Spring 5-7-2016

Twelve Best Practices for Mathematics Vocabulary Instruction for K-5 Elementary Students

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TWELVE BEST PRACTICES FOR MATHEMATICS VOCABULARY INSTRUCTION FOR K-5 ELEMENTARY STUDENTS

by

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A capstone submitted in partial fulfillment of the
requirements for the degree of Master of Arts in Teaching

Hamline University

Saint Paul, Minnesota

May 2016

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Dedicated to my loving Father God for giving me the time, resources, and brain to do this; to my family: my parents, Jim & Mary McNeil, for always believing in me, encouraging me to form good study habits and helping me pay for my schooling; my husband, Jared, for giving me the inspiration to become a teacher, for patience as I completed this, for entertaining the children countless nights, and for fueling my ideas for research; and my children, Maggie and Duncan, in hope that they accomplish even greater things in life!

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CHAPTER ONE

Introduction

Introduction and Primary Question

Vocabulary knowledge that is plentiful and comprehensible has been widely accepted as important to developing mathematical concepts (Monroe & Panchyshyn, 2005). It is important to ensure students are developing mathematical concepts through rich vocabulary. Using research-based methods to help elementary students be successful in their mathematical abilities is an area that needs to be constantly revisited because of changes in technology advances and instructional resources available to teachers and students. In light of this, my primary research question asks, what are the current best practices for mathematics vocabulary instruction for K-5 elementary students?

In order to understand the importance of this question, the development of this interest and rationale for this research needs to be told. Interest in this question first came to light during my second year of teaching, and continued to be an area of concern in following years. As I tried to develop my own strategies to confront the issue of mathematics vocabulary in my students, I realized the complexity of this issue and wanted to to more research to better meet the needs of my students.

Background

During the 2009 - 2010 school year, I began teaching a fractions unit to my first grade class. After a week into the unit, one of my students asked, “Mrs. King, what’s a denominator?” I realized that even though I had been using this vocabulary in my teaching, I had not explained the definition clearly or frequently enough. I began to realize that my students had a huge deficit when it came to mathematical vocabulary. This proved to be true after looking at their in-class assessments as well as standardized test scores. I began to wonder, “How are my students supposed to do well on these tests if they do not even know what the questions are asking?” Thus began my search for the current best practices for mathematical vocabulary instruction for K-5 elementary students.

In that teaching year of 2010-2011, I started with a bulletin board in my room dedicated to mathematics. On the board I created a word wall for mathematical vocabulary terms and concepts, complete with definitions and pictures as well as a problem-solving strategies poster. I noticed right away that students were referring to it often, as was I, during the teaching of each lesson. I then began to use the word wall as a review at the start of each daily lesson. I found students using the mathematical vocabulary more often and in the correct context when asking questions in class or discussing a concept with a partner. I began to notice an increase in understanding, made evident on in-class

assessments, both formally and informally. In the spring, I noticed a huge improvement on their standardized test scores as well. “Success!” I thought.

I continued this teaching strategy throughout the 2011-2012 school year. As I moved up to teach third grade the following year, I made my word wall even more elaborate.

However, as I began the 2013-2014 school year, I joined my school’s accreditation committee for mathematics, and again, this problem of mathematical vocabulary presented itself within our goals. I realized that mathematical vocabulary comprehension was not only a problem I had witnessed in my class in my first few years of teaching, but this is also a problem that many elementary teachers and students struggle with.

Context

My school, a private international school in Europe is working to achieve accreditation in mathematics.

Accreditation and Professional Development. Accreditation is validation by an impartial third party that an educational institution has integrity by meeting higher standards and working towards continuous improvement of the school. This is important to my school because it shows the public the school is prestigious, valuable, and reliable in their quality of education. One aspect of achieving this accreditation at my school includes developing a professional development session on best practices for mathematics vocabulary instruction for teachers to use in future mathematics lessons.

At my school, professional development sessions are offered twice a year, once in the fall and once the spring. My school uses a traditional model for professional development. This type of professional development is the most common, therefore called “traditional”. Traditional models can include lectures, conference sessions, short-term or one session workshops, or trainings. One disadvantage to this approach is that the sessions can vary in quality from year-to-year and are different depending on the person conducting the session (Porter, Garet, Desimone, Yoon, & Birman, 2000). At my school, most professional development sessions are in the workshop or lecture format. Professional development sessions are conducted by fellow teachers within the organization, and can be on whatever topic the teacher would like to present on, given that the administration approves the topic. Topics and presenters vary from year-to-year.

Identification of areas for improvement. As an accreditation committee, the faculty and teachers on my school’s mathematics accreditation committee were presented with the goal of figuring out how to help students achieve improvement in the area of mathematics. We identified two areas within mathematics education that are most problematic for students: problem-solving strategies and strategies for mathematics vocabulary instruction.

Process for Improvement. As my accreditation committee focused specifically on current best practices for mathematics vocabulary instruction to help students achieve improvement, three best practices emerged, which will be

discussed in the next paragraph. In order to find these best practices, we used three processes: (1) we discussed teaching strategies we had used in our classrooms, those that were effective, and those that were not as effective; (2) we discussed current strategies that we had been recently introduced to in professional development sessions or professional journals that we either had not had time to yet implement, or had only heard about briefly but did not know enough to use it in our classrooms; and (3) we looked at research via internet browsers, but did not have access to an academic library source.

Pre-Research on Current Best-Practices for Mathematics Vocabulary Instruction

The following three paragraphs discuss the findings from my school's committee of teachers on current best practices for mathematics vocabulary instruction for K-5 elementary students.

Strategy one: word wall. The first strategy, found by the mathematics accreditation committee at my school, was having a word wall posted in the classroom with Tier 2 and Tier 3 words (Beck & McKeown, 2007 and Beck et al., 2013). Tier 1 words are classified as the most basic words: "*warm, dog, tired, run, talk, party, swim, look, and so on*" (Beck et al., 2013, p. 8). These words usually have a high frequency because they are used in daily oral conversations, and so these words do not usually need to be directly taught. Tier 3 words are used only in specific topics and domains, so they are not as familiar and need to be directly taught. Examples of these words are: "*filibuster, pantheon, and epidermis*" (Beck et al., 2013, p. 9). These words may not be utilized often but

can be introduced as needed to understand content. Tier 2 words are utilized often and across a variety of subjects, such as: "*contradict, circumstances, precede, auspicious, fervent, and retrospect*" (Beck et al., 2013, p. 9) . Since these words are used more often in writing and less often in conversation, they are less known and need to be more directly taught. Instruction at this tier can have a huge effect on verbal skills because of the great impact they have on a language user's repertoire. Because of this, focusing on Tier 2 words will be most productive. In each lesson, teachers should identify new terms and anticipate any problematic vocabulary (which may be Tier 1, 2 or 3) or symbols, which will in turn have a positive impact on reading comprehension (Beck, Perfetti & McKeown 1982; McKeown, 1993; Monroe & Panchyshyn, 1995).

Strategy two: building student confidence. The second strategy, also found by the mathematics accreditation committee at my school, is one that Hattie (2009) concludes is the greatest predictor of student success on an assessment. Building student confidence (or self-reporting grades, as labeled and researched by John Hattie (2002, 2012)), involves the teacher finding out the student's expectations and pushing the learner to exceed these expectations. The teacher should build students' confidence to help them see more of themselves by asking students: "What grade do you think you will get on this test?" and "Why do you think that?" Hattie found students can predict their grade better than the teacher or anyone else, much like a self-fulfilling prophecy. It is the teacher's job to help them figure out why they have success (it is not luck, it is because you studied)

by asking students such questions as: “What did you do to get a good grade? What did you do at home? What did you do at school?” and “What do you need to do differently next time?” Teaching students self-talk and the skills they need (or having other successful students talk about what they did to get a good grade or get the right answer) will build this confidence. It is the teacher’s job to find out what the students’ expectations are and then push them beyond these expectations. After a student achieves success at this new level, they gain confidence in their ability to learn and do well on assessments.

Strategy three: twelve powerful words. The third strategy, also found by the mathematics accreditation committee at my school, is student knowledge of the Twelve Powerful Words and their definitions (as researched and defined by Bell (2005)). Bell (2005) found that there were twelve words used on most standardized tests. Students were not being overtly tested on these words and their definitions, but it was crucial that students knew what these words meant in order to decipher the meaning of the question. In other words, they may know the answer to the question, but not understand what the question is asking and therefore get the incorrect answer, or no answer at all because they are so confused. Bell’s Twelve Powerful Words and their meanings are listed in Appendix B.

Primary Question

Despite these three best practices that emerged from the research of the mathematics accreditation committee at my school, I felt our search was limited

and I wanted to search for best practices from deeper academic sources. So I began my research with the primary question in mind: what are the current best practices for mathematics vocabulary instruction for K-5 elementary students?

Summary

It is important to ensure that students are developing mathematical concepts through vocabulary. In my teaching experience, I have realized that most students struggle with mathematics vocabulary not only in the classroom but on assessments as well. At my school, a committee of math teachers found three strategies to help students achieve improvement in their understanding of mathematics vocabulary. These strategies were having a word wall posted in the classroom with tier 2 and tier 3 words; building student confidence by predicting their own grades; and student knowledge of the Twelve Powerful Words and definitions as researched by Larry Bell (2005). Beyond these strategies, there are more which this paper seeks to research and more completely answer the question: what are the current best practices for mathematics vocabulary instruction for K-5 elementary students? By using best practices for mathematics vocabulary instruction, we can help students increase success in mathematics in the classroom and in the years that follow elementary school. This research will aid teachers in helping students to have increased confidence in and understanding of mathematical concepts and vocabulary.

This will be the basis from which I move forward to analyze and synthesize current research in order to gain insight into my primary research question, what are the current best practices for mathematics vocabulary instruction for K-5 elementary students? In the next section, I will explore the research on challenges students face in learning mathematics vocabulary. When teachers are aware of what challenges are present for students, they can adjust their teaching strategies to meet the needs of their learners and help students be successful at learning the complicated language of mathematics. Finally, I will propose twelve current best practices for mathematics vocabulary instruction for K-5 elementary students.

CHAPTER TWO

Literature Review

Introduction

The purpose of this study is to educate and update K-5 elementary mathematics educators on best practices for instructing their students in learning mathematics vocabulary. It can be assumed that educators have the best intentions when working with elementary students to understand mathematics and that they strongly desire for their students to succeed. Despite this desire, many educators still struggle with teaching students to apply mathematical concepts and communicate in mathematical language. “Math [is a] language that must be meaningful to students if they are going to communicate and apply it productively” (NCTM, 1989). Educators desire to make mathematics meaningful for students, but are not aware of how to do so. While the curriculum they are using in their classroom is written by highly educated and qualified individuals who have done their research, and certainly helps the student to learn mathematics in many ways, teaching is also about helping students think beyond test questions and practice problems given in a textbook, and extending and applying mathematical concepts to real-world situations. If teachers were to use current best practices for mathematics vocabulary instruction, students' learning

may also improve. Therefore, my primary research question asks what are the current best practices for mathematics vocabulary instruction for K-5 elementary students?

The intent of this study is to research and summarize current best-practices for mathematics vocabulary instruction for K-5 elementary students. My primary research question asks what are the current best practices for mathematics vocabulary instruction for K-5 elementary students? Future research could then look at how using these teaching practices could also impact students ability to extend and apply mathematical concepts to real-world situations and how students' standardized test scores are affected.

