learning Activities:

**Grind Rails (60 minutes):** Students will be working in groups of 3. The three roles will be a facilitator, resource manager, and recorder/reporter. Students are given a series of 3 grind rails. Before beginning the activity, students will watch a video of skateboarders going down different grind rails. In the activity, the grind rails are lines on a dotted plane. They need to identify the vertical change and horizontal change of each of the lines. Students then need to rank the grind rails from steep to least steep and explain why they put them in that order. Students are beginning to develop an understanding of slope. After they have identified a few series of grind rails, that are tasked with creating their own grind rails. They must use straight lines and ensure that every line starts and stops on a dot. They then identify the vertical change and horizontal change of the lines they created, and rank them from steep to least steep.

**Desmos Polygraph - Lines (20 minutes):** Students log into the activity and are paired up with another student within their class. Within a pair, is a picker and a guesser. The picker selects one of 16 images on the screen. The images for this polygraph are different lines on a coordinate grid. There are positive, negative, zero, and undefined slopes. The guesser will ask a series of yes or no questions that the picker will need to answer. The guesser can eliminate images after gathering more information from the questions until they are able to accurately identify the mathematical object. This activity would be perfect for right after learning about the different types of slope.

**Which One Doesn't Belong - Lines (5 minutes):** In this routine students are presented with 4 different lines on a coordinate grid. All of the lines are straight lines. There are three positive slopes and one negative slope. One line goes through the origin. Students are posed with the question, which one doesn't belong. For this activity students need to share which one they picked and why. Each of the lines presented has a reason for not belonging. As a class, the goal is to find a reason for each line to not belong. This is a good introduction to the algebra unit when we discuss like terms.

#### Resources:

Desmos Polygraph - Lines retrieved from <https://teacher.desmos.com/polygraph-lines>

Grind Rails retrieved from

[https://robertkaplinsky.com/wp-content/uploads/2012/10/Slope\\_Printables.pdf](https://robertkaplinsky.com/wp-content/uploads/2012/10/Slope_Printables.pdf)

Which One Doesn't Belong Lines adapted from <https://wodb.ca/>

# Unit 9 - Percents

## Stage 1 - Desired Results

### Established Goal(s):

- 7.1.2.5 Use proportional reasoning to solve problems involving ratios in various contexts.
- 7.2.2.2 Solve multi-step problems involving proportional relationships in numerous
- 7.2.2.3 Use knowledge of proportions to assess the reasonableness of solutions.
- 7.2.4.2 Solve equations resulting from proportional relationships in various contexts.

### Essential Question(s):

- How do you apply discounts, taxes, and markups?

### Skill:

*Students will be able to...*

- I can find the percent of change
- I can solve markup, discount, and tax problems
- I can compute simple interest
- I can answer percent questions given a circle graph

## Stage 2 - Assessment Evidence

### Performance Task(s):

[Target Shopping Activity](#) (Discount and Sales Tax, 2014)

### Discourse Routine(s):

[Would You Rather - Discounts](#) (Would You Rather, n.d.)

## Stage 3 - Learning Plan (Learning Activities)

### Learning Activities:

**Target Shopping Activity (45 minutes):** Students will be working in groups of three. There will be a resource manager, recorder/reporter, and task manager. Students will be posed with the task of buying a variety of items from Target and trying to be the closest group to spending \$500 without going over \$500. For this activity, students will be selecting items and these items will be going on sale. During the first phase students have 20 minutes to select items from at least 5 different departments. During the first phase, students do not know what percent each department is on sale. After 20 minutes, students enter into the second phase. In this phase, the teacher reveals the percentages. Students then list the discount percent for each of their items and the sale price of the items they selected. This



second phase lasts 10 minutes. During the third phase students add up the cost of all their merchandise and add the 8% sales tax to get the total cost of their shopping trip. The group has 10 minutes for the last phase. After time is up, the groups share what their grand totals were and the group that was closest to \$500 without going over is the winner.

**Would You Rather - Discounts (10 minutes):** Students will be shown a series of would you rather questions about a pair of jeans that are on sale. The first visual will ask students if they would rather purchase a pair of jeans from store A where they are 30% off or store B where they are \$30 off. Students will move to different sides of the room to designate which store they would purchase from. After each student has decided, volunteers will share why they picked that store. The next visual asks the same question, but you find out the jeans are \$150. Students then will move to different sides of the room and share which discount they would rather have, and volunteers will explain why. In the next visual, the price of the jeans has changed to \$70. Students move to the different sides of the room to show which store they would purchase the jeans from, and students explain why they made their choice.

**Resources:**

Target Shopping Activity retrieved from <http://fawnnguyen.com/discount-sales-tax/>

Would You Rather - Discounts retrieved from <http://www.wouldyourathermath.com/discount-jeans/>

# Unit 10 - Transformations

## Stage 1 - Desired Results

### Established Goal(s):

7.3.2.4 Graph and describe translations and reflections of figures on a coordinate grid and determine the coordinates of the vertices of the figure after the transformation.

### Essential Question(s):

- What are the different ways a point, line, and shape can transform?

### Skill:

*Students will be able to...*

- I can, given a rule or word descriptions, reflect or translate a point to a new location
- I can, given a point and it's new image, be able to identify the rule in  $(x-6, y-2)$  format
- I can, given a point and it's new image, be able to identify the rule in description format
- I can reflect over the y-axis and x-axis

## Stage 2 - Assessment Evidence

### Performance Task(s):

Transformation Mystery Trip (Transformations Mystery USA Trip, n.d.)

### Discourse Routines:

[Desmos Polygraph - Transformations](#) (Transformations, n.d)

## Stage 3 - Learning Plan (Learning Activities)

### Learning Activities:

**Transformation Mystery Trip (45 minutes):** Students are placed into pairs. Within the pair, the group roles consist of a resource manager and a task manager. Students are given a map of the United States placed along a grid with various points labeled with letters and a table of various transformations. For this trip, you begin at New York City, and by performing different transformations you will end up in a new location. You perform the listed transformations in order, and keep track of where you are located on the grid. After the final transformation you will have landed in either Florida Beach, Liberty Bell, Grand Canyon, Willis Tower, Niagara Falls, or the San Diego Zoo. The transformations include

translations, reflections, and rotations.

**Desmos Polygraph - Transformations (20 minutes):** Students log into the activity and are paired up with another student within their class. Within a pair, there is a picker and a guesser. The picker selects one of 16 images on the screen. The images for this polygraph are different shapes undergoing transformations on a coordinate grid. There are dilations, reflections, translations, and rotations. The guesser will ask a series of yes or no questions that the picker will need to answer. The guesser can eliminate images after gathering more information from the questions until they are able to accurately identify the mathematical object. This activity would be perfect for right after learning about the different types of transformations.

**Resources:**

Desmos Polygraph - Transformations retrieved from

<https://teacher.desmos.com/polygraph/custom/560c53f5441172070b26220a>

Transformation Mystery Trip retrieved from

<https://www.teacherspayteachers.com/Product/Transformation-Mystery-USA-Trip-from-New-York-City-NY-3664407>

# Understanding By Design Template

Adapted from Wiggins and McTighe, 2011

## Stage 1 - Desired Results

**Established Goal(s):**

**Essential Question(s):**

**Skill:**

*Students will be able to...*

## Stage 2 - Assessment Evidence

**Performance Task(s):**

**Discourse Routine(s):**

## Stage 3 - Learning Plan (Learning Activities)

**Learning Activities:**

**Resources:**

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