Summer 2018

Developing Conceptual Place Value Intervention Curriculum For Upper Elementary Students

Angela Klint
Hamline University

DEVELOPING CONCEPTUAL PLACE VALUE INTERVENTION CURRICULUM FOR UPPER ELEMENTARY STUDENTS

By

Angela Klint

A capstone submitted in partial fulfillment of the requirements for the degree of Masters of Arts in Education.

Hamline University

Saint Paul, Minnesota

August 2018

Advisor: Susan Manikowski

Content advisor: Diane Bacon

Peer reviewer: Johanna Huang

Table of Contents

CHAPTER ONE: INTRODUCTION

Overview

My project is developing Conceptual Place Value intervention curriculum for upper elementary students. Conceptual Place Value (CPV) is the ability to flexibly increment (add) and decrement (subtract) numbers mentally (Wright, Ellemor-Collins, & Tabor, 2012). Throughout this chapter I will explain my history throughout elementary with mathematics, explain the struggles that I went through as a learner. Then explain my growth due to CPV knowledge that I obtained. After that I will explain my educating background and questions that had arose. I will then talk about the professional development that I have received on this topic. Lastly I will explain the resources that have been provided to me and issues that have arose from them.

Introduction

My role as a fourth-grade teacher is to provide grade level content to my students to ensure they are college and career ready. While teaching in an urban setting in the

upper Midwest, I have noticed that many of my students struggle with their grade level content. I teach at a school in which 57% of the population is Asian, 32% African American, 4% Hispanic, 1% American Indian, 2% white, and 2% are two or more races.

The majority of the Asian students are Hmong and that the African American students include many Somali students. My school is comprised of 49.5% English language learners, 18.7% special education students, 92% of our families qualify for free and reduced price lunches. Additionally, homeless and highly mobile students make up 10.9% of our school population. With these obstacles, while looking at my students' mathematical understandings, I have found that many of my students are well below grade level with their conceptual place value development. Many other teachers have noticed this as well. With this data, I have started to think, what can I do to help all students in their place value understandings?

Growing up

I grew up in the teaching era in which you needed to just memorize number combinations to be good at math. I didn't need to understand why 9+5= 14 I just needed to know that it did. I was not great at memorizing math facts. I remember the anxiety that math fact drills caused me. I remember sitting in class looking over the paper for the easy ones that I didn't have to work on. *Anything plus zero is its self, anything plus one is just one more*. I would see students around me almost finished with their paper at the end of the one minute while I only had the easy ones done.

While at parent teacher conferences in third grade, teachers expressed concern for my math abilities to my parents. They were worried that it was taking too long for me to recite my math facts. The teachers suggested that my parents buy flash cards and work on my memorizing them at home. My parents wanted me to do the best I could in school, of course followed their suggestions. Teachers also wanted to have me go to a remedial math fact intervention group to increase my memorization skills. My parents agreed and I was then taken out of my specialist classes to work on my math facts.

Day after day I was surrounded by these math facts. My brain could come up with the answers, but never quite quick enough for my teachers. Other students where always quicker at the around the world game (a game where you get a math fact and the person who answered correctly would win and get to move on to the next person). I felt stupid for not just knowing the answers quickly. I thought I was bad at math because I had to think about the answers. I had to know why the answer was the answer.

With constant exposure to math facts, I was able to start understanding the reason why the numbers made the answer. I remember sitting at home with my parents and saying, "*Oh, 7+9= 16 because if you add 7+10 you get 17 and 9 is just one less.*" I was able to understand why numbers worked the way they did. I understood the relationship between the addends and the sum. I could break apart the numbers to make easier problems.

When in fourth grade, I was moved out of the remedial class and put into the highest math class. I was moved because I had a conceptual understanding about multiplication. I understood how to break the numbers apart to create easier problems and then put it back together. I could do this all mentally. This deep understanding of how numbers worked had helped me accelerate my math skills.

In middle school, I tested out of sixth grade math and was put into seventh grade math classes. For the rest of my mathematical education, I was a grade level above my peers.

Even with this rough start, to this day I still love math, I love how numbers work. I love the consistency of math, 2+2 will always equal 4. However, I know that this is not the feeling of all people. This is not the feeling of all the students in my class. Math does not come easily to everyone.

Teaching Mathematics

Throughout my 5 years of teaching, I have taught primarily in upper elementary grades within high poverty areas. Within my classes, I had students who have never had formal schooling until they were in my class and others who had formal schooling their whole lives, but had not had their needs meet within their previous classrooms. Students have come to me with very limited place value knowledge and because of this they are not able to master the higher order mathematical concepts that the upper elementary grades present. The students are still using counters and fingers to add single digit numbers together.

In the past, I have tried multiple strategies to help students gain mathematical insights. I have tried using drills in the past to help students get the combinations to ten (7+3) and twenties (12+8) down solid. This helped when they were just working on small amounts; however, their knowledge fell apart when it was asked to get transferred to other topics. I have also tried using only small group instruction to meet students at their level. If students needed to learn how to subtract numbers, I taught them how to do it the algorithmic way. Always have the biggest number on top. If you can't take away in the ones place, cross out the in the tens place and give it to the ones and so on. Students struggled with this because they didn't really understand why they were doing what they were doing. It didn't make sense to them. I knew there had to be a better way of teaching these concepts; however, I didn't have those skills yet.

Professional development

This passion of improving my mathematical instruction had led me to obtain a K-5 mathematic teaching certificate. This certificate program took a year to complete along with a teaching practicum in which we went into a summer school classroom and taught small groups. This program gave me the tools that I needed to help improve my instruction for all students in my classroom. I have stopped using drilling strategies to teach combinations of tens and twenties and have switched to using activities that will lead to a fuller understanding of the numbers. When teaching whole group instruction, I have opted to using more "math talks" rather than a lecture method of teaching. My instruction has gotten better over the years, however I know there is still room for improvement.

I have also attended the National Council of Teachers of Mathematics (NCTM) conference in Duluth, Minnesota. At this conference, I was taught many different ways to improve mathematical skills in various and fun ways for students. The math specialist and I created a professional development presentation to express our learnings to the rest of the staff within our building. However, due to scheduling conflicts we were unable to present it.

Conceptual Place Value Resources

CPV is the ability to be able to flexibly increment (add) and decrement (subtract) numbers mentally (Wright, Ellemor-Collins, & Tabor, 2012). CPV is the building block to understand mathematics. With this knowledge, the school in which I work at has decided to implement CPV into all classrooms. The math specialist gave each teacher a binder full of activities for students to be working on when they are not in small group with the teacher. These activities were designed to let the students have fun while increasing the CPV knowledge.

As an educator going through a math certificate program at the time, I completely understood the need for an increase in CPV knowledge within my students. However, I found it hard to know that my students were doing these activities when I was leading small groups. I brought my concerns to other teachers and asked them how they were ensuring that students were working on the skills that they needed. Many of my coworkers had mixed feelings of the CPV binder in which we were given. Some had taught the activities in a small group setting, in which students were grouped based on their CPV level, then had them moved to a station after it being explicitly taught. Some educators would show a game to the whole class during a whole group teaching session and had students do that at a station. Some educators hadn't used the binder at all.

With this wide range of implementation amongst my coworkers, I wondered why some of the teachers had implemented this resource to fidelity while some hadn't even used the binder. I started asking my fellow educators what either made them implement or not implement this resource. A few who implemented the binder stated that they understood what it was trying to accomplish and found the importance in it. Some implemented it because they were asked to, but didn't really understand why they were asked to do so. The ones who didn't implement tended to state that they didn't see the reason for this; they didn't understand what this recourse was designed to do.