In the next section, I will explore the research on challenges students face in learning mathematics vocabulary. When teachers are aware of what challenges are present for students, they can adjust their teaching strategies to meet the needs of their learners and help students be successful at learning the complicated language of mathematics. Finally, I will propose twelve current best practices for mathematics vocabulary instruction for K-5 elementary students.

Three criteria were used in narrowing my research to apply best-practice teaching strategies to the population being studied. The first criterion is that strategies must apply to the correct age group, Kindergarten through 5th grade elementary students. The second criterion is that strategies must be for all students, not a special population, such as gifted and talented, or a special class,

such as an honors course. The third criterion is that strategies were proven by research to increase knowledge and comprehension of mathematics vocabulary.

Challenges Students Face in Learning Mathematics Vocabulary

"The essence of mathematics is not to make simple things complicated, but to make complicated things simple" (Gudder, xi). However, complications still exist. In exploring research on challenges students face when learning mathematics vocabulary, five challenges emerged.

Challenge one: reading comprehension and fluency. The first, and most clearly identified challenge to learning and understanding mathematics vocabulary is in the area of reading comprehension. "Effective reading in math is closely associated with the development of mathematics skills and concepts" (Schell, 1982). Many students struggle with reading comprehension. When students are struggling with reading comprehension in general, this difficulty will carry over to any subject, including mathematics (Aiken, 1973; Gagnon & Macini, 2001; Garbe, 1985; Gay & White, 2002; Milligan, 1983; NCTM, 1989 and 2000; Schell, 1982; Thompson & Rubenstein 2000). As Hillard (1924) and Mezynski (1983) found, a strong relationship exists between a student's level of reading comprehension and their vocabulary knowledge. Students struggling with reading comprehension suddenly have less contextual clues to help decode the meaning of new vocabulary words (Reehm & Long, 1996). Teachers intuitively know that if a student does not understand the vocabulary, they will not understand what they are reading. As McKeown, Beck, Omanson & Perfetti

(1983) state, "a difference exists between acquiring knowledge of a word's meaning and knowing the word well enough to aid comprehension of text." There are many different factors that make reading difficult for students including student readiness for reading mathematics, student interest and motivation, and student reading level. Teachers need to make sure that effective strategies for vocabulary instruction is one of their top priorities (Johnson, Pittelman & Heimlich, 1986).

In order to communicate mathematical ideas and concepts, students also need to be successful in fluency with mathematical terms, symbols, and words (Rubenstein & Thompson, 2002). Students must know extensive content knowledge before they are able to meaningfully interpret mathematical words and symbols (Schell, 1982). According to Pierce and Fontaine (2009), “the depth and breadth of a child’s mathematical vocabulary is more likely than ever to influence a child’s success in math” (239).

Challenge two: double meanings of words. The second challenge of mathematics vocabulary is that words can have double meanings. “Students' knowledge of word meanings is widely agreed upon as a significant factor in their success in comprehension” (Curtis, 1983). Words used in mathematics can have different meanings when they are used in other subjects. Words that are not used frequently, or used more often in writing than oral conversation, usually have more than one meaning, and the meanings vary from one context (or subject area) to another (Aiken, 1973; Monroe and Panchyshyn, 1995). These

words are what Beck et al. (2013) would classify as Tier 2 words. To clarify this challenge, we can look at Monroe and Panchyshyn's (1995) following four examples:

Word	Common Definition	Mathematical Definition
key	a tool with which the bolt of a lock is turned	something that gives an explanation or provides a solution
order	to command to do something	to put into sequence
another way	a different course of action	a way of equal value
table	a piece of furniture with four legs	a visual display of information

These words have importance and utility because they are in grade-level texts and appear frequently across a variety of academic domains. It is important that students understand these words have multiple definitions and know when to use which one.

Challenge three: use of non-alphabetic symbols with different meanings.

The third challenge of mathematics vocabulary is closely related to the second. Not only are there three different tiers of words which are all used in mathematical language, mathematics also uses many non-alphabetic symbols such as #, <, >, %, +, =, and -. The structure in which this mathematics language is presented is different than standard English, making further difficulties. Symbols and and specialized vocabulary are used to explain abstract concepts

instead of narrative language. Often times in mathematics, a student reads from top to bottom instead of left to right as well.

To complicate matters, mathematics symbols can have different meanings than in other contexts (Aiken, 1973; Monroe & Panchychyn, 1995; Pierce & Fontaine, 2009), as in the following four examples: 1) The symbol '-' can mean *minus* but it can also represent a negative value; the symbol '*#*' can stand for *number* or *pounds*. 2) *degrees* can refer to *degrees of temperature* or *degrees of an angle*. 3) '*2*' can be used for the value *two* but it can also mean *squared*. 4) many abbreviations do not logically correspond with the word they represent (such as *lb*. to represents *pounds*).

It is also important for the teacher to remember that different symbols and notations are expressed differently in different cultures (Furner, Yahya, & Duffy, 2005). For example, in America, the comma is used to separate place value and the period is used as a decimal point, as in the value $1,234,567.00. But in most of Europe, the comma is used as a decimal point and a space is used to separate place value as in €1 234 567,00. So if the teacher presents the number 4,000 a European student may read it as the number '*four*' (expressed to the thousandths place value) instead of the number '*four-thousand*'.

Challenge four: vocabulary in word problems. A fourth challenge is that, often times, mathematical assessments will use difficult vocabulary words in word problems to pose a particular problem. The Houghton Mifflin Mathematics Test Prep Practice website (2014) lists example test questions for each grade

level and concepts studied in each chapter. Three examples taken from this website are presented in Appendix A. In these examples, students may know the answer to the question, but may not know some of the vocabulary, and therefore may not understand the question, leading to an incorrect answer. For instance, in question number 3, if students do not know what the word '*represents*' means, they will not understand what the question is asking them to do, even though they know that 7 shaded parts out of 10 is equivalent to the decimal 0.7 (although having a picture decreases that likelihood). In question number 4, there are two challenging words '*swap*' and '*exchange*,' and in question number 7, the word '*unit*' may confuse some students. "Reading mathematical words involves both developing word meanings by using definitions and examining context clues, prefixes, suffixes, and root words, and interpreting words with special meanings" (Schell, 1982).

Challenge five: uncommon vocabulary words. The fifth challenge, as Monroe and Orme (2002) found, is that mathematics has a much higher prevalence of uncommon vocabulary words. When Monroe and Panchychyn (1995) evaluated mathematics textbooks, fifty percent of the words in them were words that were not frequently used in reading materials. Schell (1982, p. 544) concluded "math is the most difficult content area material to read, with more concepts per word, per sentence, and per paragraph than any other [content] area." Words that are specific only to mathematical language and used nowhere else (Beck, et al., 2013; Pierce & Fontaine, 2008; Rubenstein & Thompson,

2002) are classified by Beck, et al. (2013) as Tier 3 words. Tier 3 words are low-frequency words that only have one meaning, but are usually limited to specific content areas. In mathematics, one example would be the word, "*cosine*". There is no other context in which this word is used. The definition of this word only applies to mathematics. Since these low-frequency words are very important for understanding content, they should only be introduced as the need arises.

Twelve Current Best-Practices for Mathematics Vocabulary Instruction

Below are listed twelve current best-practices for mathematics vocabulary instruction for K-5 elementary students.

Strategy one: robust vocabulary. The first, and most important and incorporative strategy is "a robust approach to vocabulary [which] involves directly explaining the meanings of words along with thought-provoking, playful, and interactive follow-up" (Beck et al., 2013, p. 2). For students to deeply process word meanings, students should relate new words to ideas students already know, write sentences discussing how the word usage is different between two subjects, create a word wall (which could be color-coded to correspond to different math categories) with definitions, pictures, uses, and related words created by students, and use picture dictionaries or glossaries (Lee & Patnode, 2007; Monroe & Panchyshyn, 1995; Rubenstein & Thompson, 2002). Strategies two, three, four, five, six, ten, and eleven support this strategy and can be incorporated into it.

Strategy two: creating an interest in words. The second strategy for teaching vocabulary is helping students develop an interest in and awareness of words (Beck et al., 2013). This is one way to increase the ease of student reading, as mentioned in the challenges section. Developing an interest in words can be done through a variety of pre-reading strategies that help students understand the concept(s) behind the word (Furner, Yahya & Duffy, 2005; Johnson, Pittelman & Heimlich, 1986; Lee & Patnode, 2007; Monroe & Panchyshyn, 1995; Rubenstein & Thompson, 2002), such as:

- real objects and demonstration (beans, buttons, marbles, M&Ms®, patterned blocks)
- children's literature related to the concept
- relating words/concepts to prior knowledge and background, interdisciplinary subjects, and cross-cultural backgrounds
- skits, journal entries, and other writing assignments
- writing students' own word problems to exchange with classmates
- field trips (even internet virtual field trips)
- games
- cartoons
- songs, poems and raps
- drawings to help visualize words in word problems
- relating concepts to real-world or everyday life and experiences

Another advantage to this strategy is that students who struggle with written or verbal skills may benefit from cartoons, demonstrations, graphics, and visuals (Blachowicz & Fisher, 2000; Gay & White, 2002; Thompson & Rubenstein, 2000). Students who struggle with abstract concepts may also benefit from the concreteness of some of these strategies and may enjoy using creativity instead of worksheets (Furner, Yahya & Duffy, 2005).

Strategy three: semantic mapping. The third strategy, semantic mapping, is another pre-reading strategy that helps activate and expand prior knowledge through the use of brainstorming. “Semantic mapping has been used successfully by teachers at all levels to motivate and actively involve learners in the thinking-reading-writing processes” (Johnson, Pittelman & Heimlich, 1986). In semantic mapping, students categorize information in a graphic form, similar to a word map or mind map. Johnson, Pittelman & Heimlich (1986) present seven steps for semantic mapping, which can be used in any content area:

1. First, the teacher should choose a word that is at the center of the topic being studied.
2. The teacher should then write the word on a surface visible to the whole class (such as whiteboard, SMART board®, or chart paper). This will be the semantic map that is created as a class.
3. Then, the class should take time to brainstorm aloud words related to the topic word and list these words in categories on the whole class surface.

4. Students should then take a few minutes to think, on their own, of as many words as they can related to the topic word and write them down.
5. Students should then share aloud the words they wrote down and the teacher should add them to the class map.
6. As the class map now has a central topic word with many words surrounding in, listed in categories, the students should now suggest labels for each category.
7. The class should then discuss the words on the class map. Students should observe new words, find new meanings to old words, and make connections between new and old words.

In this way, students are “required to relate new words to their own experiences and prior knowledge” (Johnson, Pittelman & Heimlich, 1986). This categorical structuring of new and old words in a graphic form can help students see specific relationships among words, help students expand their use for known words, and understand meanings and uses of new words. In this way, students are better able to understand what they read and evaluate the information in their reading. This is especially true during the last phase of semantic mapping, in which the class discusses the words. The teacher can also check comprehension at this time. Johnson and Johnson (1986) found that semantic mapping also helps students to develop an interest in and awareness of words. When complete, semantic maps can also reveal what information students know and help the teacher to see where new concepts can be taught (Johnson, Pittelman &

Heimlich, 1986). Semantic mapping can be used as a post-reading strategy in addition to a pre-reading strategy (see Appendix C and D for examples of both).

During or after reading, students can also add words to their semantic map. Using semantic maps during post-reading is a way to reinforce main ideas and key vocabulary and also helps students to recall, organize, and graphically represent important information from the reading (Johnson, Pittelman & Heimlich, 1986).

Strategy four: introducing new vocabulary by exploration. The fourth strategy is related to how the teacher introduces new vocabulary. Even before the teacher formally introduces a new word or concept, the teacher should give students the opportunity to use materials to explore ideas, explain rationale, and invent their own terminology to build concepts, and then attach the formal vocabulary to these ideas (Rubenstein & Thompson, 2002; Thompson & Rubenstein, 2000). This gives the student a chance to reinforce ideas and understanding before being presented with a new and unfamiliar term.
Once the teacher presents formal vocabulary for these ideas, the teacher should model appropriate vocabulary usage in the correct context with correct pronunciation, spelling, illustrations and several examples (Garbe, 1985; Pierce & Fontaine, 2009; Rubenstein & Thompson, 2002; Stahl, 1998). Lee and Patnode (2007) also believe it is important to use the same terminology as state assessments and standardized tests.

Strategy five: posting tier 2 and tier 3 words on a word wall. The fifth strategy, found by the mathematics accreditation committee at my school, was having a word wall posted in the classroom with Tier 2 and Tier 3 words (Beck & McKeown, 2007 and Beck et al., 2013). Similar to Monroe and Panchyshyn's (1995) classifications, Beck et al. (2013) have classified words into three tiers. Tier 1 words are classified as the most basic words: "*warm, dog, tired, run, talk, party, swim, look, and so on*" (Beck et al., 2013, p. 8). These words usually have a high frequency because they are used in daily oral conversations, and so these words do not usually need to be directly taught. Tier 3 words are used only in specific topics and domains, so they are not as familiar and need to be directly taught. Examples of these words are: "*filibuster, pantheon, and epidermis*" (Beck et al., 2013, p. 9). These words may not be utilized often but can be introduced as needed to understand content. Tier 2 words are utilized often and across a variety of subjects, such as: "*contradict, circumstances, precede, auspicious, fervent, and retrospect*" (Beck et al., 2013, p. 9) . Since these words are used more often in writing and less often in conversation, they are less known and need to be more directly taught. Instruction at this tier can have a huge effect on verbal skills because of the great impact they have on a language user's repertoire. Because of this, focusing on Tier 2 words will be most productive. In each lesson, teachers should identify new terms and anticipate any problematic vocabulary (which may be Tier 1, 2 or 3) or symbols, which will in turn have a positive impact on reading comprehension (Beck, Perfetti & McKeown 1982;

McKeown, 1993; Monroe & Panchyshyn, 1995). "It is important, however, that students do more than merely memorize and vocalize word meanings, they must develop concepts represented by the word" (Schell, 1982).

Strategy six: differentiating definitions of words with multiple meanings. The sixth strategy is exploring and differentiating between definitions of words with multiple meanings (these are amongst the words Beck, et al. (2013) would label Tier 2 words). One way to help students differentiate between words that have one meaning in mathematics and a different meaning in a different context, is to have students use the word in sentences. Students can practice writing sentences with the word used as one meaning, and other sentences with the same word used as a different meaning. For example: "I ate dinner at the *table* last night" and "From the *table* of x- and y-values, I can make a graph." Semantic mapping could also be used in this strategy.

Strategy seven: learning Greek and Latin roots. Learning Greek and Latin Roots is the seventh strategy that will help students improve their mathematical vocabulary. Meanings of some mathematical words can be found by looking at the prefix, suffix, and/or root of the word. As students become aware of these prefixes, suffixes, and roots and become sensitive to them when reading mathematics, they can improve their awareness of word meanings. The following exercise from Schell (1982) is one such example:

Directions: Write as many words as you can think of that begin with the prefix in the column.

bi-	**tri-**	**sub-**	**poly-**

Strategy eight: translating between symbols and words. Because students are continually challenged by word problems, strategy ten looks at how to get students to succeed at this by learning to read mathematical sentences and translate between symbols and words. According to Schell (1982), there are five things students need to learn how to do to be successful at solving word problems: read the words in the problems; analyze the information in word problems; understand and use common words (such as sequence or spatial words); identify applicable, inapplicable and extra information in word problems; and use a strategy to solve the problem.

In order to read word problems successfully, students need to be ready to read and understand the words in the problem. If they cannot read the words or understand the words, the problem should be read for them and help should be given for them to understand what the words mean. Shell (1982) emphasizes that, "A math teacher cannot afford to ignore the reading levels of the students nor the difficulty of the material." The teacher also needs to take into account the difficulty of the arithmetic the word problem is requiring. Students should only be

given word problems they are able to arithmetically figure out, and the word problems should be given a little at a time. This way, as students are successful at a little, they will be motivated to conquer harder problems later.

In analyzing problems, it is important that students can state the problem in their own words and can organize their thoughts. Schell (1982) says, "It is the responsibility of the teacher to determine whether the material is too hard or whether students need help with organization." If the material is too hard, the teacher can provide an easier problem to analyze. Having students write their own problems to share with a small group is also a way to suit problems to a student's' level.

To understand symbols and words in a problem successfully, students need to correlate symbols with words or phrases; convey ideas in objects, pictures, words, and symbols; be able to interpret all symbols and put them together to give the sentence meaning; and be aware that symbols may have more than one meaning.

One way to do this is to have students write mathematical sentences in as many different ways as possible, as in the following example from Schell (1982):

398 = 300 + 90 + 8

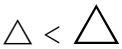
398 = (3 X 100) + (9 X 10) + (8 X 1)

three hundred ninety-eight equals 300 + 90 + 8

398 = three hundred plus ninety plus eight

Another way is to present math sentences as 'a symbol that names a thing', followed by 'a symbol that names a relationship', followed by 'symbols that name things' (Schell, 1982).

A third way is for students to correlate symbols with words and pictures as in the following example (Schell, 1982):

Symbol	Word	Picture
1/2	one-half	
<	less than	
5	five	

A fourth way is using the "direct-pure-piecemeal-complete," or DPPC, method of solving verbal problems (Dahmus, 1970). In this method, all facts are translated to the concrete and repetition of problem solving leads to success. DPPC asks students to concentrate on a few words at a time. The student translates these words from an English to a mathematical statement. By continuing this way, the student eventually translates the entire word problem into mathematical statements. Then the student can solve equations and/or systems of equations in the word problem. DPPC is a concrete sequential process that can help students who struggle with word problems.

Strategy nine: knowledge of the twelve powerful words and their definitions. The ninth strategy, as found by the mathematics accreditation committee at my school, is student knowledge of the Twelve Powerful Words and

their definitions (as researched and defined by Bell (2005)). Bell (2005) found there were twelve words used on most standardized tests. Students were not being overtly tested on these words and their definitions, but it was crucial students knew what these words meant in order to decipher the meaning of the question. In other words, they may know the answer to the question, but not understand what the question is asking and therefore get the incorrect answer, or no answer at all because they are so confused. For example, if a question asks, “Formulate a survey question to help you find the number of people who like pizza” and the student does not know the meaning of the word “formulate” they may get an incorrect answer because, even though they can create a question such as, “How many people like pizza?” they didn’t understand that they needed to create a question. Often times, test questions will also ask students to “support your answer”. If a student doesn’t understand that support means to back up with details, the student may not realize that they need to not only show the answer but also show how they got the answer. These two words, support and formulate, are just two of the twelve powerful words that Bell (2005) has identified as most commonly used, and most commonly understood, on standardized tests. The complete list of Bell’s Twelve Powerful Words and their meanings are listed in Appendix B.

Strategy ten: multi-setting repetition of vocabulary. The tenth strategy is repetition of vocabulary in different contexts. As students learn new terms and concepts, sources agree it is best to relearn the same material various times in

various contexts (Beck et al., 2013; Furner, Yahya & Duffy, 2005; Gay & White, 2002; Monroe & Panchyshyn, 1995; Renne, 2004; Sobel, Cepeda, & Kapler, 2011). Student's abilities can vary and different students may need different types of instruction and degrees of repetition to learn the same material (Lee & Patnode, 2007). Sobel et al. (2011) found that relearning the same material various times and in various contexts increases long-term retention of verbal information because there are more memory traces associated with the material. Capps (1989 as cited by Monroe and Panchychyn, 1995) even goes as far as to say that when introducing a new word or concept, students should have "six exposures during the initial lesson" and to "review terms at least 30 times during the month". There are many ways in which a teacher can review material including starting class with a review problem, acting out the vocabulary, using vocabulary flashcards, or reviewing words, definitions, illustrations, and specific examples that students have created on their word wall or in their notebooks (Lee & Patnode, 2007), in addition to the strategies and activities to help students develop an awareness and interest in words, as mentioned in strategy two.

Strategy eleven: small group and whole class discussions. The eleventh strategy is using small group work or whole class discussion. This will enrich the verbal environment in the classroom, and provide students the opportunity to explore, hypothesize, display and clarify thinking and understanding of new concepts (Beck et al., 2013; Lee & Patnode, 2007; Monroe & Panchychyn, 1995; Renne, 2004). Having a partner to confer and read material with can help

students focus on important aspects of math (Lee & Patnode, 2007; Shell, 1982). It is also important to make diverse pairing when grouping students. Grouping students heterogeneously can aid both learners by helping the student with diverse needs to be nurtured and supported and by offering the mainstream student to have their learning process and knowledge challenged (Furner, Yahya & Duffy, 2005). This peer interaction also benefits students by boosting the amount of language they understand through: using language in a meaningful context; modifying language for to those not fluent in English; aptly using paraphrase and repetition; and consistently constructing meaning of the word.

Strategy twelve: building student confidence. The twelfth strategy, as found by the mathematics accreditation committee at my school, is one that Hattie (2009) concludes is the greatest predictor of student success on an assessment. Building student confidence (or self-reporting grades, as labeled and researched by John Hattie (2002, 2012)), involves the teacher finding out the student's expectations and pushing the learner to exceed these expectations. The teacher should build students' confidence to help them see more of themselves by asking students: "What grade do you think you will get on this test?" and "Why do you think that?" Hattie found students can predict their grade better than the teacher or anyone else, much like a self-fulfilling prophecy. It is the teacher's job to help them figure out why they have success (it is not luck, it is because you studied) by asking students such questions as: "What did you do to get a good grade? What did you do at home? What did you do at school?" and

"What do you need to do differently next time?" Student confidence will be built by teaching students how to talk positively to themselves and by giving them the skills they need (or having other successful students talk about what they did to get a good grade or get the right answer). It is the teacher's job to find out what the students' expectations are and then push them beyond these expectations. After a student achieves success at this new level, they gain confidence in their ability to learn and do well on assessments.

Summary

Best-practices for mathematics vocabulary instruction cannot be looked at without considering the challenges students face in learning mathematics vocabulary. Reading comprehension and fluency; double meanings of words; symbols that have different meanings in different contexts; difficult vocabulary in word problems; and uncommon vocabulary words are all challenges students face when it comes to learning mathematics vocabulary.