With these responses, I started wondering, *what I can do to help my fellow educators understand the importance of CPV knowledge within their classroom. How can I ensure that all students are getting well implemented interventions to improve their CPV knowledge?*

Rational

Conceptual Place Value (CPV) has nine different levels of understanding. These skills range from emergent skills in which students don't have one to one correlation to level 8 where students are able to add and subtract three digit numbers mentally. Ideally, students should be able to do all these skills by the end of second grade. At the school where I teach, I analyzed the data from the third through eighth grade students and found some interesting data. 1% of our students tested at the emergent level, 22.7% tested at level one, 16.4% tested at level two, 15.3% tested at a level three, 15.9% tested at a level four, 2.2% tested at a level five, 24.4% tested at a level six, 4.5% tested at a level seven and 6.8% tested at a level eight. There is an overwhelming need to have CPV interventions implemented by educators in small groups during guided math time.

CPV levels for third through Eight grade students within my school	
Emergent	**1%**
Level 1	**22.7%**
Level 2	**16.4%**
Level 3	**15.3%**
Level 4	**15.9%**
Level 5	**2.2%**
Level 6	**24.4%**
Level 7	**4.5%**
Level 8	**6.8%**

Purpose

Conceptual Place Value (CPV) is the fundamental building blocks of mathematical understanding. With a large part of the student body being well below grade level, it is apparent there is a high need for this to be taught within small guided groups within the classroom. As an educator, I understand the struggles facing teachers, with overwhelming workloads that it is hard to implement research based interventions on a daily basis while also teaching grade level curriculum. It is made even harder when you are not knowledgeable about the subject. My purpose with this project is to provide educators with an understanding of CPV via professional development, and resources to implement small group interventions for their students.

Conclusion

Within this chapter, I outlined my mathematical educational history. I explained how I struggled through elementary school with rote memorization. My parents were able to spend time with me working on my facts and luckily, I was also able to receive additional resources from my school. This is not the experiences that the students in my school have. With high poverty within my school, many parents are working multiple jobs and have little time to work on school work with their students. As teacher of high need students, we need to provide high quality interventions while they are under the school's roof.

I then dove into my experiences with teaching math, while in upper elementary. I explained the struggles with not feeling that I was able to meet all the needs of my students. Many teachers tend to teach the way that they were taught, while this would work for students who have similar background as their teacher, this is not what will work for students with different backgrounds from their teachers. As educators, we need to better our teaching for our students.

Next, I explained the professional development in CPV while obtaining a K-5 mathematics teaching certificate and while attending NTCM. I took the time to improve my mathematical teaching abilities. Lastly, I explained the CPV resources and the benefits and short comings of these resources. As a part of the educator community, it is my duty to use my expertise to improve resources to help educators better reach the needs of their students.

The need to improve these resources is apparent from teaching the data previously stated. 1% of the third through eighth grade students tested at the emergent level, 22.7% tested at level one, 16.4% tested at level two, 15.3% tested at a level three, 15.9% tested at a level four, 2.2% tested at a level five, 24.4% tested at a level six, 4.5% tested at a level seven and 6.8% tested at a level eight. Within chapter two, I will describe where students within third through fifth grade are developmentally according to Piaget's Cognitive Development theory, Vygotsky Sociocultural theory along with his Zone of Proximal Development, and Erikson's Stages of Psychosocial Development theory. Maslow's hierarchy of needs will be included to shed light on students' needs through a holistic approach. I will also delve into how CPV understandings are formed and the levels that are attributed to them. Direct to abstract modeling of mathematical problems will be covered. I will then also explain the issues with timed test and issues with rote memorization. Lastly, how to implement quality mathematical task will be explained. I will use all this information to assist me in completing my project of developing Conceptual Place Value intervention curriculum for upper elementary students.

CHAPTER TWO: LITERATURE RIEVIEW

Overview

My project is developing Conceptual Place Value intervention curriculum for upper elementary students. Conceptual Place Value (CPV) is the ability to flexibly increment (add) and decrement (subtract) numbers mentally (Wright, Ellemor-Collins, & Tabor, 2012). Throughout this chapter, I will describe where students within third through fifth grade are developmentally according to Piaget's Cognitive Development theory, Vygotsky Sociocultural theory and Zone of Proximal Development and Erikson's Stages of Psychosocial Development theory. Maslow's hierarchy of needs will be included to shed light on students' needs through a holistic approach. I will also delve into how CPV understandings are formed and the levels that are attributed to them. Direct to abstract modeling of mathematical problems will be covered. I will then also explain the issues with timed test and issues with rote memorization. Lastly, implementing quality mathematical task will be covered.

Student Development

When teaching mathematics to students', educators must understand where their students are developmentally to avoid presenting concepts too remedial or too challenging. In this section, I will describe where students are cognitively according to Jean Piaget, Lev Vygotsky and Erik Erikson. I will also explain Maslow's hierarchy of needs to provide a holistic picture of a student. The next section will describe some theories of cognition as developed by Jean Piaget, Lev Vygotsky and Erik Erikson. Understanding the stages of human development is an integral part of teaching because when an educator can understand where a student is developmentally, they can educate the student more effectively.

Piaget

According to Piaget's theory of cognitive development, there are 4 distinct stages of development. The four stages are sensorimotor, preoperational, concrete operational and formal operational (Woolfolk, 2014). Sensorimotor stage takes place when a child is zero to two years old. At this stage the child learns through reflexes, senses and movement around their environment. Within this stage, a child gains object permanence and moves from reflexive actions to intentional actions (Woolfolk, 2014).

The Preoperational stage occurs in children around the time a child starts to talk and lasts until 7 years old. During this stage, students have difficulty understanding the concept of past and future and can only think in the present. They can understand that symbols can represent objects. For example, they understand that a picture of a car or the letters C-A-R represent a physical object. Within this stage, they have difficulties seeing other people’s point of view (Woolfolk, 2014).

The concrete operational stage starts when a student enters first grade and goes until roughly age eleven. During this stage, students learn concretely. Using manipulatives help them learn more effectively. At this stage, they are also able to mentally undo actions and understand the concepts of past, present, and future (Woolfolk, 2015).

The last stage, according to Piaget, is the formal operational stage. This stage lasts from adolescence through adulthood. During this stage, thinking becomes more organized. Students are able to think hypothetically and can consider multiple perspectives.

An educator's understanding of these levels is important to be able to address the needs of their students. Students must be presented material that corresponds with the developmental level. Additionally, teachers should present material that could reach students at previous stages as well as the next stage to ensure that all students at all developmental levels are being met. For example, when teaching a math lesson, many students within an upper elementary classroom will be in the concrete operational phase and transitioning into formal operational stages. When teaching a lesson, an educator should provide resources to promote thinking and problem solving and help students organize their thinking through multiple strategies.

Vygotsky

TheVygotsky theory of development has two aspects. One aspect is how speech is used. The other aspect is how students need to learn in their Zone of Proximal Development (Woolfolk, 2014). The first stage of development, according to Vygotsky, is pre-intellectual social speech. This stage starts at birth and lasts till around 3 years old. During this stage, speech is used for change. For example, a child will yell no when another child is reaching for their toy. (Woolfolk,2014)

The second stage is Egocentric speech. This stage starts when they are around three and end when they are around seven. During this stage, children verbalize their thoughts. For example, when playing game a child might count the squares aloud as they move their piece across the game board (Woolfolk, 2014).

The last stage is inner speech. This stage starts around 7 years old and last through adulthood. Thought is primarily done internally and when spoken it is to communicate needs and wants with others. (Woolfolk, 2014)

To better understand children's understanding, educators must understand which level of language and thought development they are at and assess their understanding accordingly.

Within an upper elementary school classroom, many students will be at the inner speech stage of development. To helps students', educators should have them articulate their inner thoughts publically to ensure that no misconceptions are happening internally.

Another theory that Vygotsky had is known as the Zone of Proximal Development (ZPD). ZPD is where a student is able to solve a problem with some assistance from another person. For example, if a child has already mastered putting together a twenty-five piece puzzle, a fifty-piece puzzle would be in their ZPD due to the fact that they might need additional support to put together the whole picture. A one-hundred-piece puzzle would most likely not be in the ZPD because they would not be able to solve it without extensive support. (Woolfolk, 2014)

Understanding the concept of ZPD for educators is integral. Educator's must be aware of where a child's understanding is and how to grow it.

Erikson

Erikson theorized that everyone goes through eight psychosocial stages in their lives. Each stage has two possible outcomes, one healthy and the other not. Erikson asserts that within each phase a person faces a developmental crisis (Woolfolk, 2014).

The first stage that a person faces is basic trust versus basic mistrust. This stage happens from birth through around eighteen months old. During this crisis, an infant can either form a loving relationship with its caregiver or develop mistrust (Woolfolk, 2014).

The second stage is autonomy versus shame/doubt. This happens around eighteen months to three years old. During this stage a child learns to walk and is learning how to use the bathroom correctly. If this development is not handled correctly, a child will still learn these skills, but could feel shame and doubt about their abilities (Woolfolk, 2014).

The third stage is initiative versus guilt. This stage happens when a child is three and last until six years of age. Within this stage, a child will become more assertive and will initiate more. During this stage, the caregiver must make sure they are providing supervision, but not stepping in too fast. If not handled properly, at this stage, a child could form a sense that everything they do is wrong (Woolfolk, 2014).

The fourth stage is industry versus inferiority. This stage happens in the elementary school years of a child. Within this stage, students must learn how to cope with the different skills they are being asked to learn. If they are unable to cope, a student can start to feel inferior to their peers. This will lead them to thinking they are unable to learn (Woolfolk, 2014).

The fifth stage is identity versus role confusion. This stage happens during the teenage years of a student. Within this stage, they are trying to answer the “Who am I” question. If this question is not answered they can feel confused about their role in society (Woolfolk, 2014).

The sixth stage is intimacy versus isolation. This stage happens in early adulthood. During this stage, a person will either form a loving relationship with others or face feeling isolated from society (Woolfolk, 2014).

The seventh stage is generativity versus stagnation. This stage happens in middle adulthood. During this stage a person will look for a way to support the next generation or feel stagnant in their lives (Woolfolk, 2014).

The eighth and final stage is ego integrity versus despair. This stage happens in late adulthood. Within this stage a person reflects back on their life and can either accept themselves and have a sense of fulfilment or feel despair of choices they have made (Woolfolk, 2014).

Educators must be aware of where their students are at this stage to ensure that they are promoting the positive outcome rather than the negative outcome. Most students in upper elementary school will be in the industry versus inferiority stage of development. That means educators must do what they can to build a students' confidence to ensure they are not falling into the inferiority side of development. To do this, we must ensure they are getting taught within their ZPD.

Maslow's Hierarchy of Needs

Maslow states that there is a Hierarchy of Needs that a person must fulfill to be able to reach self-actualization. The levels are survival needs, safety needs, belonging and loved needs, esteem needs, and finally self-actualization. (Woolfolk, 2014). If lower needs are not meet, a person cannot effectively move on to meeting the higher needs.

Survival needs are the most basic needs that a student needs to have filled. These needs include water, food, rest and feeling warm. When these needs are not met, a student will have an extremely hard time being able to learn new concepts at school. School can provide this need while the student is at school, but can do very little when they are outside of the school walls. Safety needs are also a need that is hard for an educator to fulfill outside of the school walls. While a student is in the school walls, an educator can meet the safety needs for a student by providing a safe and inclusive classroom. Social workers and other professional within the school building can work with students and families to help fulfill this need.

Understanding these needs, an effective educator could provide assistance to a child's whole being rather than just focusing on trying to teach them a single subject. A student will not be able to learn when basic needs are not meet. While there is little that an individual educator can do to ensure that all the need are met outside of the school, an educator can try and ensure that the needs are at least met when the student is in their custody.

Cognitive Development of Upper Elementary Age Children

When looking at upper elementary age students, educators must know where they are developmentally. According to Piaget, they are in the concrete operational phase and starting to move into the formal operational phases (Woolfolk, 2014). With this knowledge, educators must provide hands-on activities to facilitate learning.

Vygotsky's theory states that upper elementary school age children are in the inner speech stage in which most thought is done internally. With this knowledge, educators need to remind students to explain their thinking by speaking or writing to further asses their comprehension of material. Educators also need to understand where the student is in their understanding of a topic and scaffold to their ZPD (Woolfolk, 2014).

Erikson's theory states that upper elementary students are in the industry versus inferiority stage. With this knowledge, teachers must understand that if a student is struggling to learn new concepts, they must provide positive feedback to promote the industry side of this stage (Woolfolk, 2014). If a student struggles with n this stage, they may feel inferior to their peers and could feel as they are unable to learn. To do this, we must teach students in their ZPD to promote positive self-worth (Woolfolk, 2014).

Educators must always keep in mind the needs of the children in their classroom. While schools have very little power ensuring that the students need are being meet outside of the school, one thing that schools can do is provide breakfast. This will ensure that hunger is not stopping any student from learning. An educator must also ensure that their classroom is a safe and inclusive space so students are able to access the material.

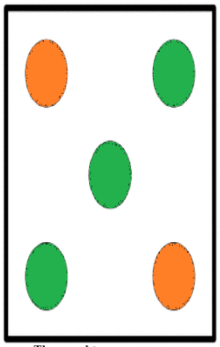
Once an educator understands the cognitive development of a student in their custody, they are then able to use this understanding to better educate the child. However, just knowing the cognitive development of a student is not enough. An educator must also understand how students learn different concepts. Within the next section, I will explain how place value understandings are developed.

CPV development

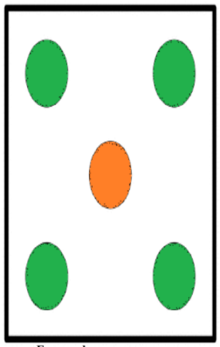
CPV is the ability to flexibly increment (add) and decrement (subtract) numbers mentally (Wright, Ellemor-Collins, & Tabor, 2012). This skill is important in its own right, but along with it they develop a sense of relative sizes, learn ways of relating multidigit numbers to each other, and are able to organize their number habitually in ones, tens, hundreds and so on (Wright, Ellemor-Collins, & Tabor, 2012). CPV development does not just happen overnight. It has to be built from the ground up with intentional instruction. When looking at Common Core standards, we are able to see the CPV progression over years of development. In kindergarten, they are asked to be able to compare groups of object and decompose numbers into 10's and one with numbers 11-19. When in 1st and 2nd grade, they are asked to compare double digit numbers and break apart those numbers into tens and ones. When in 3rd- 5th grade, they are then asked to compare numbers with decimals and understand the magnitude of these numbers (Walkowiak, 2016). All of these skills help to develop a CPV understanding. The process of building CPV understanding is discussed in the following section. To begin CPV instruction, students need to be able to *subtilize*. Subtilizing is being able to see something and instantly knowing how many there is (Clements, 1999). For example, when you roll dice you are able to instantly know how much you rolled without having to count each dot. To promote subtilizing skills within the classroom, educators can show dot cards quickly and pull them away. The educator should then ask the students how many there were. Once the students are firm in their ability to do this, an educator can ask how they saw that. For example, the number is 5 laid out on how a die, the students can say they saw it as a group of 3 and a 2 more or that they saw 4 and 1 more (See figure). Doing this helps students informally know addition facts for the numbers being subtilized. Being able to subtilize is an integral part of understanding numbers and cannot be missed. "Children who cannot subtilize conceptually are handicapped in learning such arithmetic processes" (Clements, 1999).