Once teachers are aware of what challenges are present for students, they can adjust their teaching strategies to meet the needs of their learners and help students be successful at learning the complicated language of mathematics. It is clear that teachers need to use research-based strategies for mathematics vocabulary instruction (Pierce & Fontaine, 2009). In this research, three criteria were used in narrowing research to apply current best-practice instructional strategies to the population being studied: the strategy must apply to the correct age group, (Kindergarten through 5th grade elementary students); the

strategy must be for all students (not a special population, such as gifted and talented, or a special class, such as an honors course); and the strategy must be proven by research to increase knowledge and comprehension of mathematics vocabulary. The twelve best-practices researched in this paper have been shown to improve mathematics vocabulary instruction: a "robust approach to vocabulary" instruction (Beck et al., 2013, p. 112); helping students to develop an interest and awareness of words (Beck et al., 2013); semantic mapping; introducing vocabulary by giving students time to explore words before formally introducing them (Rubenstein & Thompson, 2002; Thompson & Rubenstein, 2000); posting a word wall in the classroom with Tier 2 and 3 words (Beck & McKeown, 2007 and Beck et al., 2013); understanding different word usage; learning greek and latin roots to help understand new words with similar prefixes, suffixes, or roots; associating symbols with words and concepts; knowledge of the Twelve Powerful Words (Bell, 2005); repetition of vocabulary in different contexts (Beck et al., 2013; Gay & White, 2002; Monroe & Panchyshyn, 1995; Renne, 2004; Sobel, Cepeda & Kapler, 2011); using small group work or whole class discussion (Beck et al., 2013; Lee & Patnode, 2007; Monroe & Panchychyn, 1995; Renne, 2004); and building student confidence (Hattie, 2009). As Thompson and Rubenstein (2000) found, best results will be achieved when a variety of strategies are used. Schell (1982) puts it best by saying, "no one technique will allow us to teach easily all students to read the language of mathematics. We as teachers must be aware of the factors which make the

language of mathematics difficult to read and concentrate our efforts on strategies that will help our students understand this special language."

The next chapter will describe the methodologies used in this study. First, the rationale and description of the research design will be presented along with the description of the quantitative paradigm. Second, participants will be described and variables specified. Third, the data collection protocols will be presented including the instruments used, including threats to validity.

CHAPTER THREE

Methods

Introduction

The purpose of this research was to answer the following research question: what are the current best practices for mathematics vocabulary instruction for K-5 elementary students?

I conducted a small-group professional development training on specific best-practice strategies to help K-5 elementary teachers improve their methods of mathematics vocabulary instruction. Each teacher then self-selected the strategies they desired to implement in their classroom. This professional development session was conducted for K-5 elementary mathematics teachers from an international school in Europe with the intent to make sure teachers were educated in effectively using the current best-practices for mathematics vocabulary instruction, as researched in this paper, that they self-selected to implement in their classroom.

I used two surveys in this study. The first survey was a closed-ended questionnaire to determine which strategies teachers had experience using in their classroom recently, and which strategies they would like to try implementing in their classroom this year. The questionnaire used a simple checkbox format,

with descriptions of each strategy listed in the questionnaire and was given immediately following the professional development session on these strategies. The second survey was both closed- and open-ended. Teachers used a closed-ended format to indicate which strategies they actually used this year. This section of the questionnaire again used a simple checkbox format, with descriptions of each strategy listed in the questionnaire. Teachers were then asked to use a number scale to prioritize strategies they used with 1 being the most effective in helping their students understand mathematics vocabulary.

Overview of the Chapter

This chapter describes the methodologies used in this study. First, the rationale and description of the research design is presented along with the description of the research paradigm. Second, participants are described and variables specified. Third, the data collection protocols are presented including the instruments used, including threats to validity.

Role of the Researcher

In researching this topic, I have chosen to use my current school, an international school in Europe, where I teach 13-year-old mathematics (the equivalent of 8th grade pre-algebra in America) and secondary intensive English. This international school organization follows an American mathematics curriculum in an English language environment. The organization employs predominately American teachers to teach classroom and content courses. I have taught at two of their thirty-seven schools in two of their twenty-seven

countries. This study benefited the organization because it aligned with the current accreditation requirements of researching best-practices for mathematics vocabulary instruction and presenting these to the staff.

Qualitative Research Paradigm

The purpose of this research was to answer the following research question: what are the current best practices for mathematics vocabulary instruction for K-5 elementary students? There are three research methods: qualitative, quantitative, and mixed methods (Creswell, 2009, p. 3). Qualitative research, used in this study, involves "exploring and understanding the meaning individuals or groups ascribe to a social or human problem. The process of research involves emerging questions and procedures, data typically collected in the participant's setting, data analysis inductively building from particulars to general themes, and the researcher making interpretations of the meaning of the data" (Creswell, 2013, p. 4). Qualitative research was used in this study because it allowed for research that focuses on individual meaning and interpretation of the complex. In this study, I did not write any curriculum, I only researched best practice instructional strategies, which fits with qualitative research. This study has many characteristics of qualitative research procedures (Creswell, 2009, p. 175-176), including: using the researcher as the key instrument when collecting data by examining documents; using multiple sources of data and having the researcher review all of the data and organize it into themes; having the researcher keep a focus on learning how the problem affects the participants, not

what the researchers bring; using an emergent design to learn about the issue from participants and use the research to get that information; using a social theoretical lens of K-5 elementary students; and using interpretive inquiry to make an interpretation of what is understood.

Strategy of Inquiry. According to Denzin and Lincoln (2005, p. 25), there are eight strategies of inquiry in that are commonly used in qualitative research: (1) case study, (2) ethnographic and participant observations, (3) phenomenology, ethnomethodology and interpretive practice, (4) grounded theory, (5) biographical method, (6) historical method, (7) applied and action research, and (8) clinical models. The purpose of action research strategy of inquiry for qualitative research is to solve a particular problem and to produce guidelines for best practice (Denscombe, 2010, p. 6). This study fit into the action research strategy of inquiry because it sought to discover current best practices for mathematics vocabulary instruction for K-5 elementary students, the particular problem being mathematics vocabulary instruction for K-5 elementary students.

As will be indicated in Chapter 5, quantitative research is suggested to follow this study in order to see what current best practices for mathematics vocabulary instruction works best for certain populations and certain standardized tests.

Data Collection

Participants and Setting. Since this study presented current best practices for mathematics vocabulary instruction for K-5 elementary students, my primary

participants were elementary mathematics teachers at my school. Each participant was given a human subject review form and signed this document prior to participating in this study. A human subject review form was also given to the director of the school and was signed. The school is a two-year-old through twelfth grade private international school, and is comprised of students from all around the world, the majority from Slovakia, South Korea, or the United States. For most (eighty percent) of these students, English is their second language. There are 141 boys and 134 girls, for a total number of 275 students.

This study was done for students from grades kindergarten through fifth (ages five to ten). The total number of students ages five to ten was 105 (sixty-five boys and forty girls). The mathematics teachers in this study teach grades kindergarten through fifth (ages five to ten) and are from around the world as well, but most are licensed by the United States. There were eight female teachers and two male teachers who participated in this study. Teaching experience ranged from five years to fifteen years. The school's curriculum is American. Houghton Mifflin Mathematics series is the primary mathematics resource for ages 5 through 10-year-old, however, teachers can use older texts, if available, as additional resources.

Data Collection Technique. At the beginning of this study, participating teachers attended a professional development, where they were presented with twelve current best practices for mathematics vocabulary instruction to use in their K-5 elementary classrooms (Appendix E). At the end of the professional

development session, teachers were given a survey on their level of teaching experience and asked about their current best practices for mathematics vocabulary instruction and which ones they would like to improve upon or try this year (Appendix F). At the end of the school year, participating teachers were given a second survey asking which teaching strategies they implemented this year (Appendix G) and which strategies they thought were most effective in mathematics vocabulary instruction in their classroom this year.

Procedure

Participants. For the questionnaires on current best practices for mathematics vocabulary instruction used or to be used (first survey), teachers were given this form after a professional development session they attended on current best practices for mathematics vocabulary instruction. The second survey was given at the end of the school year. Participants were given the opportunity to complete each questionnaire on their own time in an established time frame.

Materials. A paper survey was given to participants in the fall and in the spring. The participants only needed a pencil or pen. After collecting basic background information on the teacher, the first survey asked teachers to indicate which best-practice teaching strategies for mathematics vocabulary instruction they had used in their classroom last year and which ones they would be using in their classroom this year. The second survey asked teachers to indicate which best-practice teaching strategies for mathematics vocabulary instruction they actually implemented in their classroom this year. Teachers were

also asked to rate the strategies they used on a number scale, with 1 being the most effective strategy for mathematics vocabulary instruction. Each survey was meant only to take 20 minutes of the participants' time.

Criteria. Three criteria were used in narrowing research to apply best-practice teaching strategies to the population being studied. The first criterion is strategies must apply to the correct age group, Kindergarten through 5th grade elementary students. The second criterion is strategies must be for all students, not a special population, such as gifted and talented, or a special class, such as an honors course. The third criterion is strategies were proven by research to increase knowledge and comprehension of mathematics vocabulary.

Limitations. Conducting the research in a systematic way, using the three criteria as outlined above, controlled bias. However, an increase in understanding of mathematical vocabulary in students will not necessarily match with strategies teachers thought were most effective for mathematics vocabulary instruction in their classrooms.

Summary

This qualitative study used action research procedures to discover current best practices for mathematics vocabulary instruction for K-5 elementary students. Elementary teachers grades kindergarten to fifth primarily used Houghton Mifflin Mathematics curriculum to teach mathematics in their classrooms in addition to self-selected current best practices for mathematics vocabulary instruction presented at a professional development session. Current

best practice strategies for mathematics vocabulary instruction had to meet the criteria of being applicable to elementary students who were not in a special class or grouping, and proven by research to increase knowledge and comprehension of mathematical vocabulary.

In the following chapter, results of this study will be discussed. This chapter contains the findings of the research in three sections: an overview of the professional development session, the results of the first survey, and the results of the second survey. The purpose of this study was to find the current best practices for mathematics vocabulary instruction for K-5 elementary students. All participants in this study attended a professional development session on the current best practices for mathematics vocabulary instruction for K-5 elementary students. The content of this session presented research findings from the literature review and methods sections of this paper. This professional development session took place on a weekend in October at the school's regional professional development conference, where teachers had a variety of breakout sessions to choose from. All K-5 elementary teachers at my school participated in this session, for a total of twelve participants. All twelve participants also completed two surveys for this study. These teachers also strived to implement some of the current best practices for mathematics vocabulary instruction into their K-5 mathematics classes.

CHAPTER FOUR

Results

Introduction

The purpose of this study was to find the current best practices for mathematics vocabulary instruction for K-5 elementary students. All participants in this study attended a professional development session on the current best practices for mathematics vocabulary instruction for K-5 elementary students. The content of this session presented research findings from the literature review and methods sections of this paper. This professional development session took place on a weekend in October at the school's regional professional development conference, where teachers had a variety of breakout sessions to choose from. All K-5 elementary teachers at my school participated in this session, for a total of twelve participants. All twelve participants also completed two surveys for this study. These teachers also strived to implement some of the current best practices for mathematics vocabulary instruction into their K-5 mathematics classes.

This chapter contains the findings of the research in three sections: an overview of the professional development session, the results of the first survey, and the results of the second survey. The guiding question for this study was:

what are the current best practices for mathematics vocabulary instruction for K-5 elementary students?

Summary of the Professional Development Session

During a forty-five minute session K-5 elementary mathematics teachers at my school were presented with twelve current best practices for mathematics vocabulary instruction to use in their classrooms.

Introduction. In the first part of this session, a hook was given to draw teachers' attention to the subject in three parts: (1) by making teachers aware of current school goals for mathematics; (2) by making teachers aware of current student reports of progress in the area of mathematics and (3) reflecting on the success of students in meeting personal goals or teacher-driven goals related to mathematics vocabulary.

In order to make teachers aware of current school goals for mathematics, accreditation goals were presented to teachers. Accreditation is validation by an impartial third party that an educational institution has integrity by meeting higher standards and working towards continuous improvement of the school. This is important to my school because it shows the public the school is prestigious, valuable, and reliable in their quality of education. Some of the accreditation goals for mathematics at my school that teachers were presented with were:

- a yearly 3% increase in the percent of students meeting their goal of predicted individual growth on a standardized mathematics assessment between the fall and spring testing sessions;

- an 80% success rate per year of students earning a grade of A in a mathematics unit focusing on number sense;
- an 80% success rate of students showing improvement from pretest to posttest on an internal assessment focusing on number sense.

In reflecting on data from the previous three years of student standardized testing scores and student scores on internal assessments in units focusing on number sense, our students had not met the goals of our accreditation in most areas.

In order to make teachers aware of current student reports of progress, the past three years of student standardized test scores for mathematics were presented to teachers. Our students were somewhat successful in meeting standardized test goals (set by the standardized testing company), but there was still much room for improvement.

Teachers were also asked to briefly discuss and reflect on how successful students were at meeting goals they, the teacher, set for them as well as goals students set for themselves. The general consensus was that students were more successful at these goals than the accreditation or standardized testing goals. However, overall, teachers felt the need for improvement in mathematics vocabulary instruction and in student knowledge of mathematics vocabulary.

This created a feeling of urgency in teachers to learn about these current twelve best practices for mathematics vocabulary instruction

Teaching the Twelve Best Practices. Once teachers were motivated to learn about current best practices for mathematics vocabulary instruction, each

teacher was given a page to read on one of the twelve strategies (see Appendix E), which worked perfectly because there were twelve participants. After each teacher was done reading their page, one-by-one they orally summarized and presented the strategy to the whole group. They then completed a t-chart, with the help of the other participants by facilitating a discussion about how this strategy could work, or has worked, (positively or negatively) in each teacher's classroom. This group discussion following each strategy presentation gave teachers a chance to process the strategy and help each other find ways to use it effectively in their classrooms.

Conclusion. Upon completion of this session, each teacher was given a copy of each strategy page as well as the t-chart that was created during discussion. They were then asked to complete a short survey (see Appendix F) on which of these twelve current best practices for mathematics vocabulary instruction they had used in their classroom in the past and which ones they planned on using this year in their classroom. Results of this survey are presented in the next paragraph.

Responses to the Professional Development Session

All twelve elementary mathematics teachers at my school who teach the equivalent of grades K-5 attended this professional development (PD) session. Responses were positive, and teachers were motivated to implement new best practices for mathematics vocabulary instruction into their K-5 classrooms.

Survey One: Previous Experience and Goal Setting. All twelve elementary teachers at my school also completed the first survey (see Appendix F) upon completion of the PD session. Years of teaching experience, as indicated by participating teachers, ranged from four to twenty-one years. The table below shows how many (and percentage of) participating teachers who had used the twelve teaching strategies presented (see Appendix E) in the past:

Number of Teachers	Percent of Teachers	Strategy
5	42 %	use a robust approach to vocabulary
9	75 %	create an interest in and awareness of words
1	8 %	use semantic mapping before and/or after introducing a word or concept
6	50 %	introduce new vocabulary by exploration
5	42 %	post a word wall with Tier 2 and Tier 3 words
7	58 %	differentiate between multiple meanings of words
3	25 %	teach students Greek and Latin roots
8	67 %	teach students to translate between symbols and words
1	8 %	teach students the 12 Powerful Words
4	33 %	use multi-setting repetition of vocabulary over a period of time
7	58 %	use small group and/or whole class discussion to learn vocabulary
2	17 %	build student confidence

If we analyze this more in depth, we can find percentages of teacher pre-knowledge in these strategies. If all twelve teachers had used all twelve strategies in the past, I would not be introducing any new concepts, but only

creating discussions on what has worked and not worked in the classroom with these strategies. In the table above, a '12' in all rows of the first column would indicate this complete pre-knowledge. However, this is not the case. In each strategy, there are between three and eleven teachers that are being introduced to that particular strategy. Overall, there is sixty percent room for instruction (fifty-eight total strategies known by teachers at beginning divided by one hundred forty-four total maximum of strategies to be known). This shows the importance of my research and the professional development session I created.

The table below shows how many (and percentage of) teachers indicated they would like to implement these teaching strategies (see Appendix E) this year:

Number of Teachers	Percent of Teachers	Strategy
8	67 %	use a robust approach to vocabulary
12	100 %	create an interest in and awareness of words
6	50 %	use semantic mapping before and/or after introducing a word or concept
9	75 %	introduce new vocabulary by exploration
12	100 %	post a word wall with Tier 2 and Tier 3 words
10	83 %	differentiate between multiple meanings of words
7	58 %	teach students Greek and Latin roots
12	100 %	teach students to translate between symbols and words
4	33 %	teach students the 12 Powerful Words
8	67 %	use multi-setting repetition of vocabulary over a period of time

Number of Teachers	Percent of Teachers	Strategy
12	100 %	use small group and/or whole class discussion to learn vocabulary
9	75 %	build student confidence

Analyzing this more in depth, shows that many teachers are willing to try many new strategies during this school year. If teachers were to implement all strategies that they hoped during the school year, this would be seventy-six percent of all teachers using all strategies (a eighty-eight percent increase from the strategies that were used the previous year of teaching). The five strategies that showed the greatest amount of change among teachers who desired to use the strategy this school year were: a word wall with Tier 2 and Tier 3 words (+7 teachers), building student confidence (+7 teachers), using semantic mapping (+5 teachers), small or whole group discussion (+5 teachers), teaching translation between symbols and words (+4 teachers), multi-setting repetition of vocabulary over a period of time (+4 teachers), and teaching Greek and Latin roots (+4 teachers). It may be that these five strategies were the most valuable and/or interesting to this group of elementary teachers at the time of their professional development training, and it would be interesting to pursue further research in this area.

Survey Two: Results of School Year. At the end of the school year, I sent out an email to teachers reminding them of our professional development

session we had in the fall. I made them aware that I would be distributing a follow up survey listing the current best practices for mathematics vocabulary instruction they had been introduced to during the professional development session and that they were to indicate which ones they had actually implemented in their classrooms this year. All twelve teachers completed this second survey. The table below shows how many teachers actually implemented these teaching strategies (see Appendix E) this year:

Number of Teachers	Percent of Teachers	Strategy
7	58 %	use a robust approach to vocabulary
12	100 %	create an interest in and awareness of words
3	25 %	use semantic mapping before and/or after introducing a word or concept
7	58 %	introduce new vocabulary by exploration
7	58 %	post a word wall with Tier 2 and Tier 3 words
7	58 %	differentiate between multiple meanings of words
6	50 %	teach students Greek and Latin roots
11	92 %	teach students to translate between symbols and words
2	17 %	teach students the 12 Powerful Words
6	50 %	use multi-setting repetition of vocabulary over a period of time
11	92 %	use small group and/or whole class discussion to learn vocabulary
6	50 %	build student confidence

In looking at this data, we can see the strategies that were actually implemented in the classrooms. Overall, teachers showed a forty-seven percent increase in

use of these twelve strategies in their classrooms from the previous year (with eighty-five out of one hundred forty-four, or fifty-nine percent, of the strategies being used). The five strategies that increased the most in use among teachers were: building student confidence (+4 teachers), using small or whole group discussion (+4 teachers), creating an interest in and awareness of words (+3 teachers), teaching Greek and Latin roots (+3 teachers), and translating between words and symbols (+3 teachers). This data supports the literature review that these twelve strategies are important for mathematics vocabulary instruction in K-5 elementary students.

There are many different possible reasons why teachers did not reach some of their goals for implementing instruction such as: lack of time, prioritizing other strategies or curriculum to implement, prioritizing teaching content over vocabulary, lack of support or resources, decreased interest in strategy, decrease in perceived value in strategy, no compensation for implementing strategy, or no motivation for implementing strategy.

Conclusion

In this chapter, I provided the findings of the research in three sections: an overview of the professional development session, the results of the first survey, and the results of the second survey. The guiding question for this study was: what are the current best practices for mathematics vocabulary instruction for K-5 elementary students? These findings established that most teachers were previously aware of some, if not all, the current best practices for mathematics

vocabulary instruction for K-5 elementary students, but much fewer were using them in their classroom. Also, as teachers were presented with these current best practices and talked about ways to use them in their classroom and set goals for themselves to implement them, the number of teachers using these strategies increased drastically.

In the next chapter, I will review this capstone, as well as successes and limitations. In addition, I will look at how the topic of implementing current best practices for mathematics vocabulary instruction in K-5 elementary classrooms could be further developed based on the results of this study.

CHAPTER FIVE

Conclusion

Introduction

In the previous chapter, I talked about the findings of the research via an overview of the professional development session, the results of the first survey, and the results of the second survey. In this chapter, I will look at connections to the literature review, limitations for the study, and major learnings. This will show where this study can be further developed and improved for future research.

The guiding question for this study was: what are the current best practices for mathematics vocabulary instruction for K-5 elementary students?

Connections to the Literature Review

In informal conversations with my peers who participated in this study, I found that most teachers favored the strategies of (1) creating an interest in and awareness of words, as researched by Beck et al. (2013), (specifically: using real objects and demonstration with patterned blocks, children's literature related to the concept, relating words/concepts to prior knowledge and background, games, drawings to help visualize words in word problems, and relating concepts to real-world or everyday life and experiences); (2) differentiating between multiple meanings of words, as researched by Beck et al. (2013); (3) teaching students to

translate between symbols and words, as researched by Shell (1982), specifically, correlating words with pictures and symbols; and (4) using small group and/or whole class discussion, as Beck et al. (2013), Lee & Patnode (2007), Monroe & Panchychyn (1995), and Renne (2004) noted. Many teachers had either read Beck's book or had heard of it via a citing in a professional development session, journal article, or conversation, which is why they were comfortable using some these strategies. Many of these strategies were also presented in their curriculum books.

Teachers also informally told me they noticed a change in student learning (or motivation) when using the strategies of (1) creating an interest in and awareness of words, as found by Beck et al. (2013) and (2) posting a word wall with Tier 2 or Tier 3 words, as Beck & McKeown (2007) and Beck et al. (2013) noted. This could also be because of the familiarity teachers had with Beck's work as well as how extensiveness and current it is.

Limitations for this Study

One major limitation for this study included time constraints. Having only one session to teach these strategies, and one year to observe teacher behavior, limited the extent to which assumptions could be made about how effective these current best practices were for mathematics vocabulary instruction.

This study was also limited because of the amount of teachers (and students) that were able to participate in this study. Ideally, this study should be

done in a larger school with more teachers and students participating, and multiple teachers at each level that can compare between classes.