Once a child can subtilize, students should work on learning their *basic facts (4+3,9+8).* According to Brickwedde, “Children who have not been instructionally supported in deriving and flexible choice of strategies for a sustained period of time will revert to counting on by ones when they can’t recall a fact combination because it is the only strategy choice that they have” (2012). As educators, we need to provide instruction in multiple ways to promote student thinking. Brickwedde suggests that the start of learning basic facts starts with the skills learned from subtilizing. After a subtilizing, an educator can continue on to Double Facts. Children get a hang of doubles (2+2, 5+5) quickly and near doubles are great ways to increase basic fact knowledge. After this, educators should focus on students knowing the ways to make ten. This will then lead to ten plus strategies. Throughout this whole process educators should be intertwining addition and subtraction to ensure the inverse relation is known (Brickwedde, 2012).

After students are able to subtilize and understand their basic facts, students need to become familiar with incrementing and decrementing with tens and ones. One way to do this is to use popsicle sticks in which you have bundles of tens and sticks of one. Students are asked to give what number is represented by counting by 10’s rather than ones (Wright, Ellemor-Collins, & Tabor, 2012). Once children are able to see numbers as tens and ones, an educator should then have students increment and decrement with the tens and ones.



Three and two more



Four and one more

Educators should also increment and decrement pass 110 to help students understand higher numbers (Wright, Ellemor-Collins, & Tabor, 2012).

There are three ways in which an educator can develop deeper understanding in students; Extending, increasing complexity, and distancing the materials (Wright, Ellemor-Collins, & Tabor, 2012). The concept of extending is to extend or expand the range of the numbers that they are using. For example, if you have been using numbers fewer than 100, increasing it into the 100's and 200's will increase students' abilities. After students are able to understand numbers in these ranges it can be extended into 1000's and beyond. Another way to extend students learning will be to make the increments and decrements more complex. That can be done by starting with one group of 10, you can then move to multiple groups and even 100's. Also, switching between 10's and 100's can also present a way to make a student's calculations more complex. For example, and educator can present the following task their students $158+10 \rightarrow 168+20 \rightarrow 188-100 \rightarrow 88+200 \rightarrow 288$ or extend their thinking. Lastly, putting physical distance or a barrier to the learning material can increase students' understanding. This concept is called Distancing. For example, making materials less visible by showing and then covering those up can help students move away from need the visual all together. You can also start with using mathematical notation (numbers and addition signs) before moving to the objects. Following these strategies, students CPV knowledge will increase.

Why is CPV so important? National Research Council suggests that there are five steps that children need to go through to fully understand mathematical concepts. The steps are, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. "Students who have learned only procedural skills and have little understanding of mathematics will have limited access to advanced schooling, better jobs, and other opportunities. If any group of students is deprived of the opportunity to learn with understanding, they are condemned to second-class status in society or worse" (National Research Council, 2001). Student need to be taught to think deeply and creatively, in Language art and as well in mathematics.

CPV Levels

Conceptual Place Value (CPV), which was discussed earlier in the paper, has nine different levels of understanding. These skills range from *emergent* skills in which students don't have one to one correlation to level eight (Emergent- level eight) where students are able to add and subtract three digit numbers mentally. Comprehension of these skills is part Common Core standards up until second grade. After second grade, students are assumed to be able to do these when they are working to implement harder math concepts. Below are descriptions of the various levels of CPV understanding, along with examples of how a student might understand (Masloski, 2016).

Emergent level- Students are struggling with unitizing tens and ones. They do not see 43 as four groups of ten and three ones. (Masloski, 2016)

Level One- Students at this stage understand that two digit numbers are made up of tens and ones, however they struggle with incrementing (adding) and decrementing (subtracting) tens and ones flexibly. This is not to say they are unable to do one or the other. Students would be able to start at 57 and be able to add and subtract groups of tens from that number. Their struggle here would be when an educator asks them to switch between tens and ones (Masloski, 2016).

Level two- At this level, students are able to work with tens and one flexibly, but they are struggling with adding from a decade number (Multiple of ten) and subtracting to a decade number. For example, adding to a decade number would entail a student to add 70+8 mentally without using a counting strategy. An example of subtracting to a decade is 97-7. At this stage, educators want to create a derived fact for them (Masloski, 2016).

Level three - At this level, students are able to add from a decade number and subtract to a decade number. Students are struggling with adding to a decade and subtracting from the decade. For example, students will need to be able to solve problems like 56+ __=60 as well as 70-7=___. At this stage, we are asking students to use their ten facts flexibly and in higher number groups (Masloski, 2016).

Level four- Student are struggling with adding to and subtracting through a decade number. For example, when adding 45+7, we want them to be able to break the seven into five and two so they add by doing 45+5→50+2→52. When subtracting, we want them to break the numbers apart as well. For example, 83-6 students are working on breaking apart the six into two threes so they are able to do 83-3→80-3→77 (Masloski, 2016).

Level five- Students are struggling with adding and subtracting with tens and ones from a decade number. At this level, students are working on jumping tens and then ones. For example, 40+____= 73. They need to do 40+30→70+3→73 then finding the answer is 33 from those jumps. An example of subtraction is 52-____= 30. They need to do 52-20→32-2→30 then finding the answer 22 from those jumps as well. With this level, they have to keep track of multiple jumps and knowing what they are trying to find. A hang up at this level for some students is that when they end at the final answer (73 and 30 respectively, from the above examples) they assume that is the number that they needed to find not the jumps that they made. (Masloski, 2016)

*Level Si*x- students are struggling with flexible mental strategies with two digit additions and subtraction with no materials. Students need to be able to use more than a single way to find answers. For example, when adding 56+37 they would be able to do *50+30= 80, 6+7=13, and 80+13=93* and find another way to add such as *56+30→86+7→93* (Masloski, 2016).

Level seven- students are struggling with flexible mental strategies with three digit additions and subtraction with no materials. Like level six, students need to think about numbers more flexibly. For example, 352+225 student would be able to do *300+200=500, 50+20=70 and 5+2=7. 500+70+7= 577*. Another strategy they could use would be rounding 352 to 350 then *350+225→575+2→577*. (Masloski, 2016)

Level eight- mastery of all of the aforementioned concepts. (Masloski, 2016)

Direct Modeling to Abstract Modeling

Students' mathematical understanding needs to go through different stages of complexity before they can solve problems with memorized facts (Carpenter, Fennema, Franke, Levi, Empson, 2015).

The beginning stage for most students is direct modeling. Within this stage, they are being very literal with the problem type and are unable to think multiple steps ahead in their problem solving (Carpenter, Fennema, Franke, Levi, Empson, 2015). For example, when a student is asked "Robin *has 5 toy cars and she is given 4 more toy cars,* *how many toy cars does she have now?"* A student who is direct modeling would solve this problem by drawing 5 cars (or lines) and draw 4 more and count all of them.

The next level of complexity is counting strategies. Students are able to now think more efficiently about numbers (Carpenter, Fennema, Franke, Levi, Empson, 2015). A student who is operating at this developmental level would solve the above problem by starting at 5 and saying, *5… 6,7,8,9. 9 cars.* They could be using fingers to keep track of the addition at this time.

After counting strategies, students are then moving on to flexible choice of strategies (Carpenter, Fennema, Franke, Levi, Empson, 2015). This stage is different for each student due to the fact that they are solving problems using many different strategies. For example, given the problem *there were 15 cars in a parking lot. 8 cars drove away. How many are there now?* A student might use multiple strategies to solve this problem. One way to solve this would be counting out 15 items and taking 8 away then counting what is left to get their answer. If pushed to find another way they could say that they started at 15 and counted back 8 to get their answer. When pushed even more they could solve this answer by starting at 8 and counting up to 15. With this, they are able to multiple ways to solve the same problem. While the first strategy they used was a direct modeling, they are able to use a counting strategy to solve their problem. As seen in the last example, at this stage they are able to see the inverse relationship between addition and subtraction (Carpenter, Fennema, Franke, Levi, Empson, 2015).

The last stage in development is number fact strategies. At this stage, students are able to just tell the answer because they "*just know"* it. This is when they are starting to have derived facts. Derived facts are when students are able to see the relation between operations and numbers (Carpenter, Fennema, Franke, Levi, Empson, 2015). For students to get to this point in their number sense, they have had to have numerous interactions with the number system (Carpenter, Fennema, Franke, Levi, Empson, 2015).

It is integral for students to go through this process to understand the number system. Students who have been able to direct model, use counting strategies, then create derived facts have a much better understanding of the number system than students who have learned facts through memorization. This does not only work for addition and subtraction, but multiplication and division as well (Carpenter, Fennema, Franke, Levi, Empson, 2015).

Math Anxiety and Time Test

When students are not allowed to grow their understanding through repeated exposure, and are instead asked to memorize facts without truly comprehending why things are the way they are, math anxiety can form.

Timed tests have shown to increase math anxiety amongst students as young as first grade (Boaler, 2012). Brain scans have shown that when students, as young as seven, when given math problems that there is an increase activity in areas of the brain where anxiety and fear are housed along with a decrease activity in the brain where problem solving is housed (Young, Wu & Menon, 2012). This increase in stress then leads students to be unable to recall facts that they have previously known (Boaler, 2012).

When students are given timed test, it can hinder them in math for more than just that test. Time tests can set off anxiety towards mathematics even when timed tests are not given (Boaler, 2012). Timed tests are designed to have the students work quickly, however, is that what we, as educators, are looking for in mathematics? We want students to be able to think deeply and flexibly about subject matter, not just respond rapidly. To increase mathematical understanding, educators need to move away from rote memorization techniques to tasks that enrich and promote mathematical understandings.

Math Tasks

High quality instruction is an invaluable asset to students of all backgrounds. To start, educators need to move away from the Right/Wrong paradigm in math and start understanding the ways students have come to their answers (Wells & Coffey 2005).

One way that educators can increase their understanding of a student's incorrect answer is to consider what possible question they were answering correctly (Wells & Coffey 2005). For instance, when and educator posed "Ali lived 7 blocks away from school. Jeremy lived 2 blocks closer. How many blocks away from school does Jeremy live?" When a student (or multiple students) answer 9, as reflective educators, we should point out to the students that it would have been a correct answer to if Jeremy lived 2 blocks further from school but we were wondering if he lived 2 blocks closer to school. This will link the inverse relation between closer and further and deepen their understanding and be more reflective of their answers (Wells & Coffey 2005).

Problem solving consists of four phases: (1) understanding the problem (2) devising a plan (3) carrying out the plan and (4) looking back (Pólya, 2005). Looking back in mathematics is an important part to understandings, however it is often over looked in today's classroom (Wilson, Fernandez and Hadaway 1993). If educators skip the looking back phases of problem solving, the full concept in incomplete. When educators help students find the questions they are actually answering, it is providing a chance for the students to reflect on their concepts of mathematics and increase their knowledge.

Educators need to be intentional about their implementation of math tasks in their classroom. There are three phases when it comes to implementing a high-quality task for students. Phase one is the task as it appears in curricula/ instructional materials. Phase two is the task as set up by the educator. Phase 3 is the task implemented by the students. When all phases are implemented correctly, students' learning is a result (Stein, Smith). Many educators thrive at the first two phases; however, many times this process breaks down at the implementation phase. This is due to the possibly lack of the educators understanding of what the task is meant to illicit. When students are struggling, they will ask the teacher for help with the task. At this point, if an educator is not diligent, they could give a hint that could change the difficulty of the task completely (Stein, Smith). For example, if a teacher is asking students to solve a multi-digit subtraction word problem and students need to figure out what to do when they do not have enough ones in the ones place. If a teacher reminds them to borrow, it is changing it from a high-quality task to a lower task due to they now do not need to understand why they are borrowing but just that they needed to (Smith, Bill & Hughes, 2008).

When implementing high quality mathematical task in the classroom, a teacher should follow five steps (Stein, Engle, Smith, & Hughes. 2008). The first step is *anticipating,* in which teachers should predict or anticipate possible answers that students might respond with. They should then *monitor* the students to see which strategies are being used. Next, the teacher need to purposefully *select* the answers that will be shown publicly. After picking the answers, a teacher needs to purposefully *sequence* the student's responses. Lastly, teachers needs to make connections *between* mathematical ideas and reflect on the strategies that were presented (Stein, Engle, Smith, & Hughes. 2008). When an educator plans with these steps in mind, the student's learning becomes more meaningful and will help the students grow as a result (Stein, Engle, Smith, & Hughes. 2008).

There is multiple different type of problems that students can be presented with in order to improve mathematical understandings (Carpenter, Franke, Levi, & Empson, 2015). Cognitively Guided Instruction (CGI) is a way to look at word problems as complex ideas that travel far beyond the addition, subtraction paradigm (multiplication and division as well).

There are joining problems, in which there are 3 sub categories. The sub categories are *result unknown*, *change unknown*, and *start unknown*. There are also separating problems in which they also have the same 3 sub category as the joining problems. Another addition/ subtraction problem type is Part-Part-whole problems in which it has 2 subcategories which are while unknown and part unknown. The last type of word problems are comparing problems, in which the 3 subcategories are difference unknown compared set unknown, and referent unknown. It is important for teachers to know all the different types of problems due to the fact that each problem presents different challenges to the students (Carpenter, Franke, Levi, & Empson, 2015).

Conclusion

To provide high quality education for all students, an educator must understand where students are developmentally. Educators must also understand the intricacies of mathematical learning to better meet the needs of their students.

To make student competitive in a global market, educators need to teach students to think more abstractly about math. Rote memorization will not be helpful in real life situations. Students need to be able to transfer their knowledge into real world situations. To do this, educators need to allow student to move through all the stages without pushing them too quickly.

Student in upper elementary grades are still at the concrete operational Piagetian phase (Woolfolk, 2014). With this knowledge, educators need to provide hands on activities to promote learning. Rote memorization and algorithmic teaching will not teach a student the content on a more complex level. Educators need to provide time for students to gain deep mathematical understandings which will translate over to other situations.

Educators need to avoid timed test to prevent students from forming math anxiety. With the knowledge of Erikson's research, when a student is in elementary school they are at the industry versus inferiority stage (Woolfolk, 2014). If a student consistently does poor on timed tests, they will start to feel inferior to their peers and start assuming they are just bad at math.

According to Vygotsky, students in upper elementary school are in the internal speech phase of development. Therefore, educators must prompt students to explain their thinking to truly understand what the student knows (Woolfolk, 2014). With this knowledge, educators are better equipped to provide instruction that is in the students' ZPD.

Throughout this chapter, I explained where students are in their cognitive development and how to increase CPV knowledge with students. Within this chapter, I will explain the need for this curriculum. A detailed description of the project will be presented. Then I will illustrate the environment that this project is intended for. Lastly, assessment criteria and an improvement plan will be presented to assist me in completing my project of developing Conceptual Place Value intervention curriculum for upper elementary students.