Another limitation is that participants know me, the researcher, so some of their motivation to implement new strategies in their classroom could have been because they wanted to please me, and not purely because they wanted to implement the strategy of their own motivation.

Keeping a journal of informal conversations I had with teachers would have also benefited my observations and conclusions in this study.

Conducting personal interviews with teacher participants as the study was in progress also would have greatly benefitted insights into this study.

Major Learnings/Implications

The main finding of this study is that elementary teachers are open to implementing new vocabulary strategies into their mathematics classrooms. The forty-seven percent increase in use of these twelve best-practice strategies from the previous year to current year in the K-5 mathematics elementary classes that participated, shows this. Most teachers want to be effective in their teaching. Educators desire to make mathematics instruction meaningful for students, and when given current research on best practices for mathematics vocabulary instruction and ways they can easily implement it in their classrooms, they are motivated to do so.

Also, as the first survey was completed, it was found that many teachers were already using some of the strategies in their classrooms in the past school

year(s). This helped motivated participating teachers to try strategies that their colleagues had already implemented in their classrooms because they felt like they had more support than what I could give them.

Future Study

Future research could look at how using these teaching practices could impact students ability to extend and apply mathematical concepts to real-world situations.

Looking at how using each strategy, or a combination of these strategies, affects students' standardized test scores would also be an interesting topic to explore and would give further information as to how effective each of these strategies are.

Another area of interest to be explored is to see how effective these current best practices are in mathematics vocabulary instruction at the 6-8th grade levels; 9th - 12th grade levels; and non-native English speaking populations.

As stated earlier, ideally, in the future this study would be done at a school where the researcher doesn't know the participating teachers and students, and a journal and/or interview session of teacher's and students' thoughts along the way would also be recorded and collected.

Dissemination of the Results

The results of this study will be distributed to different audiences in different ways. Hamline University published the study online, and the school

where research was conducted received a digital copy of this work via email. All teachers involved in this study were also given a digital copy.

Reflection

In reflecting on the results of this study, I was very satisfied with the success of this study. Despite the limitations mentioned above, such as time constraints and small size of the research population, this study was helpful to those who participated by giving them encouragement and refreshing new ideas in their current mathematics vocabulary instruction strategies. It also gave me, the researcher, encouragement that teachers are willing to try new things in their classroom and that my research benefited others. It also gave me insight in to what more could be done on this topic in the future and how I could develop this into further professional development topics, such as mathematics vocabulary instruction strategies for older students.

Summary

In this chapter, the following topics were discussed: connections to the literature review, limitations of the study, major learnings/implications, and ideas for future study related to this topic. A strong positive correlation was found between the findings of the literature review and the strategies teachers found to be helpful in their classrooms, although it could have been documented more formally, which was one limitation to this study. Other major limitations were time constraints, size population that participated in the study, and the researcher's involvement in the study location. Overall, participating teachers benefited from

the results of this study, which begs for further research into current best practices for teaching other age levels and content areas, and how these best practices for mathematics vocabulary instruction impact standardized test scores.

Conclusion

This study attempted to define twelve current best practices for mathematics vocabulary instruction for K-5 elementary students. Research-based methods were used to develop a professional development session K-5 elementary teachers that disseminated these results and allowed teachers a chance to discuss these strategies and how to implement them in their own classrooms. Two surveys indicated that, of the twelve current best practices for mathematics vocabulary instruction that were presented to them, most teachers were aware of some of these strategies and had used them in their classrooms in the past year, but there was a sixty percent room for instruction to implement these strategies into their classroom the current year. As teachers set goals for themselves for the school year, they showed they desired an eighty-eight percent increase from the strategies that were used the past year of teaching, and although they actually only increased by forty-seven percent in use of these twelve strategies in their classrooms by the end of the year, it was still a significant increase in the amount of current best practices for mathematics vocabulary instruction that were being used in these K-5 elementary classrooms. This shows that the research was important in educating teachers on current best practices for mathematics vocabulary instruction and in motivating them to

implement these strategies in their classrooms. It also stresses the importance of further research into current best practices for vocabulary instruction in other content areas and amongst other age levels. Overall, this paper successfully answered the question, "What are the current best practices for mathematics vocabulary instruction for K-5 elementary students?" Further research could be done on the most effective best practices for mathematics vocabulary instruction for K-5 elementary students by looking at standardized test scores or other means of testing mathematics vocabulary knowledge and growth over the course of a school year.

APPENDIX A

Houghton Mifflin Mathematics Test Prep Practice Questions

Houghton Mifflin Mathematics Test Prep Practice Questions

3. Which decimal represents the shaded portion of this model?

- 70.0
- 0.7
- 7.0
- 0.07

4. The librarian at Hamilton School held a book swap. Every student brought in 3 used books to exchange. If Hamilton School has 342 students, how many books were brought in for the swap?

- 926
- 1,116
- 1,026
- NH

7. A package of 8 balls costs $5.44. What is the unit price?

- 68
- 75
- 69
- 78

APPENDIX B

The Twelve Powerful Words and Their Definitions

The Twelve Powerful Words and Their Definitions

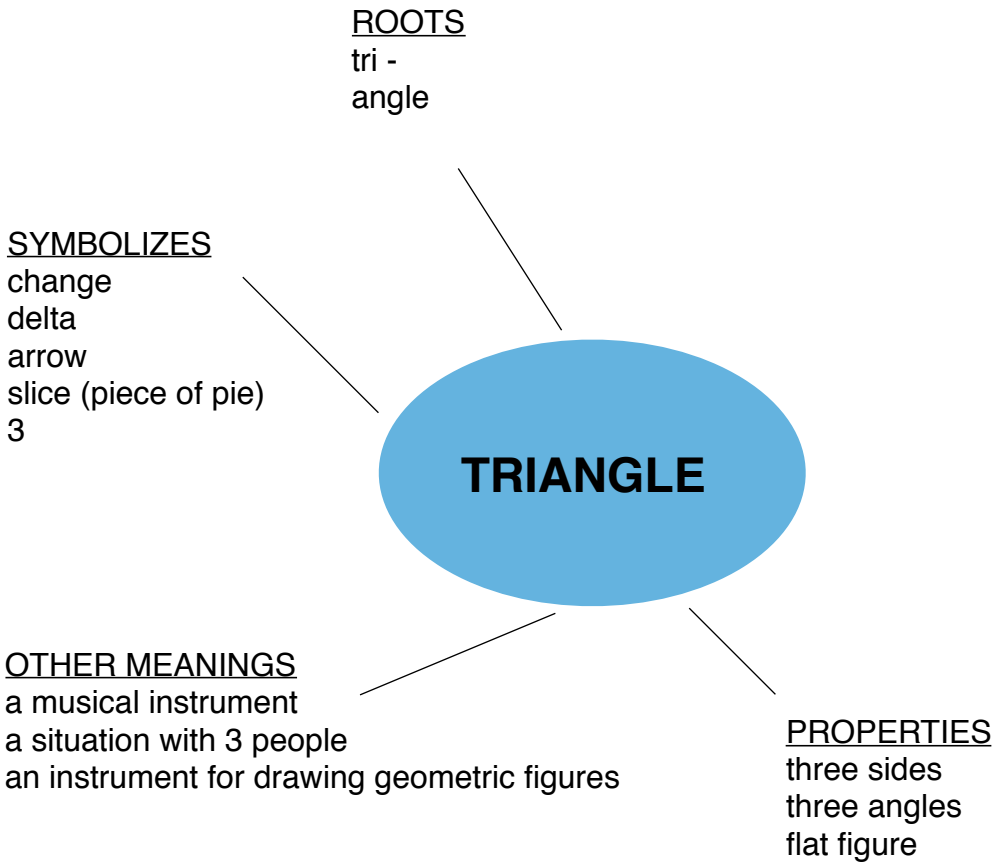
Word	Definition
analyze	to break apart
compare	ways they are the same
contrast	ways they are different
describe	tell about
formulate	create
evaluate	judge
explain	tell how
infer	read between the lines
predict	what will happen next
summarize	give a short version
support	back up with details
trace	list in steps

* from Larry Bell's book: 12 Powerful Words, That Increase Test Scores and Help Close the Achievement Gap

APPENDIX C

Pre-Reading Semantic Map

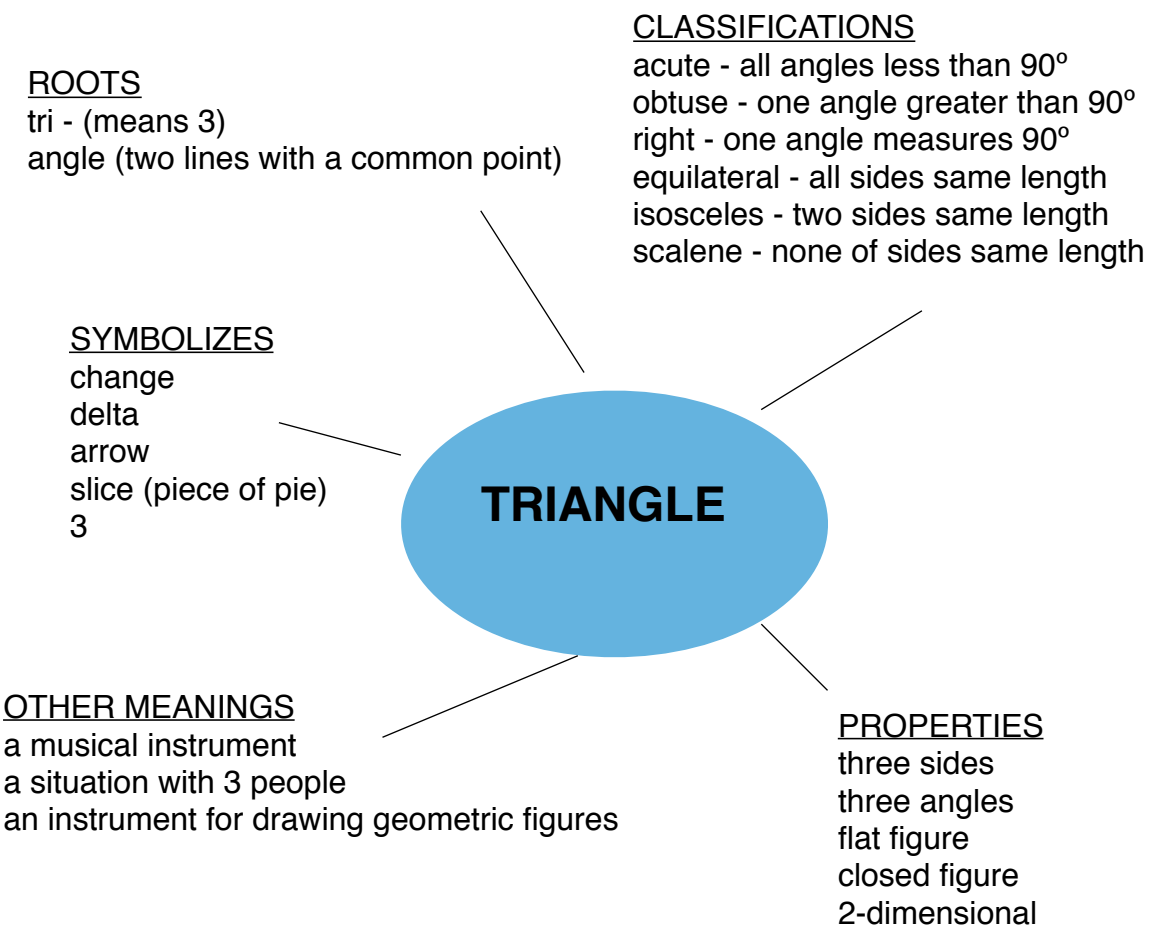
Pre-Reading Semantic Map



APPENDIX D

Post-Reading Semantic Map

Post-Reading Semantic Map



APPENDIX E

Professional Development Training Session:
Twelve Best Practices for Mathematics Vocabulary Instruction
for K-5 elementary students

TWELVE BEST PRACTICES FOR MATHEMATICS VOCABULARY INSTRUCTION FOR K-5 ELEMENTARY STUDENTS

Professional Development Fall 2014

“A robust approach to vocabulary involves directly explaining the meanings of words along with thought-provoking, playful, and interactive follow-up" (Beck et al., 2013). For students to deeply process word meanings, students should relate new words to ideas students already know, write sentences discussing how the word usage is different between two subjects, create a word wall (which could be color-coded to correspond to different math categories) with definitions, pictures, uses, and related words created by students, and use picture dictionaries or glossaries.