CHAPTER THREE: METHODS

Overview

My project is developing Conceptual Place Value intervention curriculum for upper elementary students. Conceptual Place Value (CPV) is the ability to flexibly increment (add) and decrement (subtract) numbers mentally (Wright, Ellemor-Collins, & Tabor, 2012). I have designed this curriculum to improve CPV instruction within my school. From conversations with my peers, the resources that were provided previously were not implemented as regularly and with the fidelity as intended. With my project, I created lesson plans in which all educators can implement with ease. With researched based lessons, students will increase in their CPV levels. Within this chapter, I will explain the need for this curriculum. A detailed description of the project will be presented. Then I will illustrate the environment that this project is intended for. Lastly, assessment criteria and improvement plan will be presented.

Project Background

CPV understandings are Common Core standards through second grade. After second grade, standards assume that students have mastery of CPV concepts. However, not all students have mastered CPV concepts and need additional support in these areas. Math curriculum follows the standards therefore CPV concepts are not practiced with regularity. With this in mind, educators need to supplement their curriculum. Supplemental resources are available online through multiple websites, both paid and unpaid options; however, these resources may not be researched based. Greene (2016) comments about the rise of various websites that sell lesson plans that are created by teachers, and the possible pitfalls arising from them. A pitfall is when educators rely on these type of websites, instruction can become choppy and incoherent. The intervention lesson plans that I created have been designed to be cohesive from one lesson to the other. While the lessons may be cute and fun, some may be culturally inappropriate and/ or not best practice. These lesson plans I created were done so by using scholarly resources to ensure they are best practice and relevant.

Learning math is best when presented by a knowledgeable instructor. The more understanding that an educator has about the concept they are teaching the better they will be able to teach the concept thereby increasing student success. Many educators have limited understandings of mathematical concepts (Ball, Hill and Bass, 2005). With this knowledge, educators must use well researched materials to support student learning until they become more knowledgeable about CPV.

I used UbD planning format to create the lesson plans for this project. UbD is a format in which an educator backwards plans to ensure that all lessons are linked to the overarching goal. When using UbD, an educator starts with establishing a goal or goals. Then they are asked to find the understanding that are needed to reach the goal, create an essential question that students are working towards answering and create *students will know* statements to insure understanding. When the first steps are completed, an educator creates performance tasks to evaluate if the students are understanding the instruction and plan for what other evidence will be collected along the way. After all previous steps have been taken; learning activities are then created (Wiggins &McTighe, 2003). Using UbD framework focuses on true understanding of the overarching goal. I chose to use this framework because of the focus on students create deeper understandings rather than memorization or surface level knowledge.

Project description

Within this project, I created three week lesson plans for each of the CPV levels excluding level 8. Below are descriptions of the various levels of CPV, along with examples of what how a student might understand

Emergent level- Students are struggling with unitizing tens and ones. They do not see 43 as four groups of ten and three ones. (Masloski, 2016)

Level One- Students at this stage understand that two digit numbers are made up of tens and ones, however they struggle with incrementing (adding) and decrementing (subtracting) tens and ones flexibly. This is not to say they are unable to do one or the other. Students would be able to start at 57 and be able to add and subtract groups of tens from that number. Their struggle here would be when an educator asks them to switch between then tens and ones. (Masloski, 2016)

Level two- At this level, students are able to work with tens and one flexibly, but they are struggling with adding from a decade number (multiple of ten) and subtracting to a decade number. For example, adding to a decade number would entail a student to add 70+8 mentally without using a counting strategy. An example of subtracting to a decade is 97-7. At this stage educators want to create a derived fact for them (Masloski, 2016).

Level three - At this level, students are able to add from a decade number and subtract to a decade number. Students are struggling with adding to a decade and subtracting from the decade. For example student will need to be able to solve problems like 56+ __=60 as well as 70-7=___. At this stage, we are asking students to use their ten facts flexibly and in higher number groups (Masloski, 2016).

Level four- Student are struggling with adding to and subtracting through a decade number. For example, when adding 45+7, we want them to be able to break the seven into five and two so they add by doing 45+5→50+2→52. When subtracting, we want them to break the numbers apart as well. For example 83-6 students are working on breaking apart the six into two threes so they are able to do 83-3→80-3→77 (Masloski, 2016).

Level five- Students are struggling with adding and subtracting with tens and ones from a decade number. At this level, students are working on jumping tens and then ones. For example, 40+____= 73. They need to do 40+30→70+3→73 then finding the answer is 33 from those jumps. An example of subtraction is 52-____= 30. They need to do 52-20→32-2→30 then finding the answer 22 from those jumps as well. With this level, they have to keep track of multiple jumps and knowing what they are trying to find. A hang up at this level for some students is that when they end at the final answer (73 and 30 respectively, from the above examples) they assume that is the number that they needed to find not the jumps that they made (Masloski, 2016).

*Level Si*x- students are struggling with flexible mental strategies with two digit additions and subtraction with no materials. Students need to be able to use more than a single way to find answers. For example, when adding 56+37 they would be able to do

50+30= 80, 6+7=13, and 80+13=93 and find another way to add such as *56+30→86+7→93* (Masloski, 2016).

Level 7- students are struggling with flexible mental strategies with three digit additions and subtraction with no materials. Similar level six, students need to think about numbers more flexibly. For example, 352+225 student would be able to do *300+200=500, 50+20=70 and 5+2=7. 500+70+7= 577.* Another strategy they could use would be rounding 352 to 350 then *350+225→575+2→577* (Masloski, 2016).

Level eight- mastery of all of the aforementioned concepts (Masloski, 2016).

I used UbD planning format to create the lesson plans for this project. UbD is a format in which an educator backwards plans to ensure that all lessons are linked to the overarching goal. When using UbD, an educator starts with establishing a goal or goals. Then they are asked to find the understanding that are needed to reach the goal, create an essential question that students are working towards answering and create *students will know* statements to insure understanding. When the first steps are completed, an educator may create performance tasks to evaluate if the students are understanding the instruction and plan for what other evidence will be collected along the way. After all previous steps have been taken; learning activities are then created (Wiggins &McTighe, 2003). Using UbD framework focuses on true understanding of the overarching goal. I chose to use this framework because of the focus on students create deeper understandings rather than memorization or surface level knowledge.

These lessons plans are designed to be used 5 days a week, for 15 minutes each day. Lessons should not be presented as whole group instruction, but in groups of three to six students at a time. Each Level will have overarching learning objectives, lesson objectives and formative assessments. Students should be tested using the CPV assessment (See appendix A) and placed in groups with students of like scores. If students show mastery of the level they are on during formative assessment, educator should retest student on CPV assessment. The educator will then move student out of the intervention group and student will start lesson one on the next CPV level.

Participation

My intended audience for this project is educators of students in the third through fifth grade. I created these lesson plans for the students at the school in which I currently teach at in mind. I teach at a school in the Midwest in which 57% of the population is Asian, 32% African American, 4% Hispanic, 1% American Indian, 2% white, and 2% two or more races. The majority of the Asian students are Hmong and the African American students include many Somali. My school is comprised of 49.5% English language learners, 18.7% special education students, 92% of our families qualify for free and reduced price lunches and 10.9% of our students are homeless. The intent for this project is to be used for a classroom educator to be able to provide interventions with small groups of student during their math block. Within my school, 1% of the third through eighth grade students tested at the emergent level, 22.7% tested at level one, 16.4% tested at level two, 15.3% tested at a level three, 15.9% tested at a level four, 2.2% tested at a level five, 24.4% tested at a level six, 4.5% tested at a level seven and 6.8% tested at a level eight. I have created the lesson plans to move all these students through the multiple levels of CPV.

Assessment

To determine if my project is effective, I will ask educators to complete a one question survey on how often they are using the lesson plans with their students. If it shows that a majority of teachers are not using the curriculum, I will have a second survey to determine the cause of lack of use then change the curriculum accordingly. I will also look at the CPV school data and look for correlations between classrooms in which the CPV lessons were implemented and growth of students' achievement. If in classrooms where teachers are using the CPV curriculum are growing at the same rate of classrooms in which are not using the curriculum, I will examine the lesson plans with peers and improve the lessons.

Conclusion

Within chapter 3, the reasons why this projected is needed within my school is explained. The percentage of students who are below or well below grade level in the CPV understanding is worrisome. Third through fifth grade educators are already supplementing their curriculum to meet these student's needs. However, due to many educators having limited understandings themselves (Ball, Hill and Bass, 2005), the supplements gathered may not be best practice or appropriate (Greene, 2016). I put forth a detailed explanation of how I intended to create my project along with a detailed description on how I will examine if this project is a success.

Within chapter 4, I reflect on my experiences creating this project and I will reflect on my learning from the literature. I will state the implications that my project has. Then I will delve into the benefits as well as the limitation of my project. Next steps on how I will share this project with the education community and suggestions for future research will be provided.

CHAPTER FOUR: CONCLUSION

For my project, I developed Conceptual Place Value intervention curriculum for upper elementary students. Conceptual Place Value (CPV) is the ability to flexibly increment (add) and decrement (subtract) numbers mentally (Wright, Ellemor-Collins, & Tabor, 2012). Throughout this chapter I will reflect on my learnings throughout the process of creating the curriculum and review the literature I used to create my project. I will also talk about the implication and limitations that my project has. Lastly, I will talk about what this project means for me and the professional community.

Reflection on the Process

When looking back on my time creating this project, I have concerns about the state of math instruction in the United States, but I also have hope as well. Throughout the researching and collecting literature process, I was told to keep narrowing down my topic and search words to help me find useable information for my project. I was told that if I did not narrow it down, I would be drowning in text and much would not be relevant to what I was trying to accomplish. For this project, I did not find this to be the case. Many of the times that I went looking for information on my topic, I only got a few with the key words that I was looking for. I would have to think of multiple ways to find information on my topic. I would also have to look for information that was similar to my question. When entering this project, I had a vast amount of knowledge from receiving my math certificate which helped me with being able to find resources that I would be able to use.

While looking for research that connected with my project was difficult; it also gave me hope for the future of mathematics teaching. Article after article insisted that rote memorization was not the way to teach math any more. All the articles stated that creating a conceptual understanding of the topics will lead to students being successful and more prepared in the future. While there is not as much information and research out there about mathematic teaching, the research that is out there is consistently saying that conceptual understanding is the way that it should be taught.

Reflection on the Literature

Conceptual understanding of what is happening in a problem is fundamental to students' future success in mathematics. Throughout the articles, they all insisted that understanding what and why they were doing a step in a problem helped them transfer their knowledge to other situations. As educators, we need to make sure that their knowledge will transfer to ensure that they are success as adults. We are currently preparing them for jobs that haven't even been invented yet. The ability to adapt and transfer is integral for the students to be successful.

While I used many sources to create my project, the ones that I found most impactful were *Childrens mathematics: Cognitively Guided Instruction* (Carpenter, Fennema, Franke, Levi, & Empson, 2015), *Developing number knowledge: Assessment, teaching and intervention with 7-11 year olds* (Wright, Tabor, & Ellemor-Collins, 2012) *and Teaching Number in the Classroom with 4-8 Year Olds. (*Wright, Stanger, Stafford, & Martland, 2006).

When writing the openings of the lesson plans, I wanted to start each session with a word problem. Students need to see how they can use the mathematical knowledge in their day to day lives; Using word problems does just that. Students are able to see math used in their day to day lives they are more invested in the lesson itself. Carpenter's text goes into detail of all the different ways that word problems can be presented. As adults, we are able to see if a problem is an addition or subtraction problem right off the bat, but students are not always able to see that. Students tend to see each type of word problem as a completely different problem that needs a completely different way of solving it. When students are exposed to all the different types of problems, they are able to see the connections between the different types of addition and subtraction problems. This leads to a deeper understanding which will in turn allow them to transfer knowledge in different situations.

The Wrightbooks go extensively into the scope and sequence of mathematical development in children. These books show from the early stages of development with subitizing and one to one correlation all the way to multiplication and division. They also include examples of students thinking, along with ways to progress them into the next level. When writing the main body of my lesson plans, I focused on these texts to ensure that they were following the same progression.

Implications

When designing this curriculum, I created it to be able to use for as tier two intervention to be used by a math interventionist educators to use during pull-out groups. While I am aware that not every school has a math interventionist, the hope is that will soon change. The way this curriculum is set up, a classroom educator will be able to use it for small group instruction time.

Due to cognitive development stages and the fact that it is delivered in a small group setting, students should be able to move through the CPV levels at a relatively quick pace. However, if students are not making progress through this curriculum, students should be moved to a one on one intervention or referred to special education services.

Benefits

This project is designed to help students who are struggling in mathematics at a fundamental level. While some children are able to be successful with only an algorithmic understanding of mathematical concepts, many children are not. Conceptual understanding helps students understand why the algorithm way works and will help them transfer their knowledge into other areas.

Additionally, when paired with ongoing PD this will help educators improve their mathematical presentation and understandings as well. It has been my experience that many educators struggle with conceptual understanding when it comes to complex addition and subtraction problems. With continuing PD and this curriculum, I believe this will help educators grow in their practice and their mathematical abilities. When students are able to understand the material on a more in-depth level, they are able to educate their students more effectively.

Limitations

This curriculum is designed to be used by an educator who has some background in Cognitively Guided Instruction and some understanding on the different types of strategies that students might discover during conversations. Educators who are only use to teaching students the algorithmic way of solving problems will struggle with implementing this curriculum to its fullest potential. Educators also need to know how to lead a mathematical discussion with a group of students.

In relation to having an educator who understand how to effectively teach mathematics, educators must have access to the math manipulatives. Hands on learning and exploration is integral to gaining a deeper understanding in any subject. Without manipulatives, these lessons will not be usable or effective.

This curriculum is designed to be used as a small group instructions for third through fifth grade students. Much of the progress, even more in the higher levels depend on conversation between students. While lessons could be adapted to a whole group setting, students will not show as much progress as they would in a small group setting. Additionally, students who are younger than third grade will not be able to move through the levels as quickly as the older children will.

Sharing

When thinking about how I will share my findings in this project, I have always thought about sharing the final result with my fellow educators. Within my school, we had a push over the past few years for implementing conceptual place value lessons within our instruction, however professional development (PD) within this area has been limited and poorly attended. My hope with this project is to provide my fellow coworkers and educators a starting point for their small group instruction to be paired with ongoing CPV PD.

Suggestions for Ongoing Research

When thinking about the future research in relation to CPV education, there are areas that could be improved. As I stated earlier, when looking for research on CPV topics, it was hard to find research out there. More in-depth research in the elementary math field is necessary. While many schools may be doing data cycles or action research projects on this topic, scholarly level data is needed to improve instruction on a national scale.

Additionally, conceptual understanding of multiplication and division is needed for students to be successful. For this project, I created a curriculum to help students who are below grade level in addition and subtraction. Many of those students also struggle with multiplication and division problems. Like addition and subtraction, for years' educators have push rote memorization with multiplication and division fact. While this strategy may work for some students, it is not what is best practice. Many educators struggle with understanding multiplication and division conceptually so additional PD and accompanying curriculum is necessary for students to be successful.