Developing an interest in and awareness of words can be done through a variety of pre-reading strategies that help students understand the concept(s) behind the word, such as:

- real objects and demonstration (beans, buttons, marbles, M&Ms®, patterned blocks)
- children’s literature related to the concept
- relating words/concepts to prior knowledge and background, interdisciplinary subjects, and cross-cultural backgrounds
- skits, journal entries, and other writing assignments

- writing students' own word problems to exchange with classmates
- field trips (even internet virtual field trips)
- games
- cartoons
- songs, poems and raps
- drawings to help visualize words in word problems
- relating concepts to real-world or everyday life and experiences

Semantic mapping is a pre-reading strategy that helps activate and expand prior knowledge through the use of brainstorming. In semantic mapping, students categorize information in a graphic form, similar to a word map or mind map. Seven steps for semantic mapping are:

First, the teacher should choose a word that is a the center of the topic being studied.

The teacher should then write the word on a surface visible to the whole class (such as whiteboard, SMART board®, or chart paper). This will be the semantic map that is created as a class.

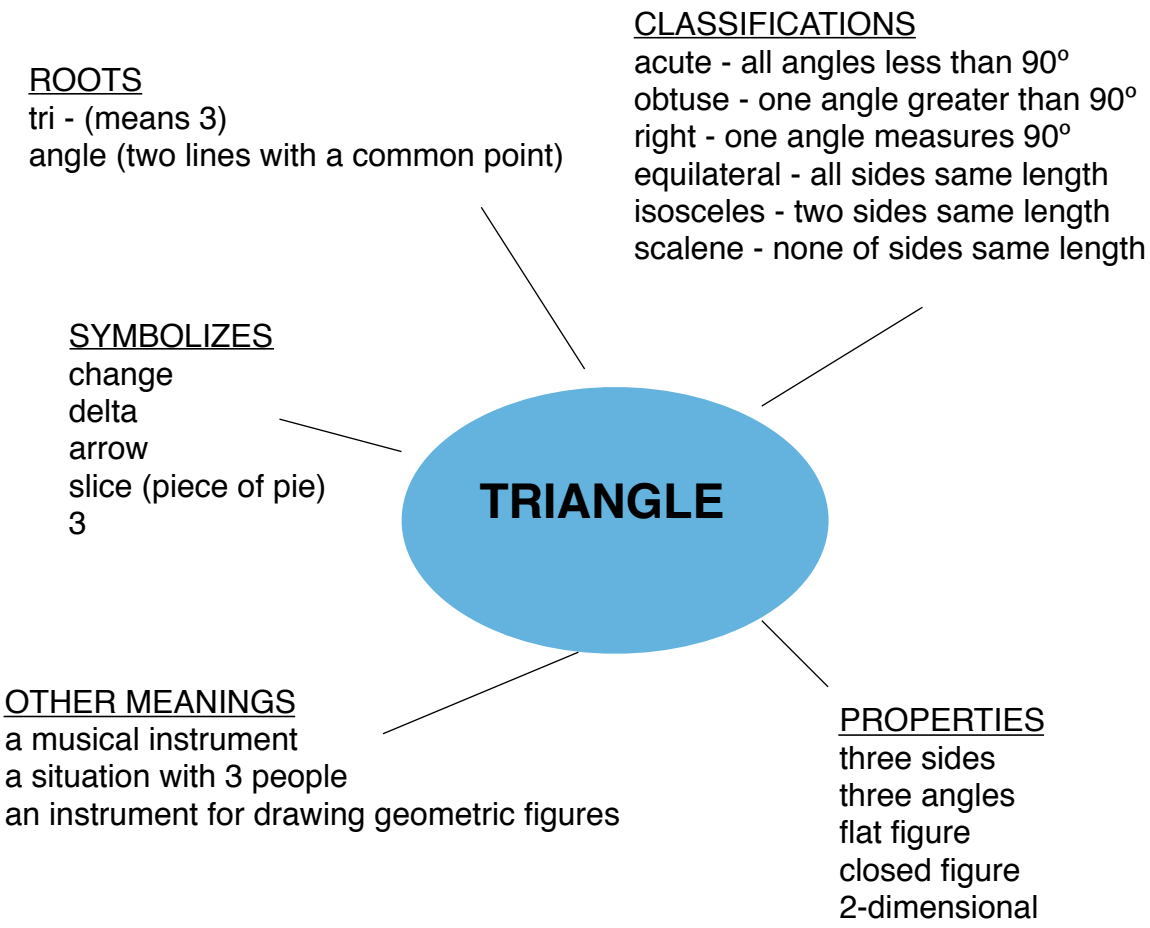
Then, the class should take time to brainstorm aloud words related to the topic word, list these words in categories on the whole class surface.

Students should then take a few minutes the think on their own of as many words as they can related to the topic word and write them down.

Students should then share aloud the words they wrote down and the teacher should add them to the class map.

As your class map now has a central topic word with many words surrounding in, listed in categories, the students should now suggest labels for each category.

The class should then discuss the words on the class map. Students should observe new words, find new meanings to old words, and make connections between new and old words.



Introducing new vocabulary by exploration. Before formally introducing a new word or concept, the teacher gives students the opportunity to use materials to explore ideas, explain rationale, and invent their own terminology to build concepts, and then attach the formal vocabulary to these ideas. The point is for students to reinforce ideas and understanding before being presented with a new and unfamiliar term.

Posting a Word Wall with tier 2 and tier 3 words gives students a place to visually see the words and their definitions in print (and maybe even picture). The three tiers are defined as:

Tier 1 words are classified as the most basic words: "*warm, dog, tired, run, talk, party, swim, look, and so on*" (Beck et al., 2013). These words usually have a high frequency because they are used in daily oral conversations, and so these words do not usually need to be directly taught.

Tier 3 words are used only in specific topics and domains, so they are not as familiar and need to be directly taught. Examples of these words are: "*filibuster, pantheon, and epidermis*" (Beck et al., 2013). These words may not be utilized often but can be introduced as needed to understand content.

Tier 2 words are utilized often and across a variety of subjects, such as: "*contradict, circumstances, precede, auspicious, fervent, and retrospect*" (Beck et

al., 2013) . Since these words are used more often in writing and less often in conversation, they are less known and need to be more directly taught.

In each lesson, the teacher identifies new terms and anticipates any problematic vocabulary (which may be Tier 1, 2 or 3) or symbols.

Differentiating between definitions of words with multiple meanings means talking about the two (or more) definitions for one word. Sometimes the words have different definitions within mathematics, but most of the time, these words have one meaning in mathematics and a different meaning in a different context. One example of an activity a teacher would do for this is have students practice writing sentences with the word used as one meaning, and other sentences with the same word used as a different meaning. For example: "I ate dinner at the *table* last night" and "From the *table* of x- and y-values, I can make a graph."

Teaching Greek and Latin roots. Meanings of some mathematical words can be found by looking at the prefix, suffix, and/or root of the word. The following exercise is one such example:

Directions: Write as many words as you can think of that begin with the prefix in the column.

bi-	tri-	sub-	poly-

In teaching students to translate between symbols and words, the teacher helps students learn how to do to be successful at solving word problems though five steps: read the words in the problems; analyze the information in word problems; understand and use common words (such as sequence or spatial words); identify applicable, inapplicable and extra information in word problems; and use a strategy to solve the problem. The teacher guides students to correlate symbols with words or phrases; convey ideas in objects, pictures, words, and symbols; be able to interpret all symbols and put them together to give the sentence meaning; and be aware that symbols may have more than one meaning. The teacher takes into account the difficulty of the arithmetic the word problem is requiring. Students are only given word problems they are able to arithmetically figure out, and the word problems are be given a little at a time. If the material is too hard, the teacher provides an easier problem to analyze. Having students write their own problems to share with a small group is also a way a teacher might fit problems to a student's' level.

Overtly teaching these Twelve powerful words and their definitions:

Word	Definition
analyze	to break apart
compare	ways they are the same
contrast	ways they are different
describe	tell about
formulate	create
evaluate	judge
explain	tell how
infer	read between the lines
predict	what will happen next
summarize	give a short version
support	back up with details
trace	list in steps

Using multi-setting repetition of vocabulary means using different types of instruction and degrees of repetition for the same material. When introducing a new word or concept, students have “six exposures during the initial lesson” and “review terms at least 30 times during the month”. Reviewing material includes starting class with a review problem, acting out the vocabulary, using vocabulary flashcards, or reviewing words, definitions, illustrations, and specific examples

that students have created on their word wall or in their notebooks in addition to many other strategies.

Using small group and whole class discussions is providing students the opportunity to explore, hypothesize, display and clarify thinking and understanding of new concepts. One example is having students pair together to confer and read material.

Building student confidence (or self-reporting grades, as labeled and researched by John Hattie (2002, 2012)), involves the teacher finding out the student's expectations and pushing the learner to exceed these expectations. The teacher should build students' confidence to help them see more of themselves by asking students: "What grade do you think you will get on this test?" and "Why do you think that?" Hattie found students can predict their grade better than the teacher or anyone else, much like a self-fulfilling prophecy. It is the teacher's job to help them figure out why they have success (it is not luck, it is because you studied) by asking students such questions as: "What did you do to get a good grade? What did you do at home? What did you do at school?" and "What do you need to do differently next time?" Student confidence will be built by teaching students how to talk positively to themselves and by giving them the skills they need (or

having other successful students talk about what they did to get a good grade or get the right answer). It is the teacher's job to find out what the students' expectations are and then push them beyond these expectations. After a student achieves success at this new level, they gain confidence in their ability to learn and do well on assessments.