Conclusion

When reflecting on my experience with creating this project, I have mixed feelings about the field of teaching mathematics. I am frustrated that there is not more research out there. Thinking about what an educator is expected to teach, reading, writing and arithmetic are the largest part of the equation. While there is, what feels like endless amount, of research on reading and writing, math research is much harder to come by. With math being such a large part of the day, and of testing, it is surprising that this is the case.

This project also gave me hope for the future of math instruction. When looking at the content of the research, they all insisted that conceptual understanding of mathematics was important to student success. With the consistent message, future research will follow proving that conceptual teaching practices are more effective than rote memorization. Additionally, more scholarly articles and research has been coming out within the past 10 years and the common core standards push conceptual understanding in mathematics. This leads me to believe that more research is not far behind. When more research is available and when there is more focus on mathematic instruction, improved teaching and learning is not far behind.

REFERENCES

Ambrose, R., Baek, J., Carpenter, T.P., (2003). Children's invention of multiplication and division algorithms. In A. Baroody and A. Dowker (Eds.) *The development of arithmetic concepts and skills: Recent research and theory.* Mahwah, NJ: Erlbaum.

Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator,* Fall 2005, 14-46.

Brickwedde, J. (2012a). Learning the basic facts: Opportunities to develop important mathematical understandings. Minneapolis, MN: Project for Elementary Mathematics.

Brickwedde, J. (2012b). Children's development of place value and base ten understanding: Building a multiplicative rate of ten. Minneapolis, MN: Project for Elementary Mathematics.

Carpenter, T.P., Ansell, E., Franke, M.L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education, 24*(5), 428-441.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (1999). *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Carpenter, T.P. & Franke, M.L. (1998). Teachers as learners. *Principled Practice in Mathematics & Science Education. 2* (2), 1 – 3. Madison, WI: National Center for Improving Student Learning & Achievement in Mathematics & Science.

Carpenter, T.P. & Franke, M.L., Jacobs, V.R., Fennema, E., and Empson, S.B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal from Research in Mathematics Education, 29*(1), 3-20.

Carpenter, T.P., Franke, M.L. & Levi, L. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School.* Portsmouth, NH: Heinemann.

Chan, W. W. L., Au, T. K., & Tang, J. (2014). Strategic counting: A novel assessment of place-value understanding. *Learning and Instruction, 29*, 78-94.

Clements, D.H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics, 5*(7), 400-405.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Oliver, A., and Human, P. (1997). Chapter 1: Introducing the critical features of classrooms. *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Oliver, A., and Human, P. (1997). Chapter 3: The role of the teacher. *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Graven, M., Stott, D., Mofu, Z., & Ndongeni, S. (2015). Identifying stages of numeracy proficiency to enable remediation of foundational knowledge using the learning framework in number. Paper presented at the *Proceedings at the 23rd Annual Conference of the Southern African Associafion for Research in Mathemafics, Science and Technology Educafion in Maputo, Mozambique,* 13-16.

Greene, K., kim.greene@ascd.org. (2016). FOR SALE your lesson plans. *Educational Leadership, 74*(2), 28-33. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=eft&AN=118666290&site=ehost-live

Jacobs, V.R. & Ambrose, R.C. (2008). Making the most of story problems. *Teaching Children Mathematics, 15*(5), 260-266.

Jacobs, V.R., Ambrose, R.C., Clement, L., & Brown, D. (2006). Using teacher-produced videotapes of student interviews as discussion catalysts. *Teaching Children Mathematics, 13*(5), 276-281).

Jacobs, V.R., Lamb, L.L.C., & Phillipp, R.A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41* (2), 169-202.

Jacobs, V.R. & Phillipp, R.A. (2010). Supporting children's problem solving. *Teaching Children Mathematics, 17*(2), 99-105.

Kamii, C. & Rummelsburg, J. (2008). Arithmetic for first graders lacking number concepts. *Teaching Children Mathematics, 14*(7), 389-394

McLeod, S. A. (2017). Maslow's hierarchy of needs. Retrieved from www.simplypsychology.org/maslow.html

Michel, L. (2015). *Supporting development of mathematics teaching practices in the context of the common core: An action research model of professional development for upper elementary teachers* (Ed.D.). Available from ProQuest Dissertations & Theses Global. (1698504816).

Mononen, R., & Aunio, P. (2014). A mathematics intervention for low-performing finnish second graders: Findings from a pilot study. *European Journal of Special Needs Education, 29*(4), 457-473. doi:10.1080/08856257.2014.922794

National Research Council (2001). Chapter 4: The strands of mathematical proficiency. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.) *Adding it up: Helping children learn mathematics*, pp. 115 – 155, Washington, DC: National Academies Press.

O'Connell, S., & SanGiovanni, J. (2011). Mastering the basic math facts in addition and subtraction : Strategies, activities & interventions to move students beyond memorization / Susan O'Connell and John SanGiovanni. Portsmouth, NH: Heinemann.

Sun, W. & Zhang, J.Y. (2001). Teaching addition and subtraction facts: A Chinese perspective. *Teaching Children Mathematics, 8*(1), 28-31.

Rogers, A. (2012). Steps in developing a quality whole number place value assessment for years 3-6: Unmasking the" experts". *Mathematics Education Research Group of Australasia,*

Royea, D. A. (2012). No title. *Effects of Lesson Sequencing on Preservice Teachers' Mathematical Knowledge of Place-Value,*

Shavelson, R. J., Young, D. B., Ayala, C. C., Brandon, P. R., Furtak, E. M., Ruiz-Primo, M., . . . Yin, Y. (2008). On the impact of curriculum-embedded formative assessment on learning: A collaboration between curriculum and assessment developers. *Applied Measurement in Education, 21*(4), 295-314. doi:10.1080/08957340802347647

Stott, D., & Graven, M. (2013). Quantifying qualitative numeracy interview data. Paper presented at the *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa, , 1* 194-208.

Thouless, H. (2014). No title. *Whole-Number Place-Value Understanding of Students with Learning Disabilities,*

Thouless, H. R. (2014). *Whole-number place-value understanding of students with learning disabilities* (Ph.D.). Available from ProQuest Dissertations & Theses Global. (1530195709).

Wiggins, G. P., & McTighe, J. (2008). *Understanding by design*. Alexandria, VA: Association for Supervision and Curriculum Development.

Weitz, M., & Venkat, H. (2017). Using assessments to explore early and later performance in mathematics. *Improving primary mathematics education, teaching and learning* (pp. 27-43) Springer.

Wells, P.J. & Coffey, D.C. (2005). Are they wrong: Or did they just answer a different question? *Teaching Children Mathematics, 12*(4), 202-207.

Woolfolk, A. (2014). *Educational psychology*. Harlow, England: Pearson Education.

Wright, R. J., Ellemor-Collins, D., & Lewis, G. (2007). Developing pedagogical tools for intervention: Approach, methodology, and an experimental framework. *Mathematics: Essential Research, Essential Practice, 2*, 843-851.

Wright, R. J., Tabor, P. D., & Ellemor-Collins, D. (2012). *Developing number knowledge: Assessment, teaching and intervention with 7-11 year olds*. Los Angeles: Sage.

Wright, R. J. (2006). *Teaching number: Advancing childrens skills and strategies*. London: Chapman.

Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2015). *Teaching number in the classroom: With 4-8 year olds*. Thousand Oaks, CA: Sage Publications.

Wright, R. J., Ellemor-Collins, D., & Tabor, P. D. (2011). *Developing number knowledge: Assessment, teaching and intervention with 7-11 year olds. math recovery*