APPENDIX F

FALL TEACHER SURVEY

FALL TEACHER SURVEY

Vocabulary Teaching Strategies Survey
for Elementary Mathematics Teachers
Fall 2014

Your name: _____ Grade Level of Math You Teach: _____

How many years teaching experience do you have? _____

Which strategies did you use last year in teaching mathematics vocabulary? Which will you use this year? (for a description of these, see the following pages matching the letter in parentheses)

Last Year	This Year	
☐	☐	use a robust approach to vocabulary (A)
☐	☐	create an interest in and awareness of words (B)
☐	☐	use semantic mapping before and/or after introducing a word or concept (C)
☐	☐	introduce new vocabulary by exploration (D)
☐	☐	post a word wall with Tier 2 and Tier 3 words (E)
☐	☐	differentiate between multiple meanings of words (F)
☐	☐	teach students Greek and Latin roots (G)
☐	☐	teach students to translate between symbols and words (H)
☐	☐	teach students the 12 Powerful words (I)
☐	☐	use multi-setting repetition of vocabulary over a period of time (J)
☐	☐	use small group and/or whole class discussion to learn vocabulary (K)
☐	☐	build student confidence (L)
☐		I did not teach last year

Thank you!
A follow-up survey will be given at the end of the school year.

APPENDIX G

SPRING TEACHER SURVEY

SPRING TEACHER SURVEY

Vocabulary Teaching Strategies Survey
for Elementary Mathematics Teachers
Spring 2015

Your name: _____ Grade Level of Math You Teach: _____

In the first column, please check the strategies you used this year in teaching mathematics vocabulary (for a description of these, see the following pages matching the letter in parentheses).

In the second column, please rank only the strategies you used this year in order of their effectiveness in your classroom to help students achieve better understanding of mathematics vocabulary, with 1 being the most helpful.

This Year	Your Ranking	
☐		use a robust approach to vocabulary (A)
☐		create an interest in and awareness of words (B)
☐		use semantic mapping before and/or after introducing a word or concept (C)
☐		introduce new vocabulary by exploration (D)
☐		post a word wall with Tier 2 and Tier 3 words (E)
☐		differentiate between multiple meanings of words (F)
☐		teach students Greek and Latin roots (G)
☐		teach students to translate between symbols and words (H)
☐		teach students the 12 Powerful words (I)
☐		use multi-setting repetition of vocabulary over a period of time (J)
☐		use small group and/or whole class discussion to learn vocabulary (K)
☐		build student confidence (L)
		I did not teach last year

Thank you!

REFERENCES

Aiken, L. R. (1973). Education-Mathematics. *Journal of Learning Disabilities, 6(4)*, 226-227.

Beck, I. L., & McKeown, M. G. (2007). Increasing young low-income children's oral vocabulary repertoires through rich and focused instruction. *The Elementary School Journal, 107(3),* 251-271.

Beck, I. L., McKeown, M. G., & Kuncan, L. (2013). *Bringing words to life: Robust vocabulary instruction.* New York: Guilford Press.

Beck, I. L., Perfetti, C. A., & McKeown, M. G. (1982). Effects of long-term vocabulary instruction on lexical access and reading comprehension. *Journal of Educational Psychology, 74 (4),* 506-521.

Bell, L. I. (2005). *12 Powerful Words That Increase Test Scores and Help Close the Achievement Gap: A Resource for Educators and Parents.* Manassas, VA: Multicultural America.

Bickmore-Brand, J. (1990). *Language in Mathematics.* Portsmouth, NH: Heinemann.

Blachowicz, C., & Fisher, P. (2000). *Vocabulary Instruction.* In M. Kamil, P. Mosenthal, P.D. Pearson, & R. Barr (Eds.), *Handbook of reading research, Volume 3* (pp. 503 - 524). Mahwah, NJ: Erlbaum.

Bloom, K. C., & Shuell, T. J. (1981). Effects of Massed and Distributed Practice on the Learning and Retention of Second-Language Vocabulary. *Journal of Educational Research, 74(4).*

Capps, L. R. (1989, April). *Problem solving: Is language a problem?* Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Orlando, FL.

Capps, L. R., & Pickreign, J. (1993). Language connections in mathematics: A critical part of mathematics instruction. *Arithmetic Teacher, 41(1),* 8-12.

Creswell, J. W. (2013). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (4th ed.).* Thousand Oaks, CA: Sage Publications.

Creswell, J. W. (2009). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (3rd ed.).* Thousand Oaks, CA: Sage Publications.

Curtis, M. E. (1983). *Vocabulary Testing and Vocabulary Instruction.* In M. G. McKeown & M. E. Curtis (Eds.), The nature of vocabulary acquisition (37-51). Hillsdale, NJ: Erlbaum.

Denscombe, M., (2010). *The good research guide: for small-scale social research projects (4th ed.).* Maidenhead, UK: Open University Press.

Denzin, N. K., & Lincoln, Y. S. (2005). *The SAGE handbook of qualitative research.* Thousand Oaks, CA: Sage Publications.

Earp, N. W., & Tanner, F. W. (1980). Mathematics and Language. *Arithmetic Teacher, 28(12),* 32-34.

Furner, J. M., Yahya, N., Duffy, M. L. (2005). Teach Mathematics: Strategies to Reach All Students. *Intervention in School and Clinic, 41(1),* 16-23.

Gagnon, J. C. & Maccini, P. (2001). Preparing students with disabilities for algebra. *Teaching Exceptional Children, 34(1),* 8-16.

Garbe, D. G. (1985). Mathematics vocabulary and the culturally different student. *Arithmetic Teacher, 33,* 39-42.

Gay, A. S., & White, S. H. (2002). Teaching vocabulary communicate mathematically. *Middle School Journal, 34 (2),* 33-38.

Gudder, S. (1994). *A Mathematical Journey.* New York, NY: McGraw Hill Companies.

Hattie, J. (2009). *Hattie Ranking: Influences and Effect Sizes Related to Student Achievement.* Retrieved from http://http://visible-learning.org/hattie-ranking-influences-effect-sizes-learning-achievement/

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* London: Routledge.

Hattie, J. (2012). *Student Self-Reported Grades.* Retrieved from http://visible-learning.org/glossary/

Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning.* London: Routledge.

Hattie, J. and Cognition Education (2012, May 2). *Self Reported Grades with John Hattie* [video file]. Retrieved from http://vimeo.com/41465488

Hilliard, G. (1924). *Probable Types of Difficulties Underlying Low Scores in Comprehension Tests* (Doctoral dissertation). Retrieved from Sleepy Hollow Books.

Houghton Mifflin (2014). *Houghton Mifflin Mathematics Test Prep Practice.* Retrieved from http://www.eduplace.com/kids/mhm/testprep/gr3/1203/02.html?qseq=0,1,2,3,4,5,6,7&at=0&fb=tr&score=20&curq=2 http://www.eduplace.com/kids/mhm/testprep/gr3/1203/03.html?qseq=0,1,2,3,4,5,6,7&at=0&fb=tr&score=30&curq=3 http://www.eduplace.com/kids/mhm/testprep/gr3/1204/06.html?qseq=0,1,2,3,4,5,6,7&at=0&fb=tr&score=60&curq=6

Johnson, D. D., & Johnson, B. H. (1986). Highlighting vocabulary in inferential comprehension instruction. *Journal of Reading, 29,* 622-625.

Johnson, D. D., Pittelman, S. D., & Heimlich, J. E. (1986). Semantic Mapping. *The Reading Teacher, 39(8),* 778 - 783.

Lee, H., & Herner-Patnode, L. (2007). What Works for Me: Teaching Mathematics Vocabulary to Diverse Groups. *Intervention in School and Clinic, 43(2),* 121-126.

McKeown, M. G. (1993). Creating Effective Definitions for Young Word Learners. *Reading Research Quarterly, 28(1),* 16-31.

McKeown, M.G., Beck, I.L., Omanson, R.C., & Perfetti, C.A. (1983). The effects of long-term vocabulary instruction on reading comprehension: A replication. *Journal of Reading Behavior, 15(1)*, 3-18.

McKeown, M.G., Beck, I.L., Omanson, R.C., & Pople, M.T. (1985). Some effects of the nature and frequency of vocabulary instruction on the knowledge and use of words. *Reading Research Quarterly, 20(5),* 522-535.

McKeown, M. G. & Curtis, M. E. (1983). *The nature of vocabulary acquisition.* Hillsdale, NJ: Erlbaum.

Mezynski, K. (1983). Issues Concerning the Acquisition of Knowledge: Effects of Vocabulary Training on Reading Comprehension. *Review of Educational Research, 53,* 253-279.

Milligan, C.F. (1983). A Linguistic Approach to Learning Mathematics Vocabulary. *The Mathematics Teacher, 76 (7),* 488-490.

Monroe, E. E., & Orme, M. P. (2002). Developing Mathematical Vocabulary. *Preventing School Failure: Alternative Education for Children and Youth, 46(3),* 139-142.

Monroe, E. E., & Panchyshyn, R. (1995-1996). Vocabulary Considerations for Teaching Mathematics. *Childhood Education, 72(2),* 80-83.

Moore, D. W., & Readence, J. E. (1984). A quantitative and qualitative review of graphic organizer research. *Journal of Educational Research, 78(1),* 11-17.

National Council of Teachers of Mathematics(NCTM) (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM, 1989.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM, 2000.

Noonan, J. (1990). Readability problems presented by mathematics text. *Early Child Development and Care, 54,* 57-81.

Pierce, M. E., & Fontaine, L. M. (2008). *Let's talk math: Identifying and teaching math vocabulary on the MCAS.* Poster session presented at the annual meeting of the New England Educational Research Organization, Hyannis, MA.

Pierce, M. E., & Fontaine, L. M. (2009). Designing Vocabulary Instruction in Mathematics. *The Reading Teacher, 63(3)*, 239-243.

Porter, A.C., Garet, M.S., Desimone, L., Yoon, K.S., & Birman, B.F. (2000) *Does professional development change teaching practice? Results from a three-year study.* American Institutes for Research in the Behavioral Sciences, Washington, DC: U.S. Department of Education.

Reehm, S.P., & Long, S. A. (1996). Reading in the mathematics classroom. *Middle School Journal, 27 (5),* 35-41.

Renne, C.G. (2004). Is a Rectangle a Square? Developing Mathematical Vocabulary and Conceptual Understanding. *Teaching Children Mathematics, 10(5),* 258.

Rohrer, D., & Taylor, K. (2006). The effects of overlearning and distributed practise on the retention of mathematics knowledge. *Applied Cognitive Psychology, 20(9),* 1209-1224.

Rubenstein, R.N., & Thompson, D.R. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics, 9(2),* 107-112.

Schell, V. J. (1982). Learning Partners: Reading and Mathematics. *Reading Teacher, 35,* 544-548.

Shaftel, J., Belton-Kocher, E., Glasnapp, D. & Poggio, J. (2006). The Impact of Language Characteristics in Mathematics Test Items on the Performance of English Language Learners and Students With Disabilities. *Educational Assessment, 11(2),* 105-126.

Sobel, H. S., Cepeda, N. J., & Kapler, I. V. (2011). Spacing effects in real-world classroom vocabulary learning. *Applied Cognitive Psychology, 25(5),* 763-767.

Stahl, S. A. (1998). *Four questions about vocabulary knowledge and reading and some answers*. In C.R. Hynd (ed.), *Learning from text across conceptual domains* (pp 73-94). Mahwah, NJ: Erlbaum.

Thompson, D. R., & Rubenstein, R. N. (2000). Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies. *Mathematics Teacher, 93,* 568 - 574.

Whitin, P. E. and D. J. (1997). Ice Numbers and Beyond: Language Lessons for the Mathematics Classroom. *Language Arts, 74(2),* 108-115.

Zinchenko, P. I., & Sereda, G. K. (2011). Involuntary Memory and Classroom Instruction. *Journal of Russian and East European Psychology, 49(1),* 17-30.